AVAZ inversion for anisotropy parameters of a fractured medium: A physical modeling study

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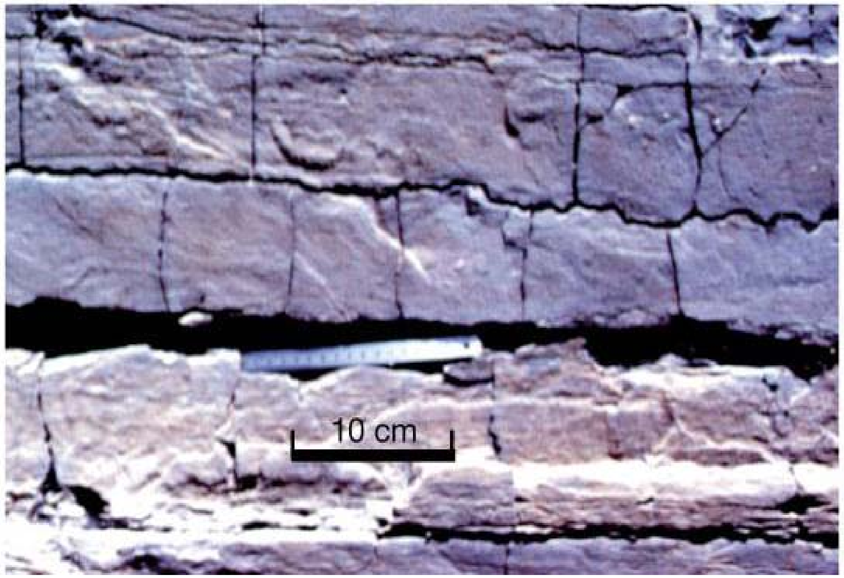
Outline

- Fractures, azimuthal anisotropy
- Anisotropy parameters
- Background theory, AVAZ inversion
- Physical model data
- Amplitude corrections
- Conclusions
- Acknowledgements

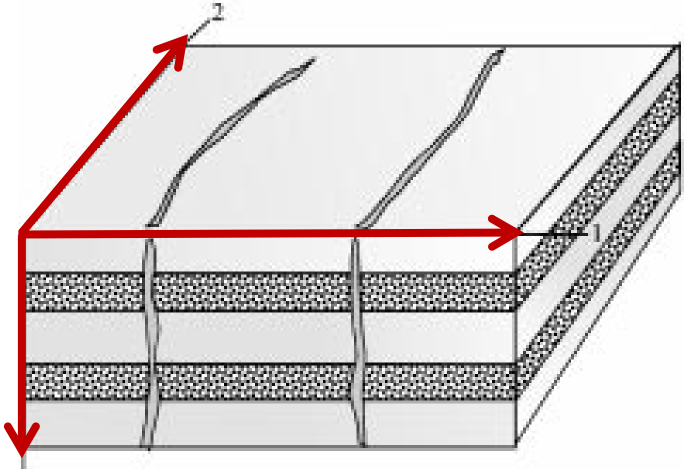
Naturally fractured medium

- Macroscopic cracks
- Vertical to the bedding
- Dominant strike



Orthorhombic symmetry

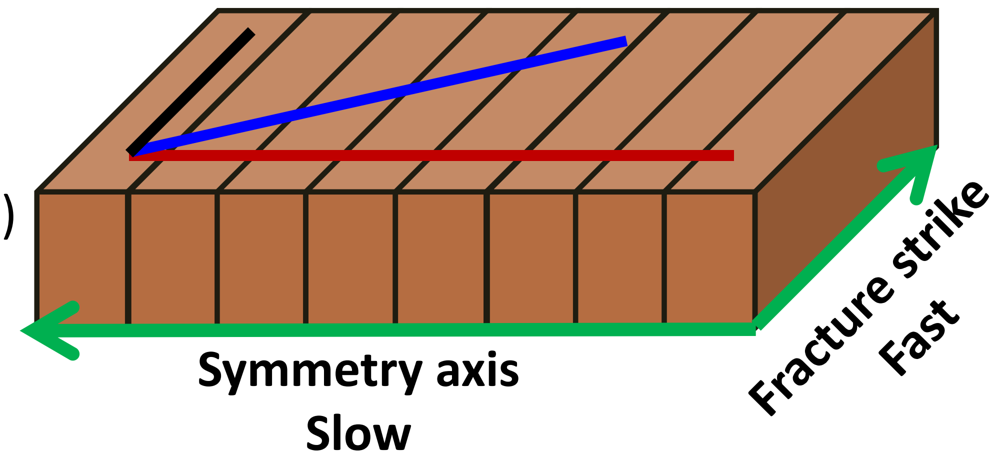
Vertical cracks in horizontal layering



(Schoenberg and Helbig 1997)

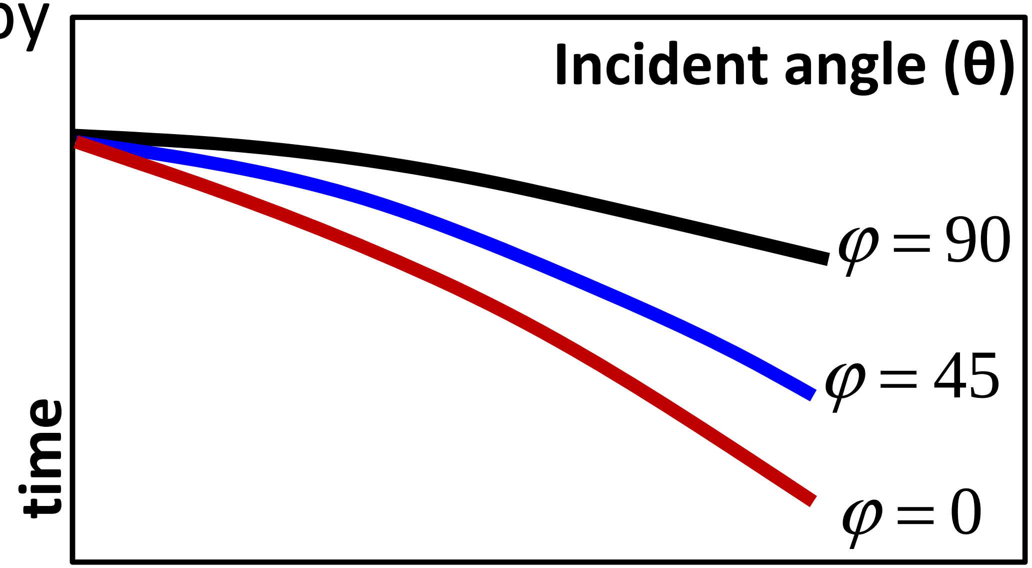
Fractured medium

- HTI: horizontal transverse isotropy
  (two independent directions)



- Azimuthal anisotropy

$$V_{NMO}(\theta,\varphi)$$



Azimuthal anisotropy

P-wave NMO-effect

$$V_{NMO}(\theta,\varphi) = V_0\left(1 + \delta A(\theta,\varphi) + \varepsilon B(\theta,\varphi)\right)$$

θ: incident angle

ϕ: azimuth

$$\varepsilon = \frac{V_P(fast) - V_P(slow)}{V_P(slow)}$$

ε: Far-offset

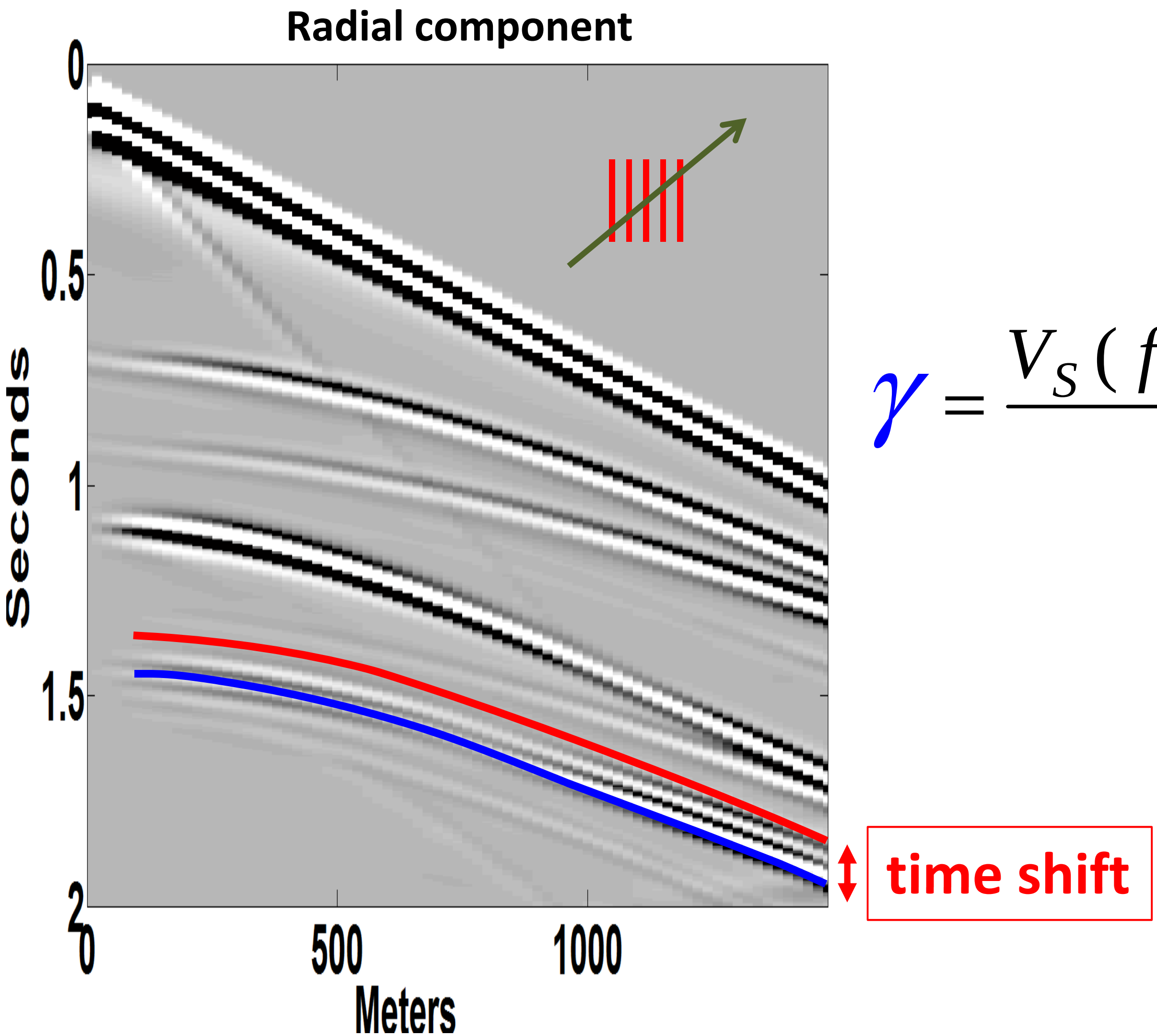
$$\delta = \frac{(C_{13}+C_{55})^2 - (C_{33}-C_{55})}{2C_{33}(C_{33}-C_{55})}$$

δ: near offset

$$V_{NMO}(\theta,\varphi) \approx V_0\left(1 + \delta A(\theta,\varphi)\right)$$

Azimuthal anisotropy

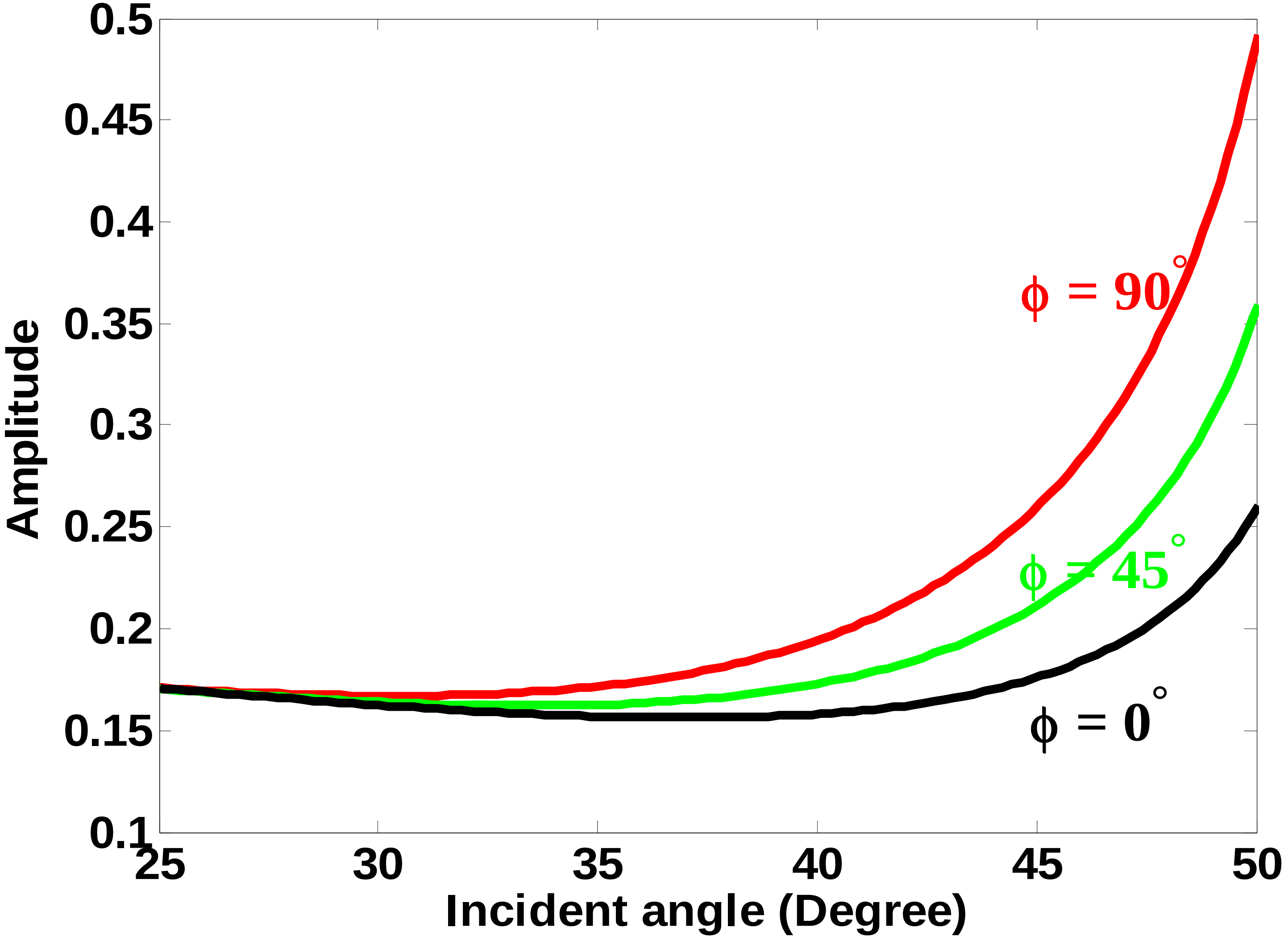
Shear-wave splitting



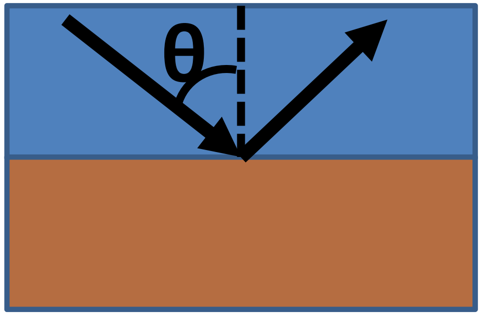
$$\gamma = \frac{V_S(fast) - V_S(slow)}{V_S(slow)}$$

Azimuthal anisotropy

PP reflection amplitude



PP reflection coefficient



Isotropy (Aki and Richards)

$$R_{PP}^{iso}(\theta) = A(\theta)\frac{\Delta\alpha}{\bar{\alpha}} + B(\theta)\frac{\Delta\beta}{\bar{\beta}} + C(\theta)\frac{\Delta\rho}{\bar{\rho}}$$

α: P-velocity $\quad$ β: S-velocity $\quad$ ρ: Density

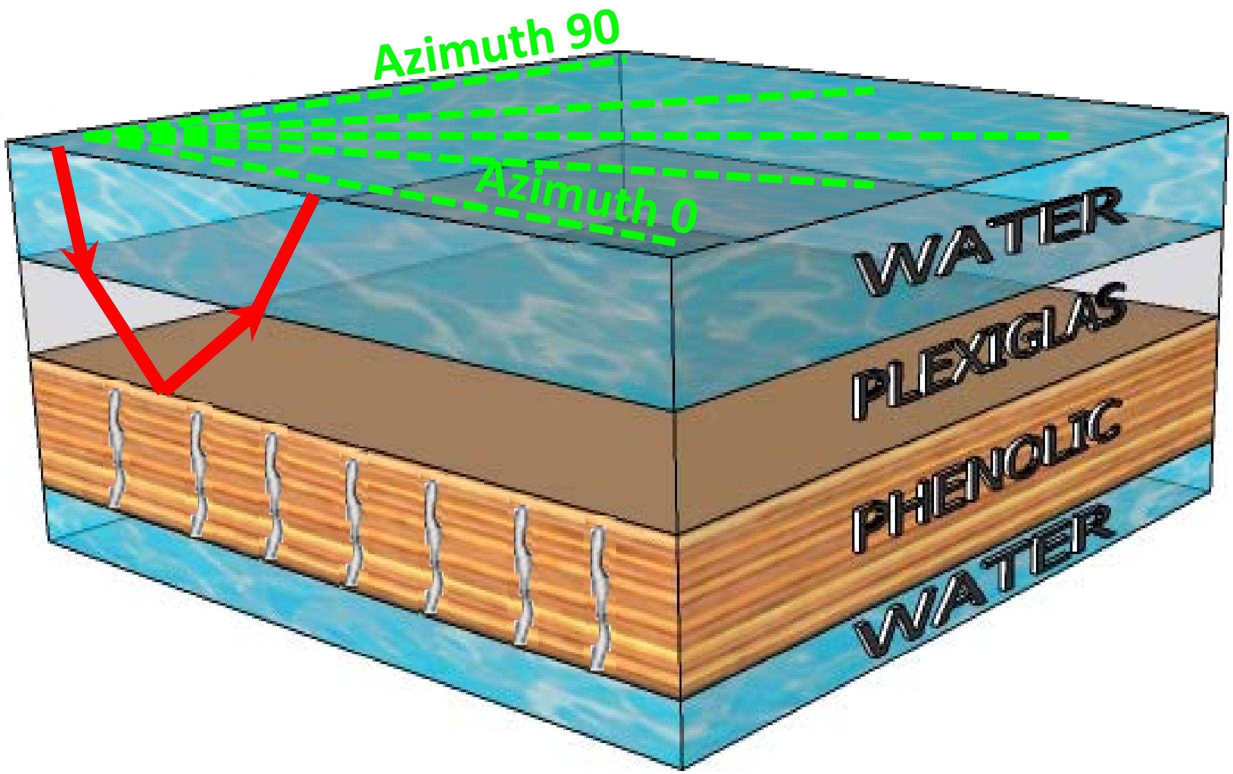
HTI (Rüger, 1996)

$$R_{PP}^{HTI}(\theta,\varphi) = A(\theta)\frac{\Delta\alpha}{\bar{\alpha}} + B(\theta)\frac{\Delta\beta}{\bar{\beta}} + C(\theta)\frac{\Delta\rho}{\bar{\rho}} +$$

$$D(\theta,\varphi)\Delta\delta + E(\theta,\varphi)\Delta\varepsilon + F(\theta,\varphi)\Delta\gamma$$

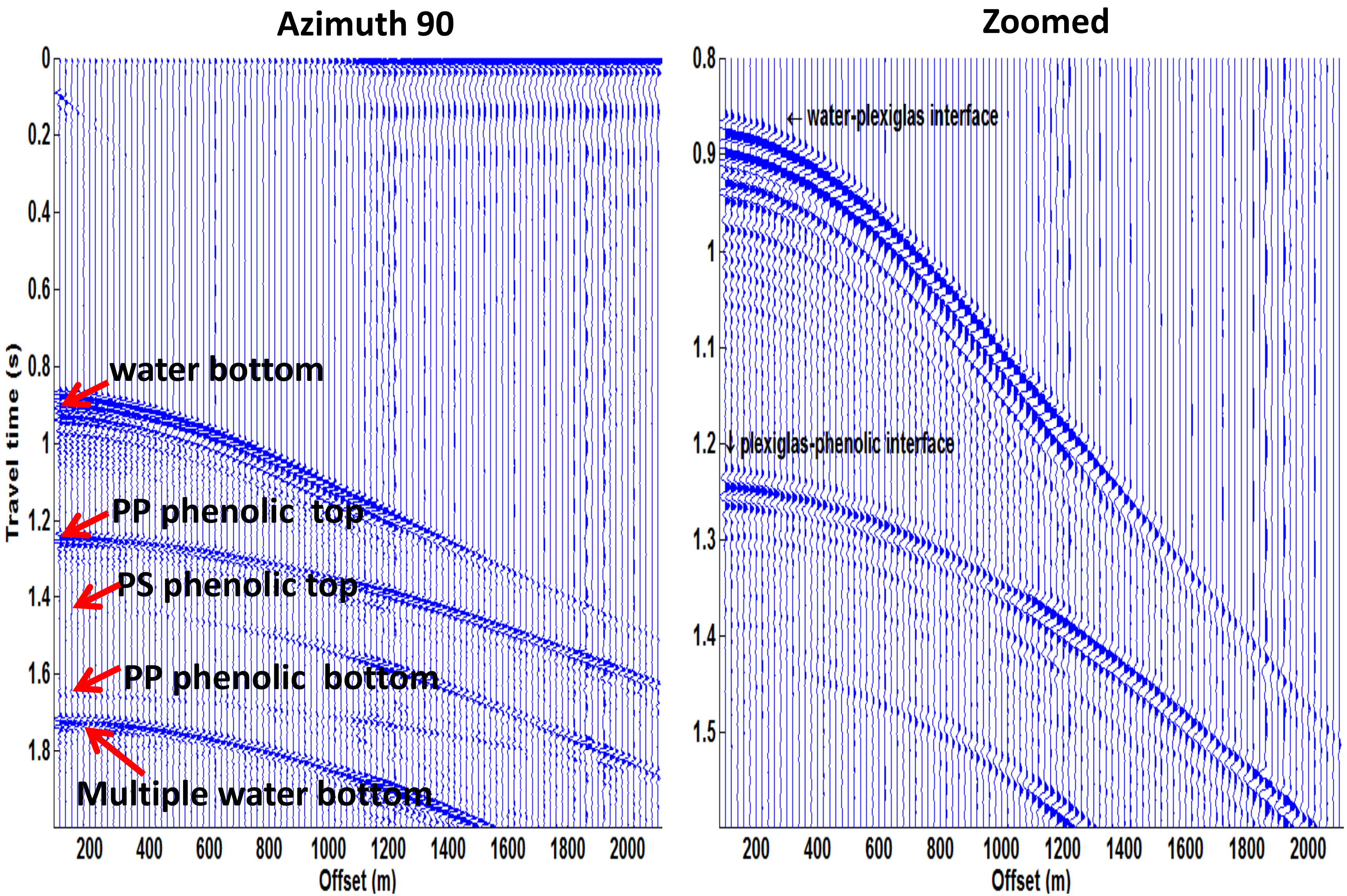
Physical modeling

Phenolic material ≈ HTI



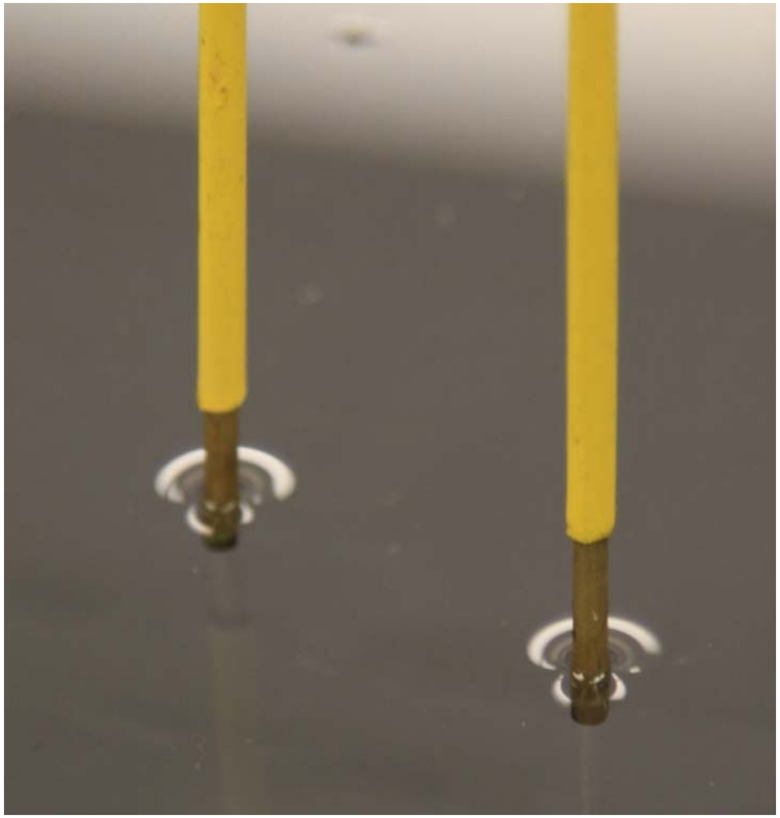
Mahmoudian et al., 2010, Determining elastic constants of an orthorhombic material by physical seismic modeling, CREWES report.

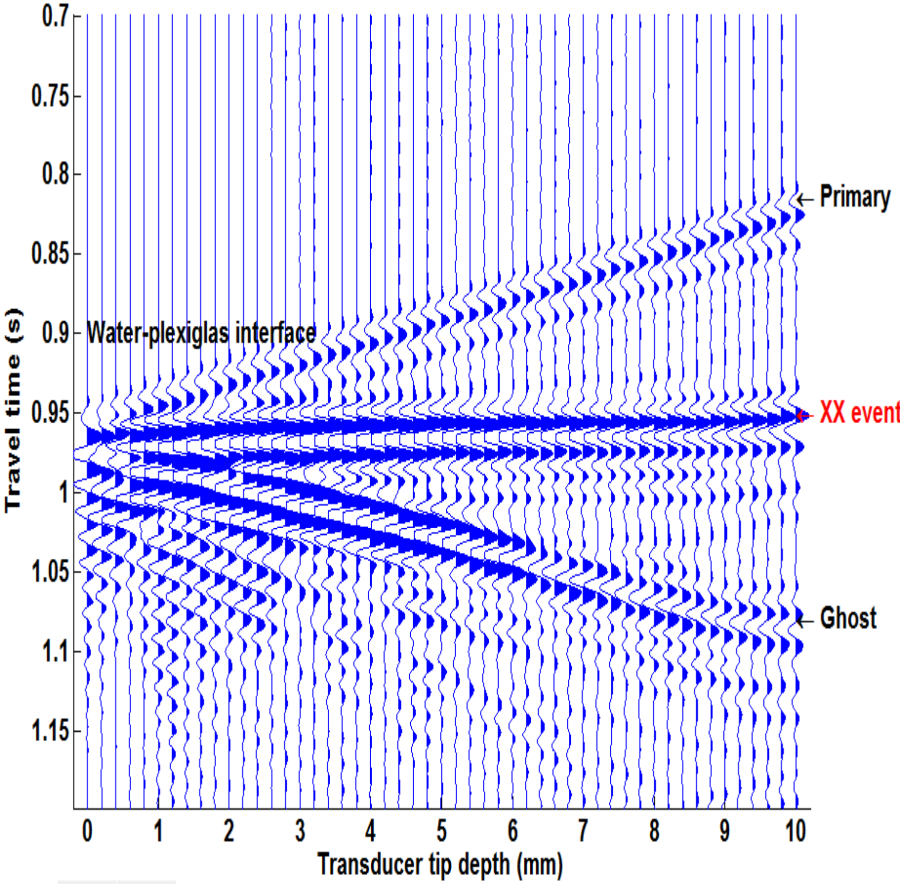
Physical model data



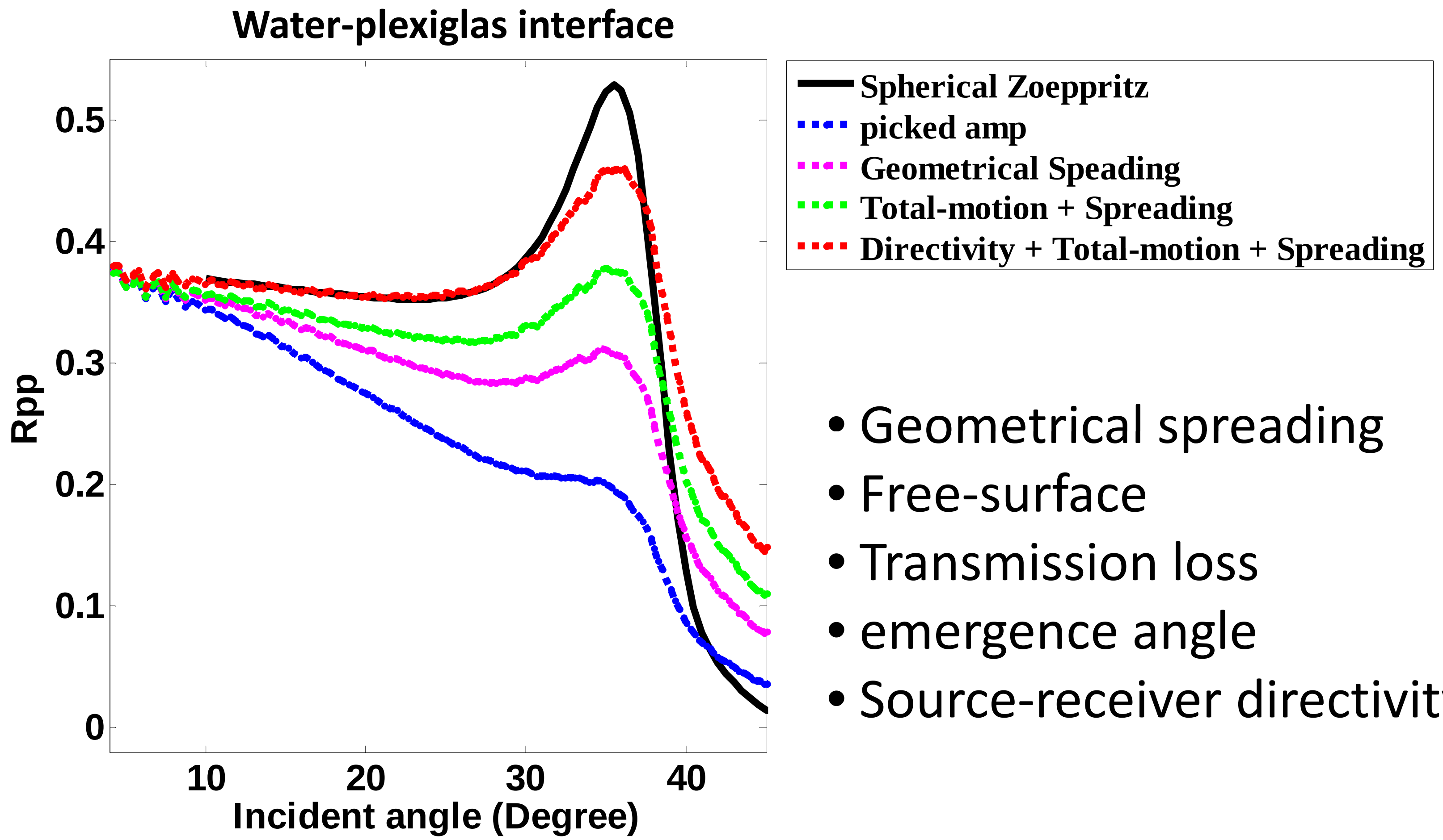
Amplitude picking

- Avoiding primary and ghost overlapping
- Unexpected phenomena marked by **XX**



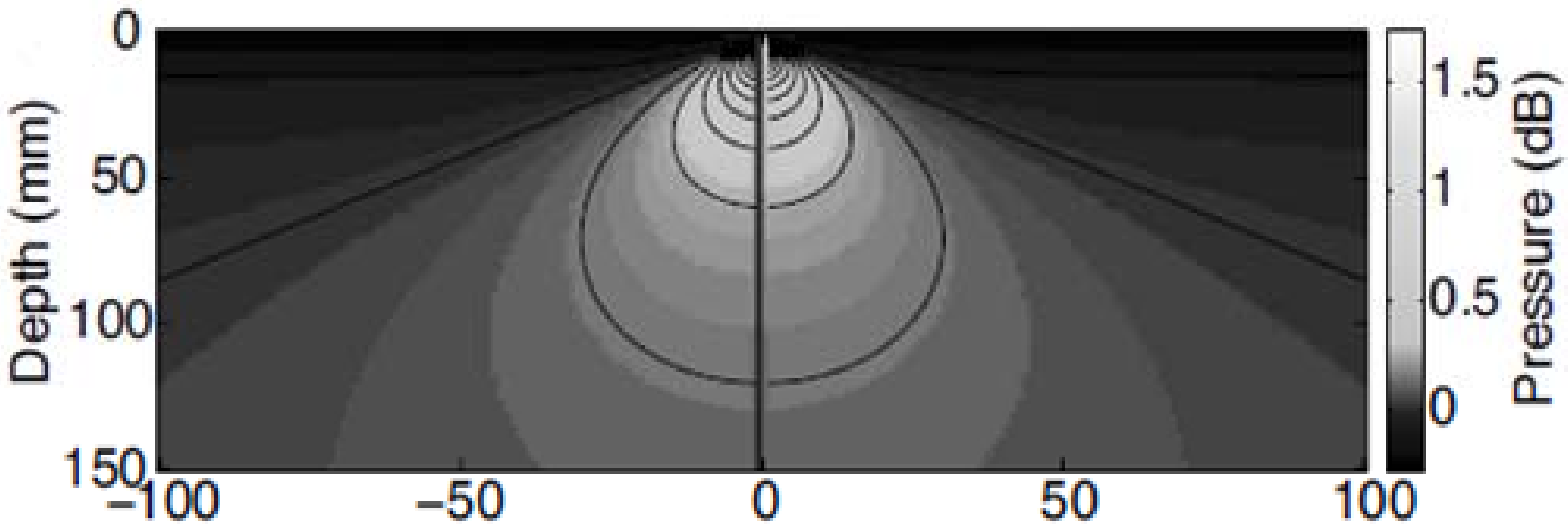


How to make reflection amplitudes to represent reflectivity?



- Geometrical spreading
- Free-surface
- Transmission loss
- emergence angle
- Source-receiver directivity

Source-receiver directivity

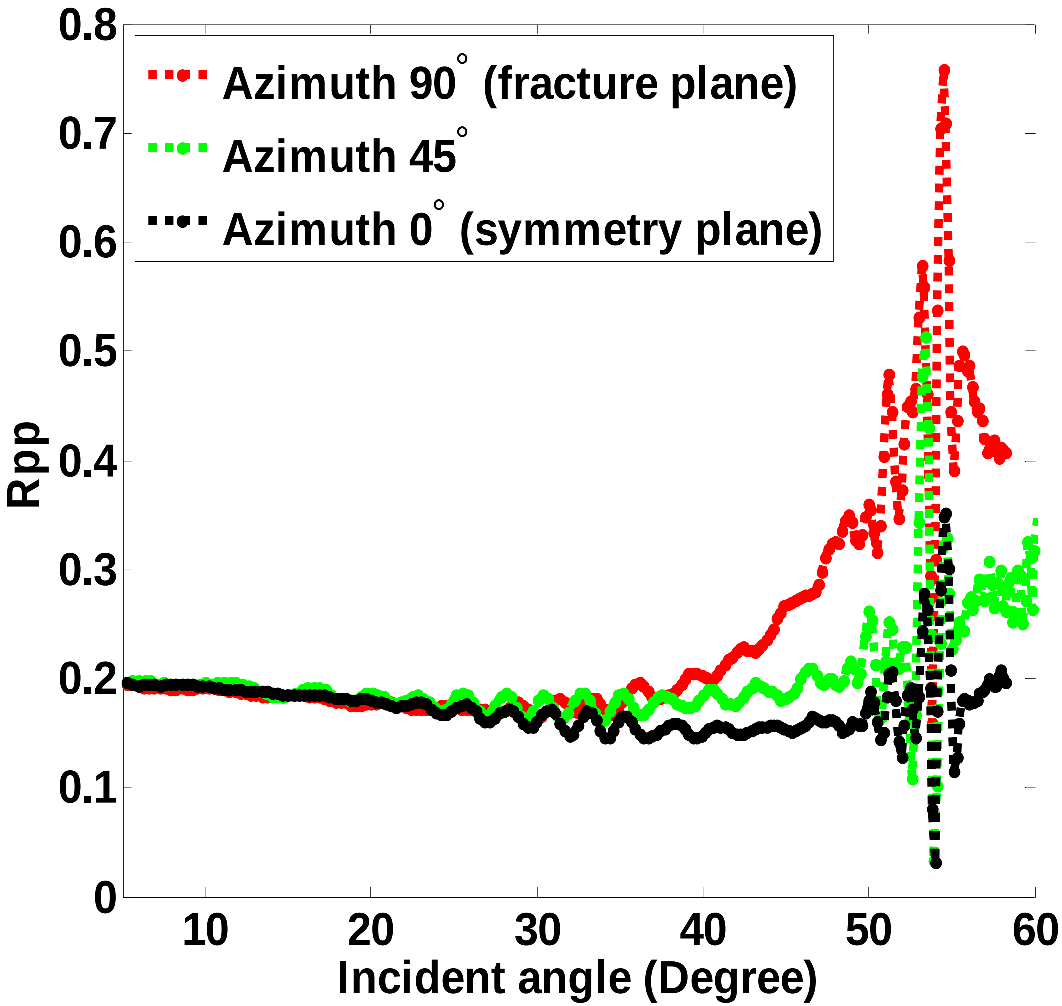


(Courtesy of Buddensiek et al., 2009)

$$A = 4A_0 \frac{J_1(X)}{X} \sin\left(\frac{\pi D}{8\lambda z}\right), \qquad X = \frac{\pi D}{\lambda} \sin\gamma$$

Wong and Mahmoudian: Physical modeling ii: directivity patterns of disc transducers, 2011, CREWES report.

Phenolic top reflector



Incident angle < 30^o

- Subtle AVA
- Subtle azimuthal variation

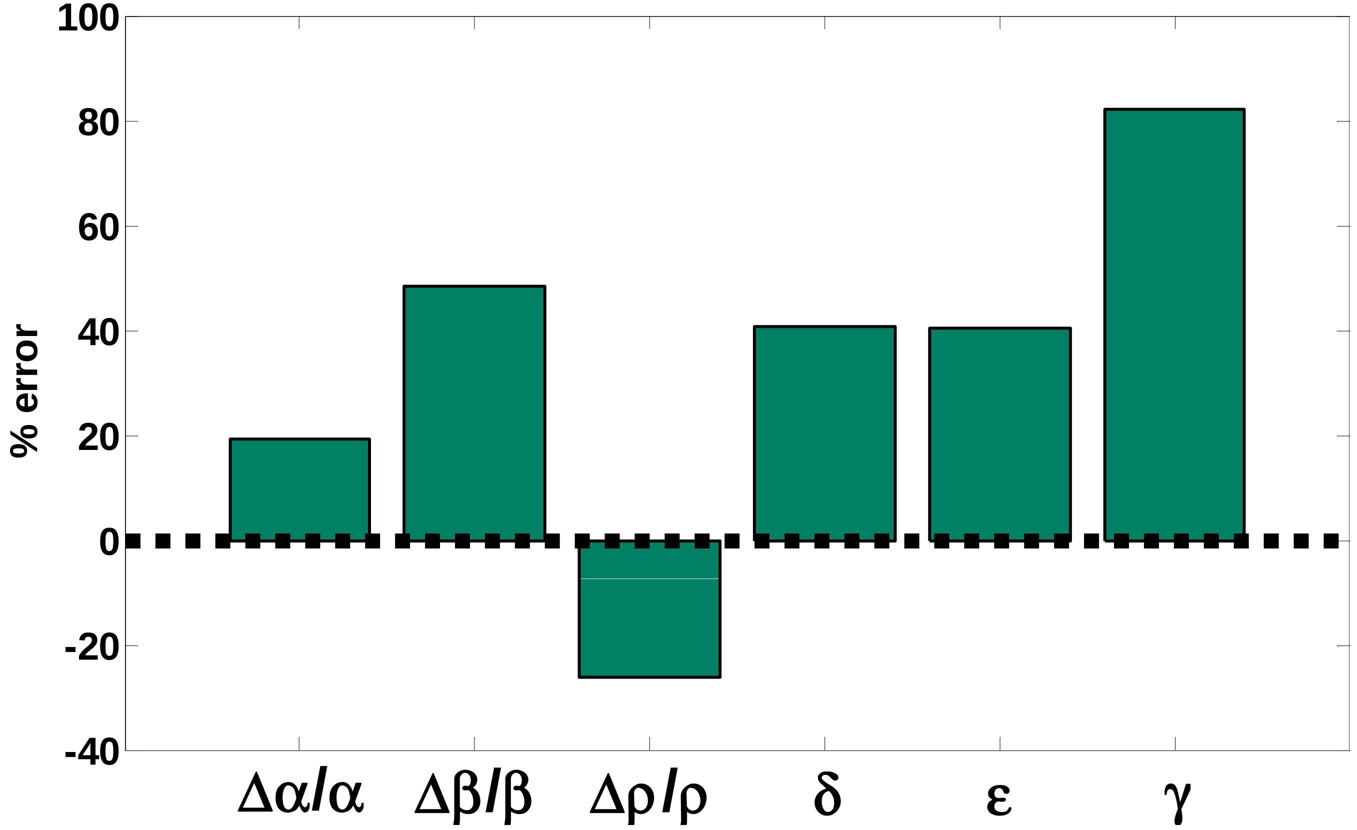
AVAZ inversion

$$\begin{bmatrix} A_{1\varphi_1} & B_{1\varphi_1} & C_{1\varphi_1} & D_{1\varphi_1} & E_{1\varphi_1} & F_{1\varphi_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{n\varphi_1} & B_{n\varphi_1} & C_{n\varphi_1} & D_{n\varphi_1} & E_{n\varphi_1} & F_{n\varphi_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{1\varphi_m} & B_{1\varphi_m} & C_{1\varphi_m} & D_{1\varphi_m} & E_{1\varphi_m} & F_{1\varphi_m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{n\varphi_m} & B_{n\varphi_m} & C_{n\varphi_m} & D_{n\varphi_m} & E_{n\varphi_m} & F_{n\varphi_m} \end{bmatrix}_{(nm\times 6)} \begin{bmatrix} \Delta\alpha/\alpha \\ \Delta\beta/\beta \\ \Delta\rho/\rho \\ \Delta\delta \\ \Delta\varepsilon \\ \Delta\gamma \end{bmatrix}_{(6\times 1)} = \begin{bmatrix} R_{11} \\ \vdots \\ R_{n1} \\ \vdots \\ \vdots \\ R_{1m} \\ \vdots \\ R_{nm} \end{bmatrix}_{(nm\times 1)}$$

$$Gm = d$$

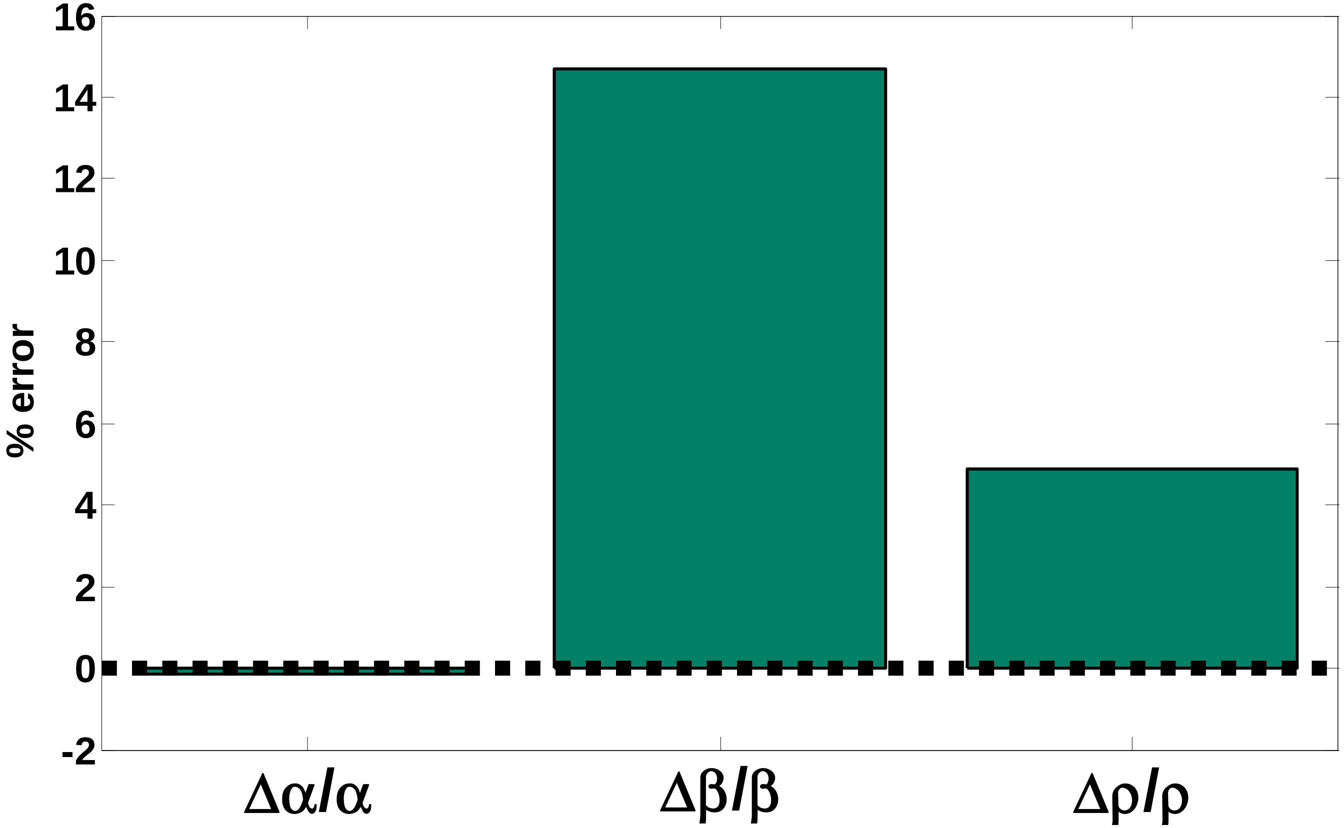
$$m_{est} = (G^T G + \mu)^{-1} G^T d$$

AVAZ inversion results



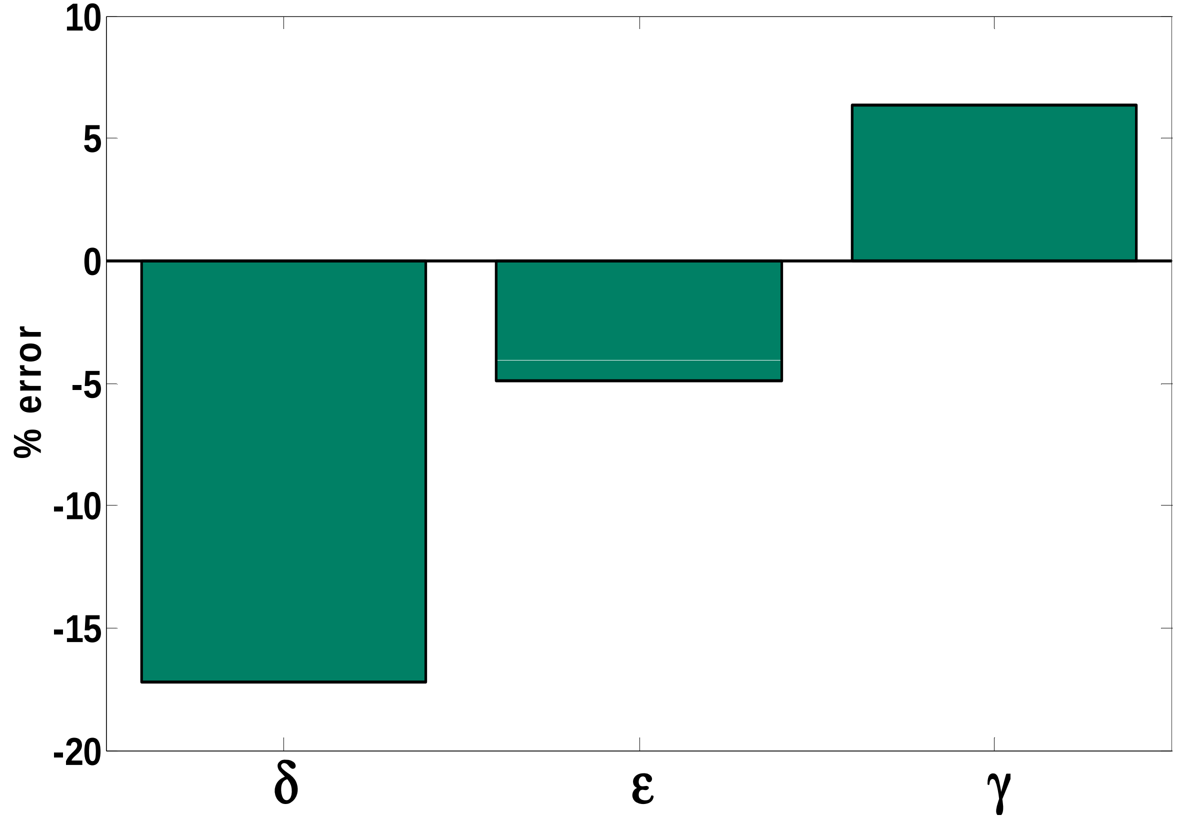
- Reasonable results for α, ρ, ε, and δ
- Not favourable for β and γ

Constraints on α, β, and ρ



- Azimuth 90 º = isotropic plane
- From AVA inversion of azimuth 90º data

Constrained AVAZ inversion



Favourable results compared to those obtained previously by traveltime inversion

Orthorhombic

	ε	γ	δ
(x_2,x_3) plane	$\varepsilon^{(1)}$	$\gamma^{(1)}$	$\delta^{(1)}$
(x_1,x_3) plane	$\varepsilon^{(2)}$	$\gamma^{(2)}$	$\delta^{(2)}$
(x_1,x_2) plane	$\varepsilon^{(3)}$	$\gamma^{(3)}$	$\delta^{(3)}$

Vavryčuk and Psenčik (1999)

$$R_{PP}(\theta,\varphi) = R_{PP}^{iso}(\theta) +$$

$$\frac{1}{2}\left[\Delta\left(\frac{A_{13}+2A_{55}-A_{33}}{A_{33}}\right)\cos^2\varphi + \left(\Delta\left(\frac{A_{23}+2A_{44}-A_{33}}{A_{33}}\right) - 8\Delta\left(\frac{A_{44}-A_{55}}{2A_{33}}\right)\right)\sin^2\varphi\right]\sin^2\theta +$$

$$\frac{1}{2}\left[\Delta\left(\frac{A_{11}-A_{33}}{2A_{33}}\right)\cos^4\varphi + \Delta\left(\frac{A_{22}-A_{33}}{2A_{33}}\right)\sin^4\varphi + \Delta\left(\frac{A_{12}+2A_{66}-A_{33}}{A_{33}}\right)\cos^2\varphi\sin^2\varphi\right]\sin^2\theta\tan^2\theta$$

Orthorhombic AVAZ inversion

$$R_{PP}(\theta,\varphi) = R_{PP}^{ISO}$$

$$+ \left(\frac{1}{2}\cos^2\varphi\sin^2\theta\right)\Delta\delta^{(2)} + \left(\frac{1}{2}\sin^2\varphi\sin^2\theta\right)\Delta\delta^{(1)}$$

$$+ \left(\frac{1}{2}\cos^2\varphi\sin^2\varphi\sin^2\theta\tan^2\theta\right)\Delta\delta^{(3)}$$

$$+ \left(\frac{1}{2}\cos^4\varphi\sin^2\theta\tan^2\theta\right)\Delta\varepsilon^{(2)} + \left(\frac{1}{2}\sin^4\varphi\sin^2\theta\tan^2\theta\right)\Delta\varepsilon^{(1)}$$

$$- \left(\frac{4\beta^2}{\alpha^2}\sin^2\varphi\ \sin^2\theta\right)\Delta\gamma^{(3)}$$

Conclusions and discussions

- Successful AVAZ inversion of a HTI medium for Thomsen anisotropy parameters.
- Estimation of shear-wave splitting parameter from PP data.
- Pre-knowledge of the fracture orientation.
- Physical model suitable for quantitative amplitude analysis.
- AVAZ inversion can be applied to real world data, knowing enough info about the overburden.
- Better results if stronger azimuthal or AVA variations.
- Incorporating larger incident angles, but not close to critical angle.
- Orthorhombic AVAZ inversion is possible.

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Source-receiver directivity correction