CREWES

# A Gassmann consistent rock physics template


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# Introduction

- In this talk, we will discuss a new approach to the calculation of a rock template using the pore space stiffness method.
- We first explain the concept of pore space stiffness and use the Betti-Rayleigh reciprocity theorem to derive Gassmann's equation from the dry and saturated pore space stiffnesses.
- We then discuss the Ødegaard and Avseth approach to the rock physics template and show how the new approach differs from their method.
- Using lab measurements on sandstones, and log and inverted seismic data from the Colony sand of central Alberta, we will then compare the two methods.

# Pressure and compressibility

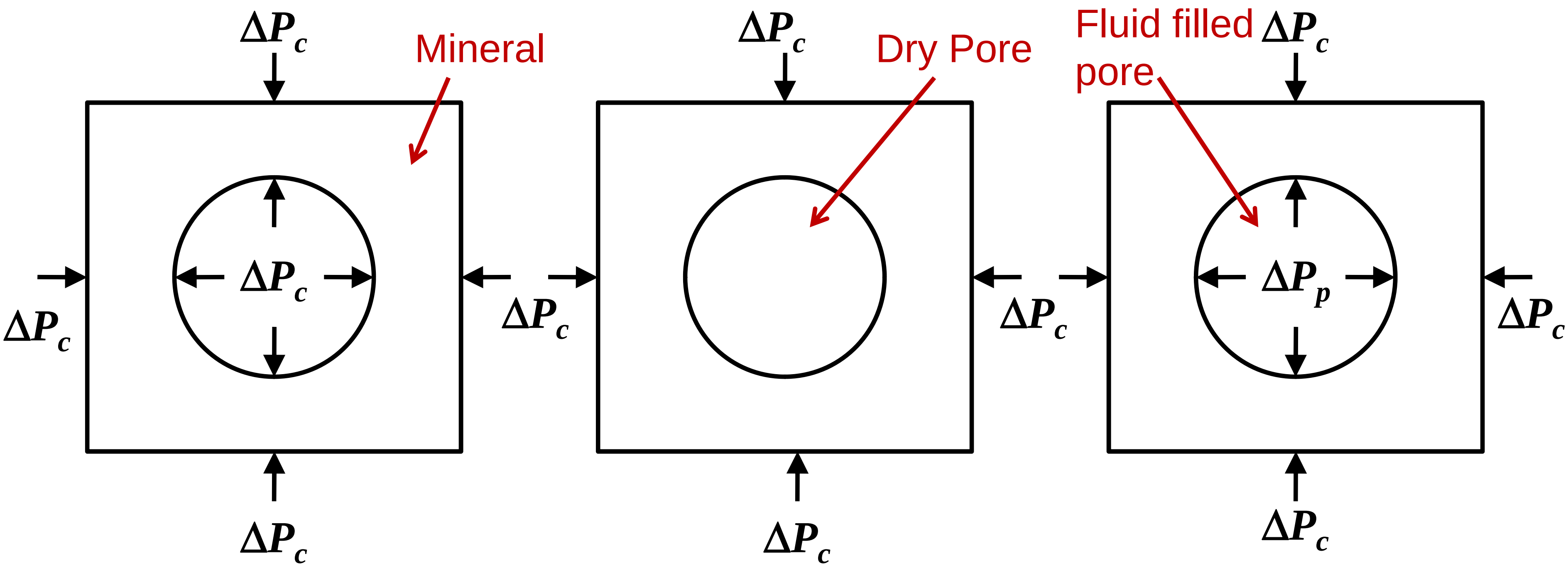
- Pressure is one of the key parameters in rock physics, and leads directly to the concept of compressibility.
- The compressibility of the rock, $C$, which is the inverse of the bulk modulus $K$, is the change of the volume of the rock with respect to pressure, divided by the volume:

$$C = \frac{1}{K} = -\left(\frac{1}{V}\right)\frac{dV}{dP}, \text{ where : } V = \text{volume}, \quad P = \text{pressure}.$$

- In the above equation, there are two fundamental types of pressure: confining pressure, $P_C$, and pore pressure, $P_P$.
- Also, there are three different volumes to consider: the volume of the bulk rock, the mineral and the pore space.

# Three models of a porous rock

Utilizing these concepts, we can build three simple models of the rock volume, as shown here (Mavko and Mukerji, 1995):




A. Mineral case B. Dry case C. Saturated case

In A, we compress the mineral, in B we compress the mineral and dry pore, and in C the mineral and saturated pore.

# Betti-Rayleigh Reciprocity

- The Betti-Rayleigh reciprocity theorem states: “For an elastic body acted on by two different forces, the work done by the first force acting on the displacements caused by the second force equals the work done by the second force acting on the displacements caused by the first force.”
- Using the Betti-Rayleigh reciprocity theorem to compare cases A and B gives the following equation:

$$\frac{1}{K_{dry}} = \frac{1}{K_m} + \frac{\phi}{K_\phi}, \quad \text{where :}$$

$K_{dry} = \text{dry rock bulk modulus}, \; K_m = \text{mineral bulk modulus},$

$K_\phi = \text{dry pore space stiffness}, \; \text{and } \phi = \text{porosity}.$

# Pore space stiffness and compressibility

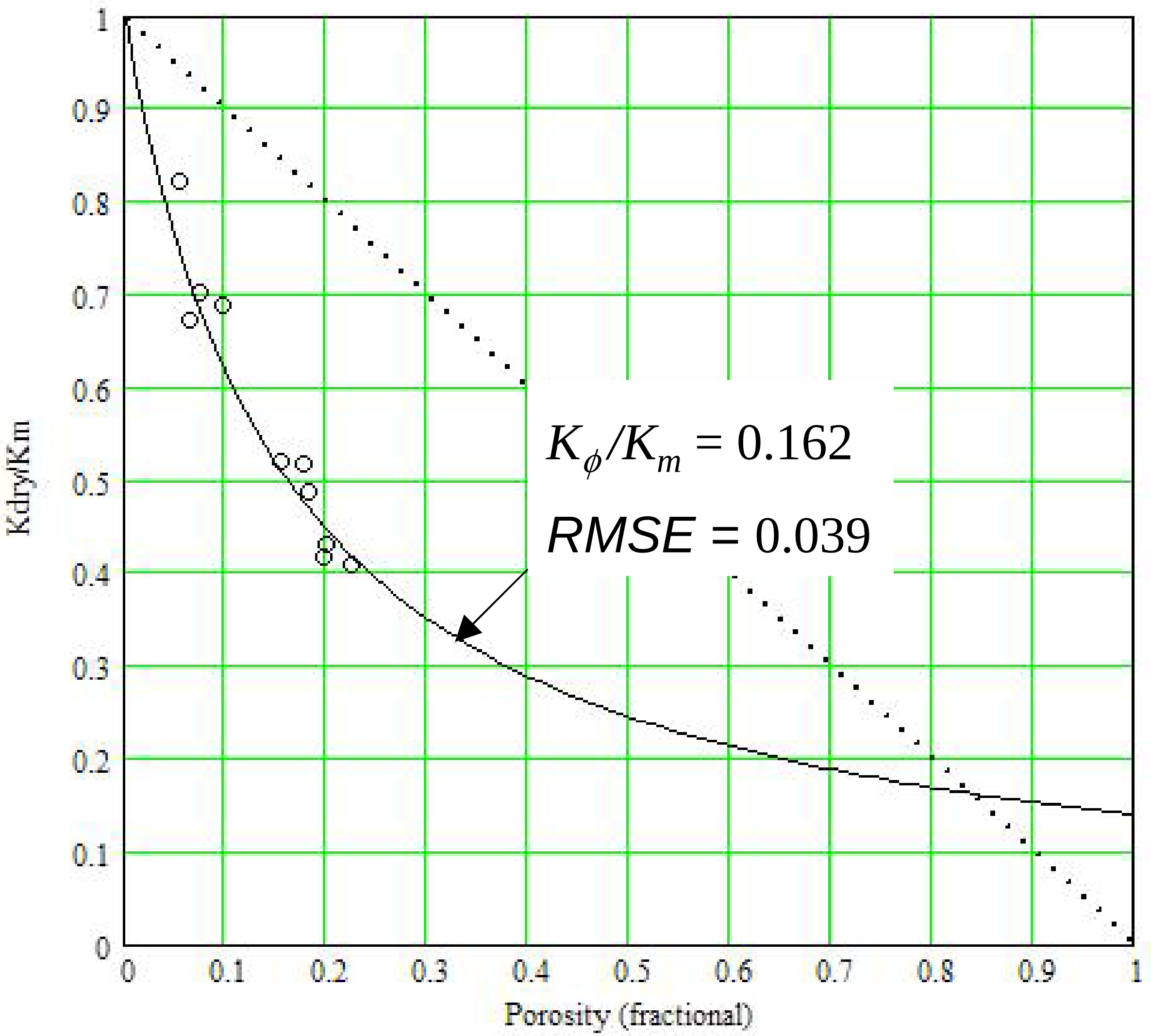
- The pore space stiffness is the inverse of the pore space compressibility, which is given as:

$$C_\phi = \frac{1}{K_\phi} = -\left(\frac{1}{V_p}\right)\frac{dV_p}{dP_c}\bigg|_{P_p}$$

- That is, the pore space compressibility represents the change in pore volume with respect to confining pressure, with the pore pressure held constant.
- The key point to note from this is that if the confining pressure is constant (i.e. no depth change), the pore space compressibility (and stiffness) will stay constant for a range of porosities.
- This is shown empirically on the next slide for a fit to measured data by Han (1986).

# Empirical fit to Han's dataset

This figure (from Russell and Smith, 2007) shows the fit of pore space stiffness to a set of measured values at constant confining pressure and differing porosity (Han, 1986), where $K_{dry}$ and $K_{\phi}$ have been normalized by dividing by $K_m$.



# Modeling $K_{dry}$ versus porosity

- To model $K_{dry}$ at different porosities, the equation for the in-situ, or calibrated, $K_{dry}$ can be re-arranged as follows:

$$\frac{1}{K_\phi} = \frac{1}{\phi_{cal}}\left(\frac{1}{K_{dry\_cal}} - \frac{1}{K_m}\right)$$

- The new value can be written:

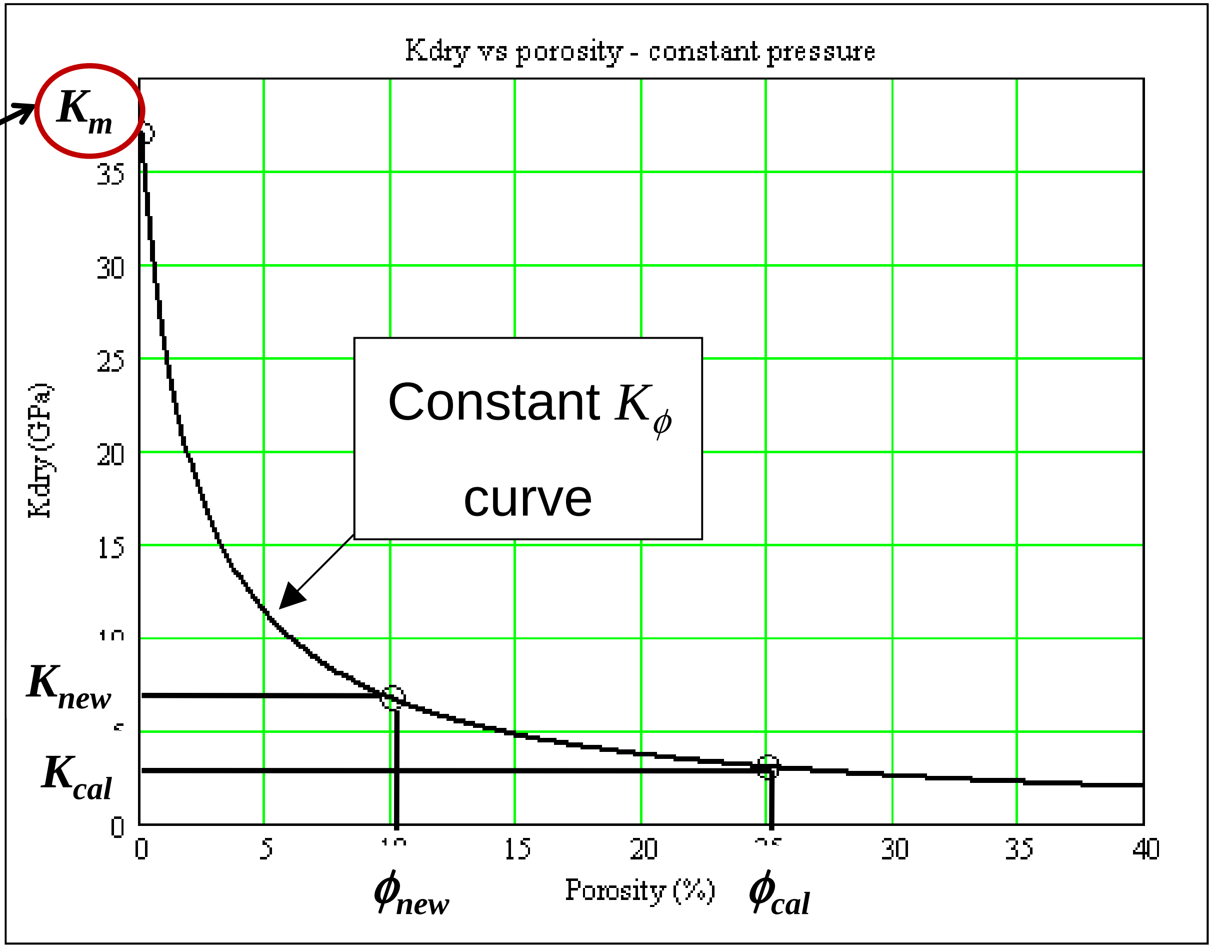
$$\frac{1}{K_\phi} = \frac{1}{\phi_{new}}\left(\frac{1}{K_{dry\_new}} - \frac{1}{K_m}\right)$$

- These equations allows us to eliminate the pore space stiffness term and thus compute a new $K_{dry}$:

$$\boxed{\frac{1}{K_{dry\_new}} = \frac{1}{K_m} + \frac{\phi_{new}}{\phi_{cal}}\left(\frac{1}{K_{dry\_cal}} - \frac{1}{K_m}\right)}$$

# Modeling $K_{dry}$ versus porosity

Note that $K_\phi$ reduces to $K_m$ at 0% porosity, as it should.




Graphically, this shows that we can thus model $K_{dry}$ at a new porosity $\phi_{new}$ using a calibration porosity $\phi_{cal}$.

# Fluid pore space stiffness

- Using the Rayleigh-Betti reciprocity theorem to compare the A (mineral) and C (fluid) cases shown earlier gives an equation involving $K_{sat}$, the saturated bulk modulus:

$$\frac{1}{K_{sat}} = \frac{1}{K_m} + \frac{\phi}{\tilde{K}_\phi}, \quad \text{where :}$$

$$K_{dry} = \text{dry rock bulk modulus}, \; K_m = \text{mineral bulk modulus},$$

$$\tilde{K}_\phi = K_\phi + \frac{K_m K_f}{K_m - K_f} = \text{fluid pore space stiffness},$$

$$K_f = \text{fluid bulk modulus}, \quad \text{and } \phi = \text{porosity}.$$

- Note that this equation is identical in form to the dry pore space stiffness equation, and for $K_f = 0$ it reduces to the dry equation.

# Deriving the Gassmann equation

- We now have two relationships that relate the dry, saturated, fluid and mineral bulk moduli to porosity and pore space stiffness.
- These can be re-arranged for $K_\phi$ as follows:

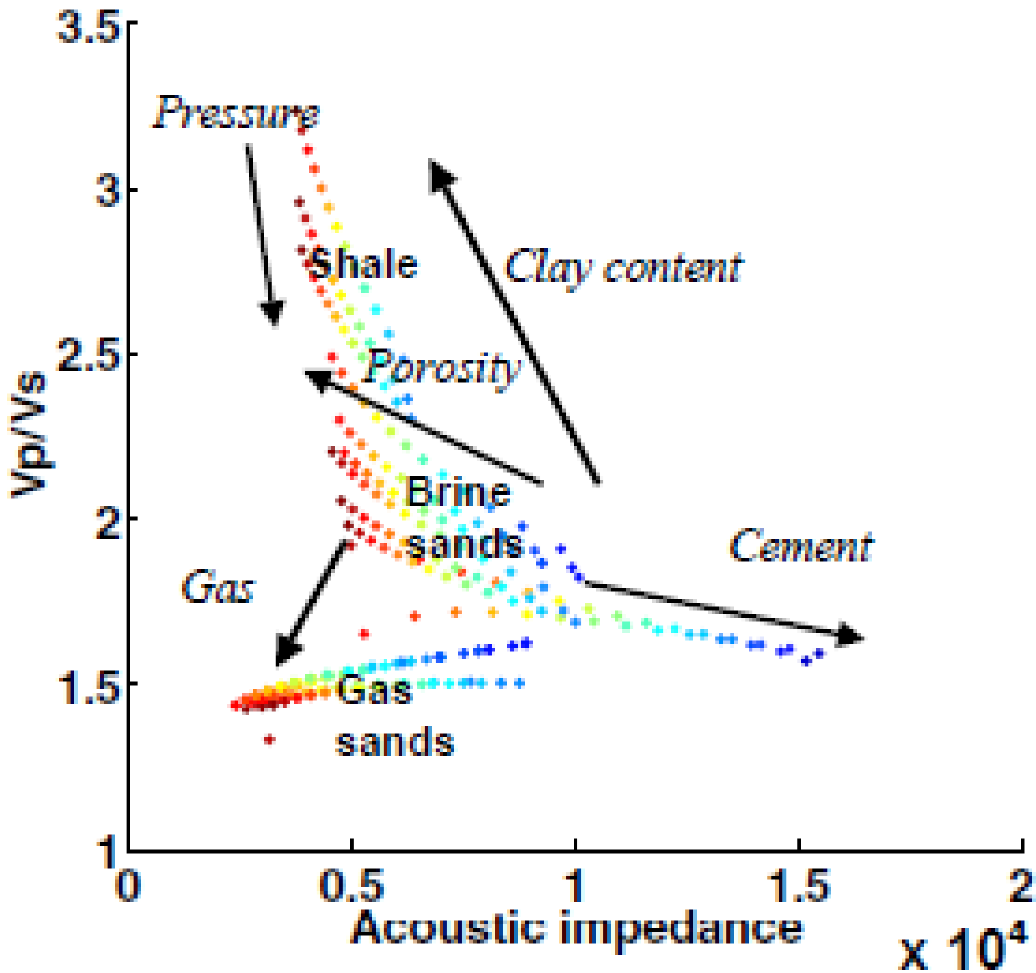
$$\mathrm{Dry}: K_\phi = \phi\left(\frac{K_m K_{dry}}{K_m - K_{dry}}\right), \quad \mathrm{Sat.}: K_\phi = \phi\left(\frac{K_m K_{sat}}{K_m - K_{sat}}\right) - \frac{K_m K_f}{K_m - K_f}$$

- Eliminating $K_\phi$ and dividing through by $\phi$ and $K_m$ gives us the famous Gassmann (1951) equation:

$$\frac{K_{sat}}{K_m - K_{sat}} = \frac{K_{dry}}{K_m - K_{dry}} + \frac{K_f}{\phi(K_m - K_f)}$$

# The rock physics template (RPT)

Ødegaard and Avseth (2003) proposed a technique they called the rock physics template (RPT), in which the fluid and mineralogical content of a reservoir could be estimated on a crossplot of Vp/Vs ratio against acoustic impedance, as shown here.




from Ødegaard and Avseth (2003)

# The Ødegaard/Avseth RPT

- Ødegaard and Avseth (2003) compute $K_{dry}$ and $\mu_{dry}$ as a function of porosity $\phi$ using Hertz-Mindlin (HM) contact theory and the lower Hashin-Shtrikman bound:

$$K_{dry} = \left[ \frac{\phi/\phi_c}{K_{HM} + (4/3)\mu_{HM}} - \frac{1-\phi/\phi_c}{K_m + (4/3)\mu_{HM}} \right]^{-1} - \frac{4}{3}\mu_{HM}$$

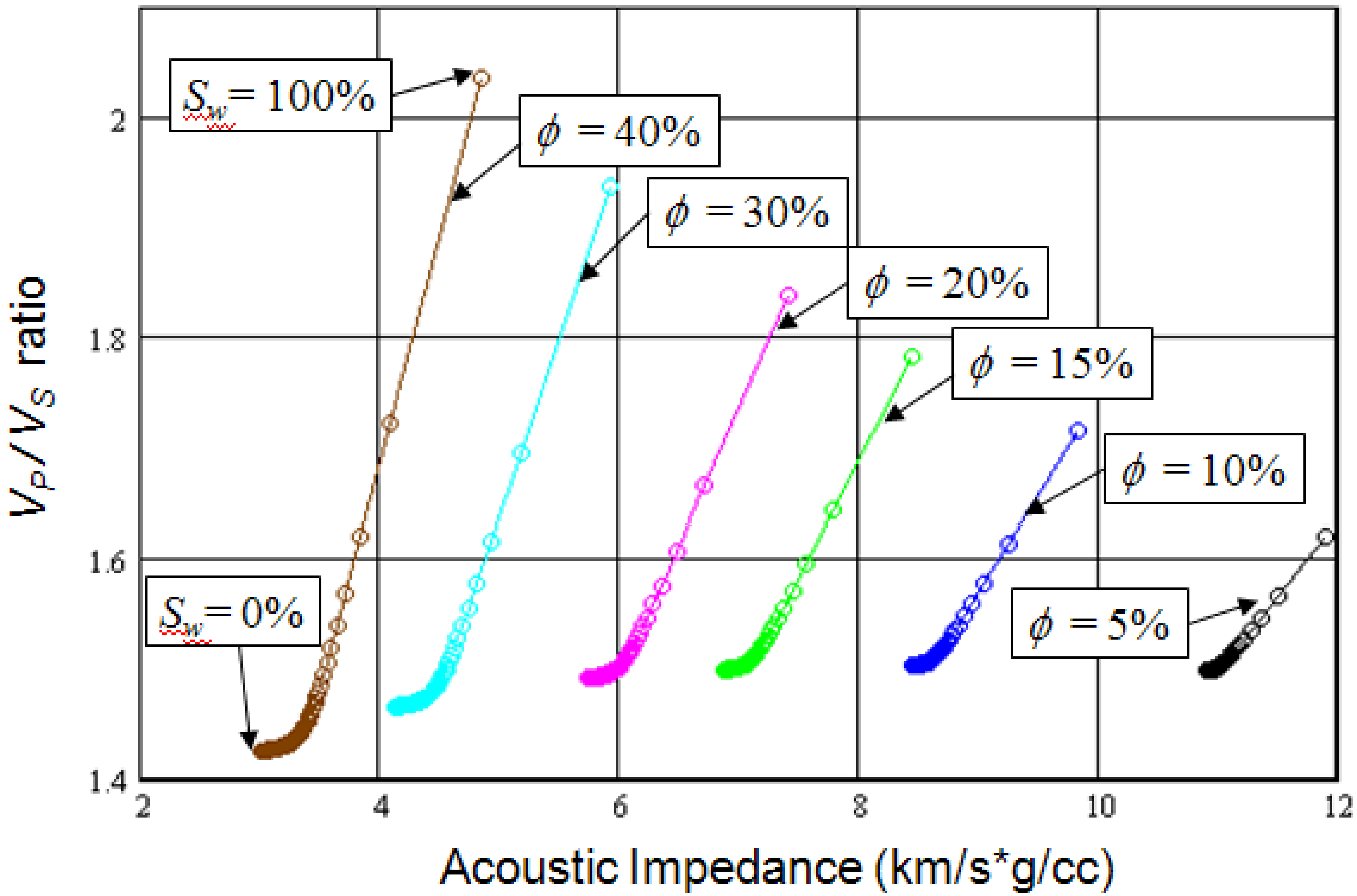
$$\mu_{dry} = \left[ \frac{\phi/\phi_c}{\mu_{HM} + z} - \frac{1-\phi/\phi_c}{\mu_m + z} \right]^{-1} - \frac{4}{3}\mu_{HM}, \text{ where } \; z = \frac{\mu_{HM}}{6}\left( \frac{9K_{HM} + 8\mu_{HM}}{K_{HM} + 2\mu_{HM}} \right),$$

$$K_{HM} = \left[ \frac{n^2(1-\phi_c)^2\mu_m^2}{18\pi^2(1-\nu_m)^2} P \right]^{\frac{1}{3}}, \mu_{HM} = \frac{4-4\nu_m}{5(2-\nu_m)} \left[ \frac{3n^2(1-\phi_c)^2\mu_m^2}{2\pi^2(1-\nu_m)^2} P \right]^{\frac{1}{3}},$$

$\boxed{P = \text{confining pressure}}$, $K_m, \mu_m =$ mineral bulk and shear modulus, $n =$ contacts per grain, $\nu_m =$ mineral Poisson's ratio, and $\phi_c =$ high porosity end-member.

- They then use standard Gassmann theory for the fluid replacement process.

# The Ødegaard/Avseth RPT

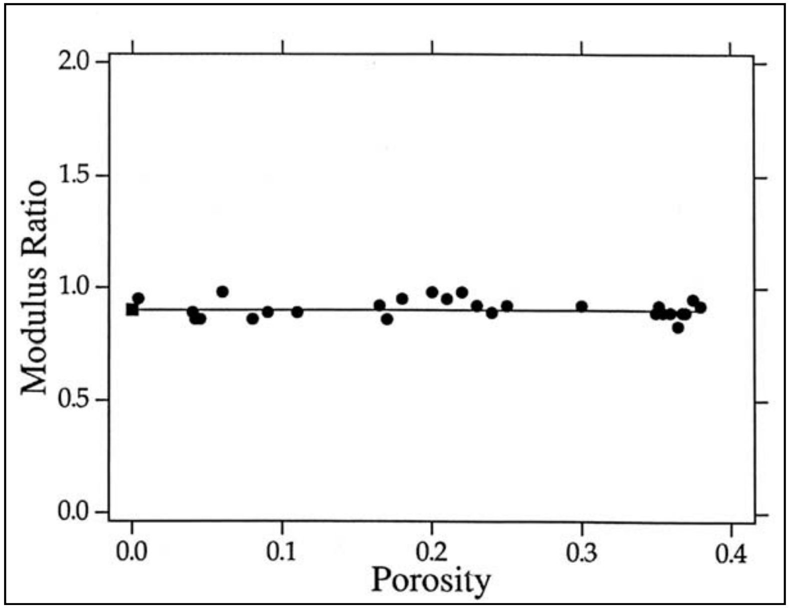



Here is the Ødegaard/Avseth RPT for a range of porosities and water saturations, in a clean sand case.

# A pore space stiffness RPT

- We propose a new approach to the rock physics template, in which we still use Gassmann for saturation change but use pore space stiffness to compute the porosity change.
- The only other thing we need is a method of computing shear modulus change.

- Murphy et al. (1993) measured $K_{dry}$ and $\mu$ for clean quartz sandstones, and found a constant of 0.9 for their ratio:



# Modeling $\mu$ versus porosity

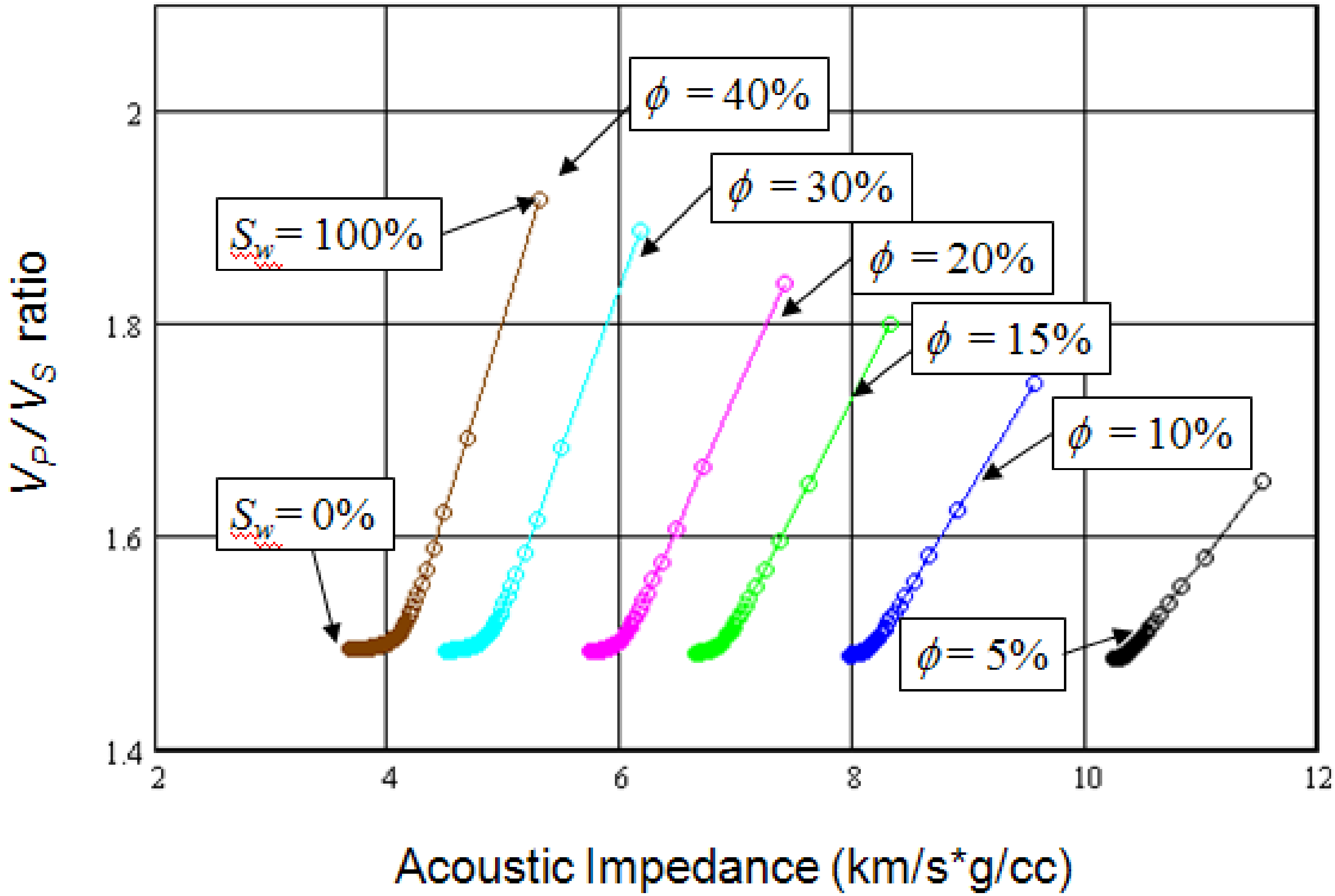
- As shown in the previous figure, the ratio of $K_{dry}/\mu$ is constant for varying porosity. Therefore, we could compute the new value of $\mu$ using the equation:

$$\mu_{new} = \mu_{in-situ} \frac{K_{dry\_new}}{K_{dry\_in-situ}}$$

- However, the formula above does not correctly predict the mineral value at 0% porosity. Our new approach is to use the same formulation as for the bulk modulus:

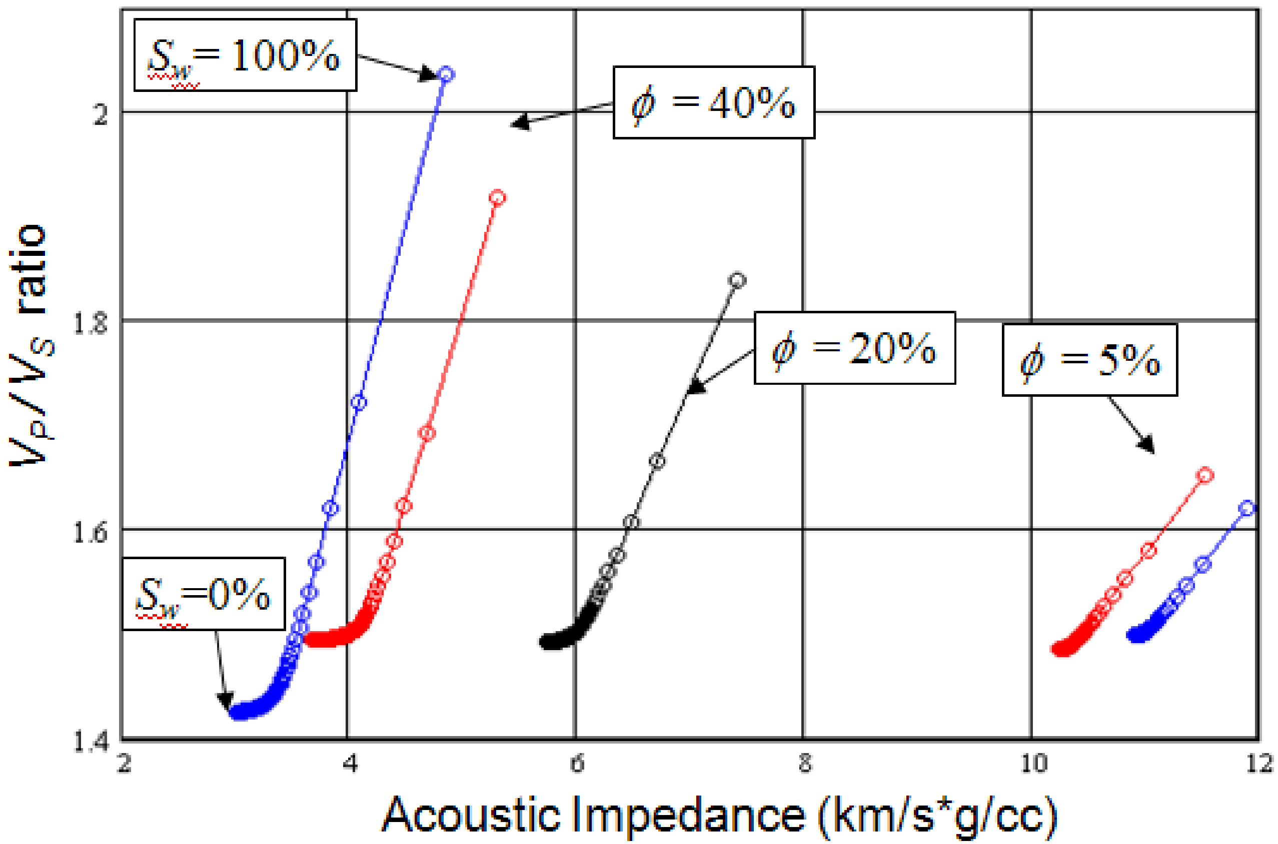
$$\frac{1}{\mu_{dry\_new}} = \frac{1}{\mu_m} + \frac{\phi_{new}}{\phi_{cal}} \left( \frac{1}{\mu_{dry\_cal}} - \frac{1}{\mu_m} \right)$$

# The pore space stiffness RPT




Here is the pore space stiffness RPT for a range of porosities and water saturations, where we have calibrated the curves at 20% porosity.

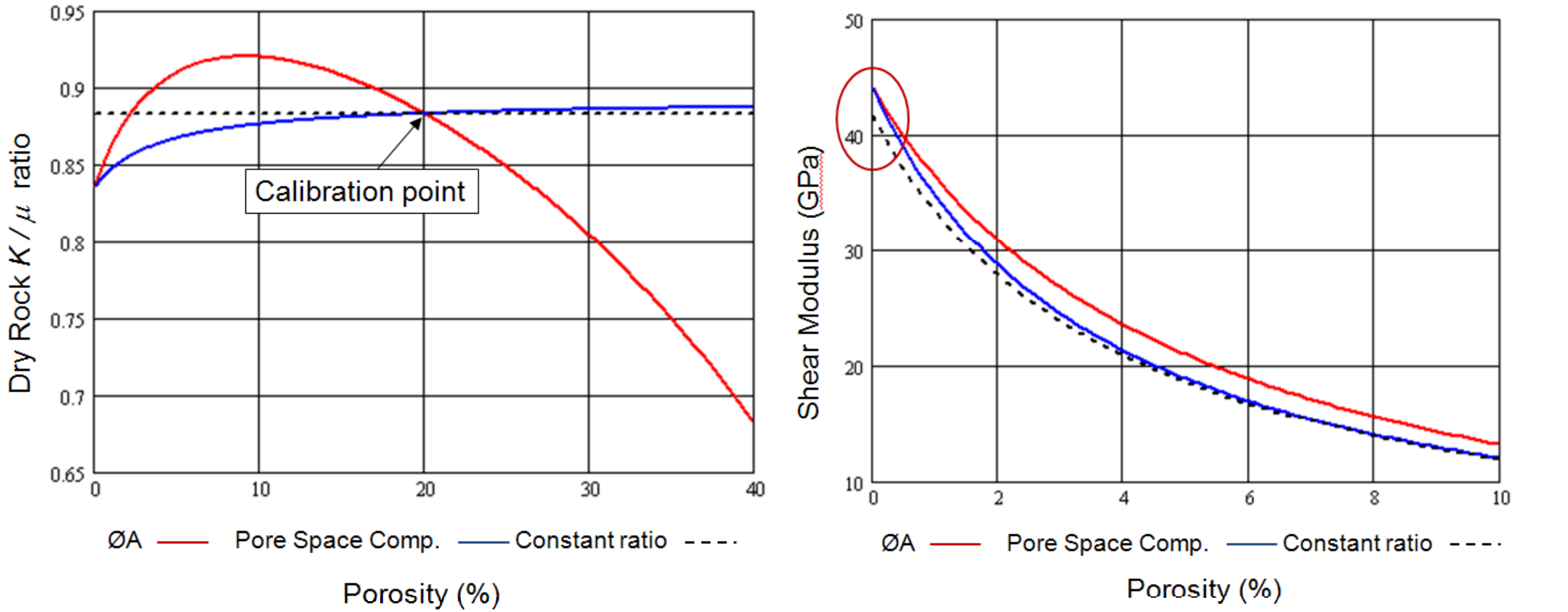
# A comparison of the two methods




Here is a comparison of the pore space stiffness method (red) and the Ødegaard and Avseth method (blue). At the calibration value for a porosity of 20% (black), the curves are identical.

# Comparison of the methods for the modulus ratio

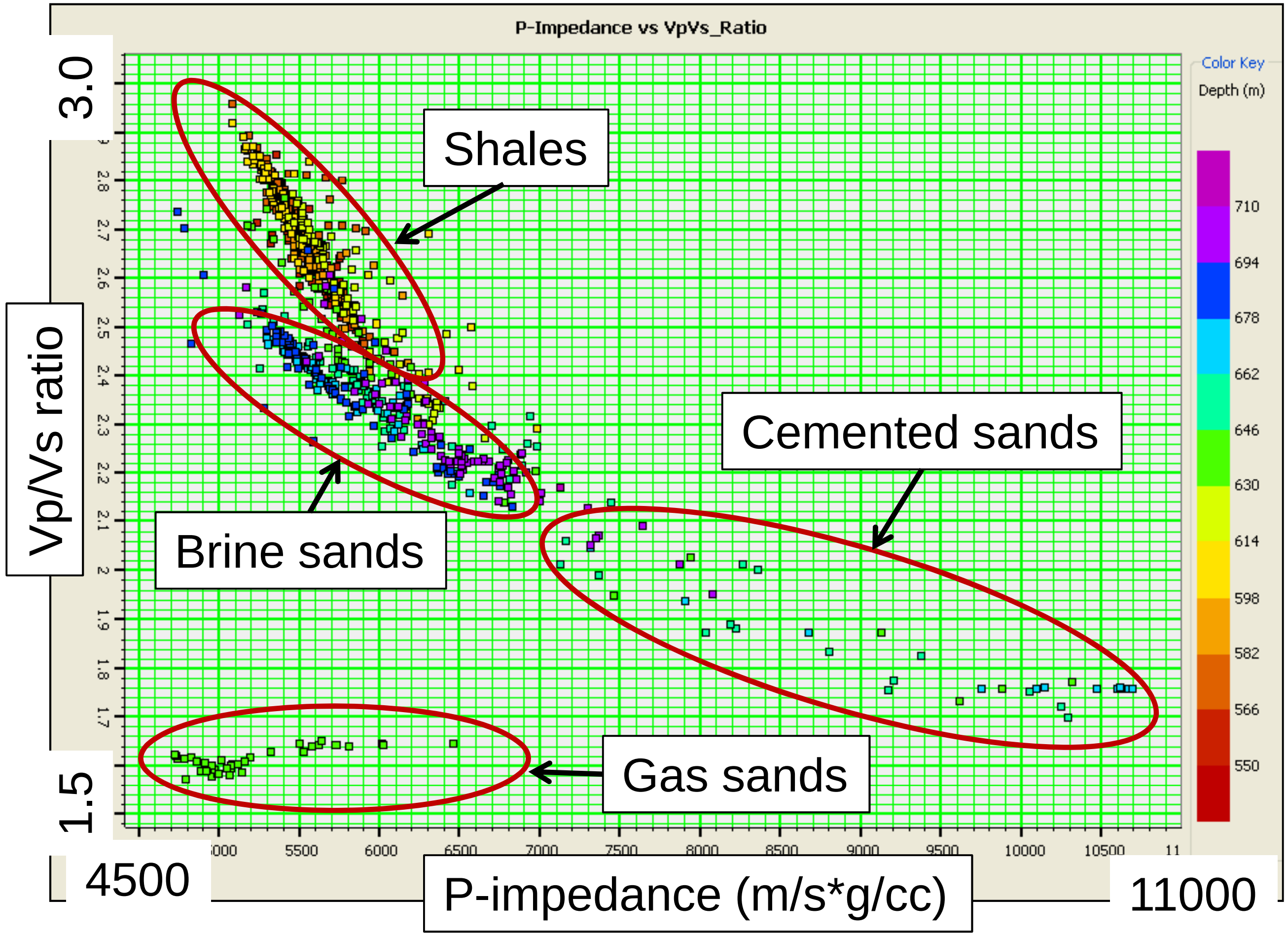
- A comparison between the two methods and the constant ratio empirical result. The plot on the left shows the dry rock $K/\mu$ ratio as a function of porosity (0 to 40%) and the plot on the right shows the dry shear modulus as a function of porosity for only the first 10% of porosity:




- The new approach is closer to the experimental results of Murphy et al., except near 0% porosity, where it correctly predicts the mineral value.

# Vp/Vs vs P-impedance from logs

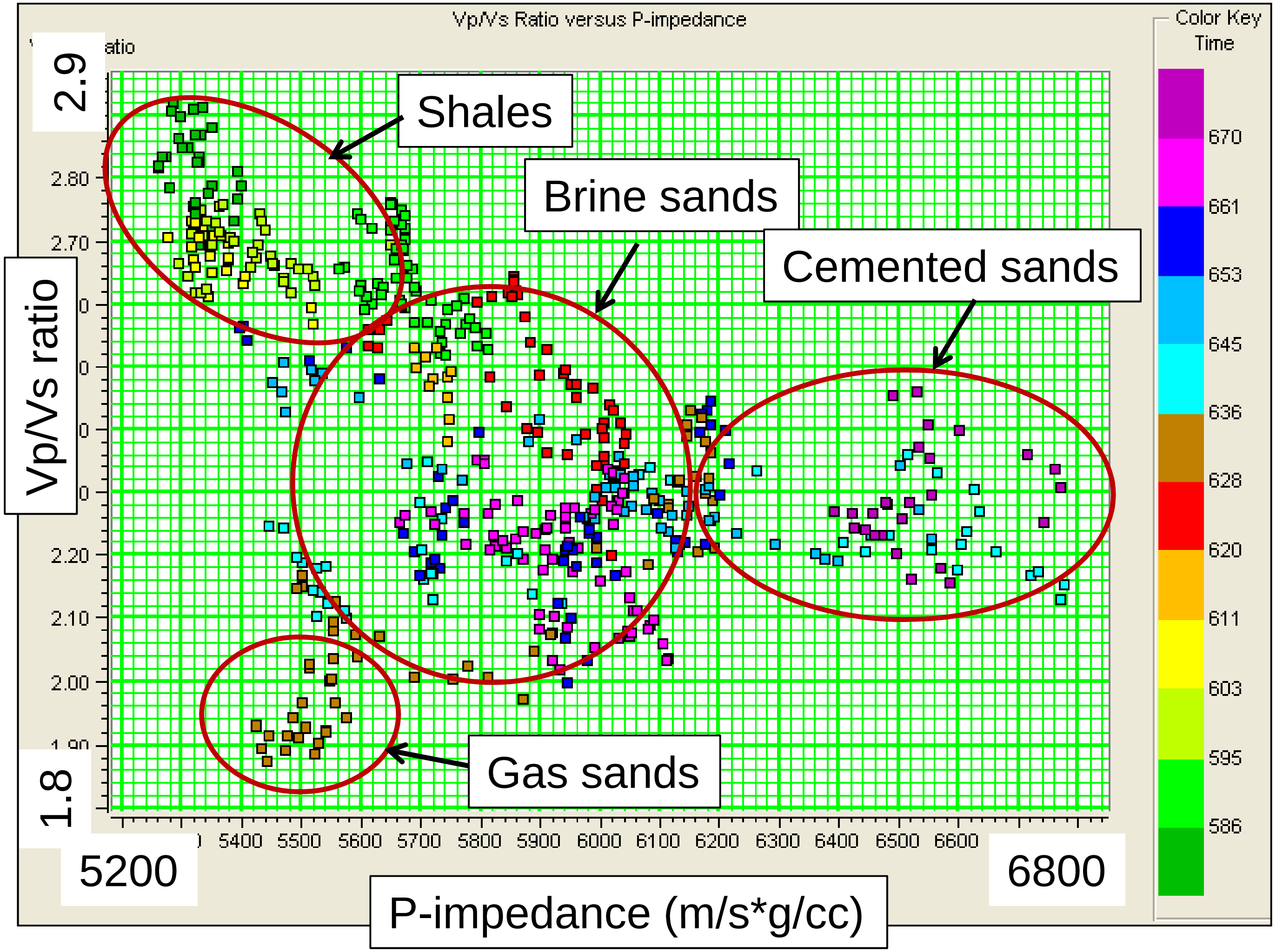
Now we will compare our templates to real data. This plot shows well log data from a gas sand in the Colony area of Alberta.




The *Vp* and $\rho$ logs were measured and *Vs* was computed using the mud-rock line in the shales and wet sands and the Gassmann equations in the gas sands.

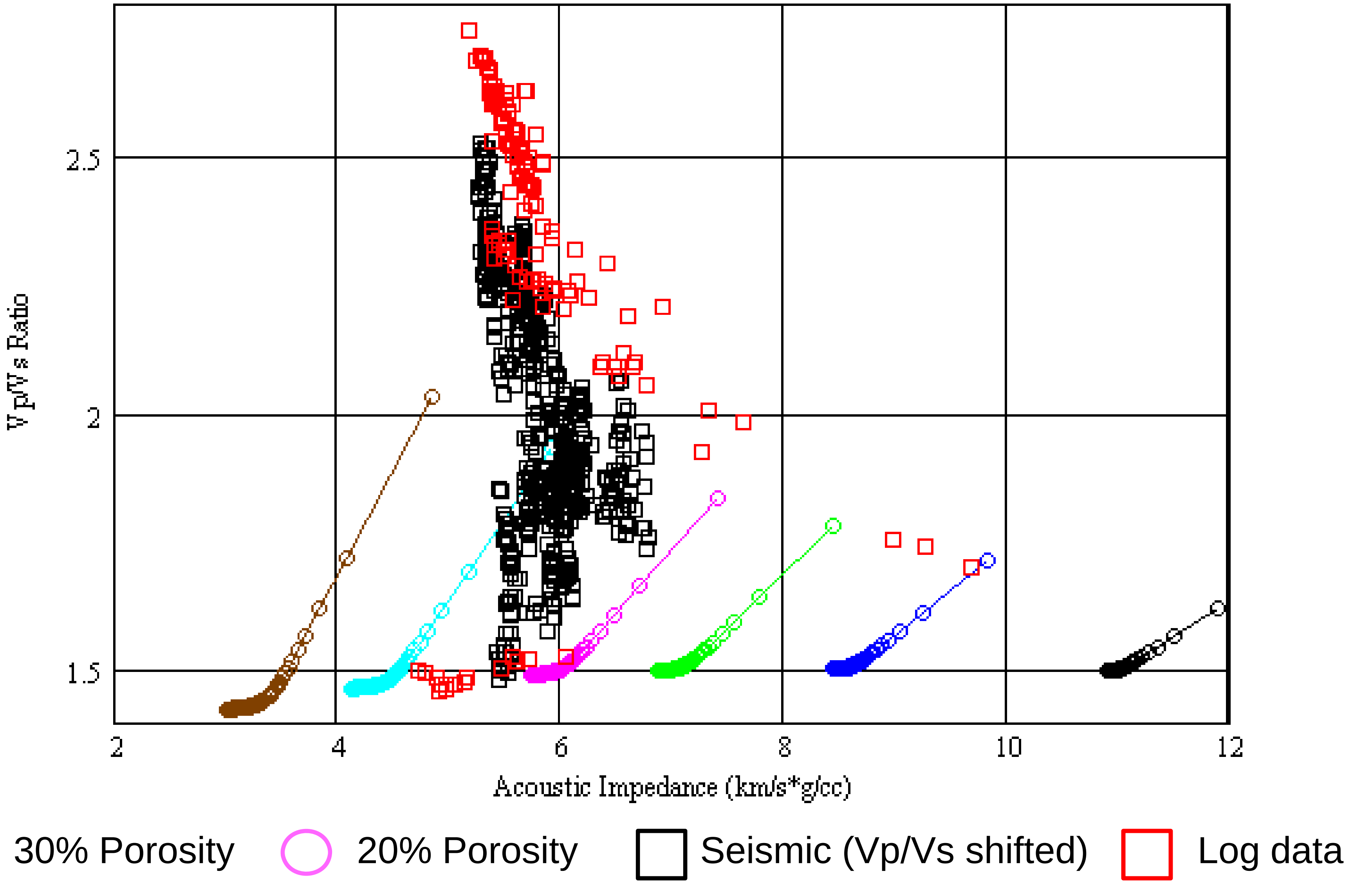
# Vp/Vs vs P-impedance from inversion

The results of a simultaneous pre-stack inversion from the same area. Note that the range of values is less extreme than on the log data due to the bandlimited nature of the seismic data.

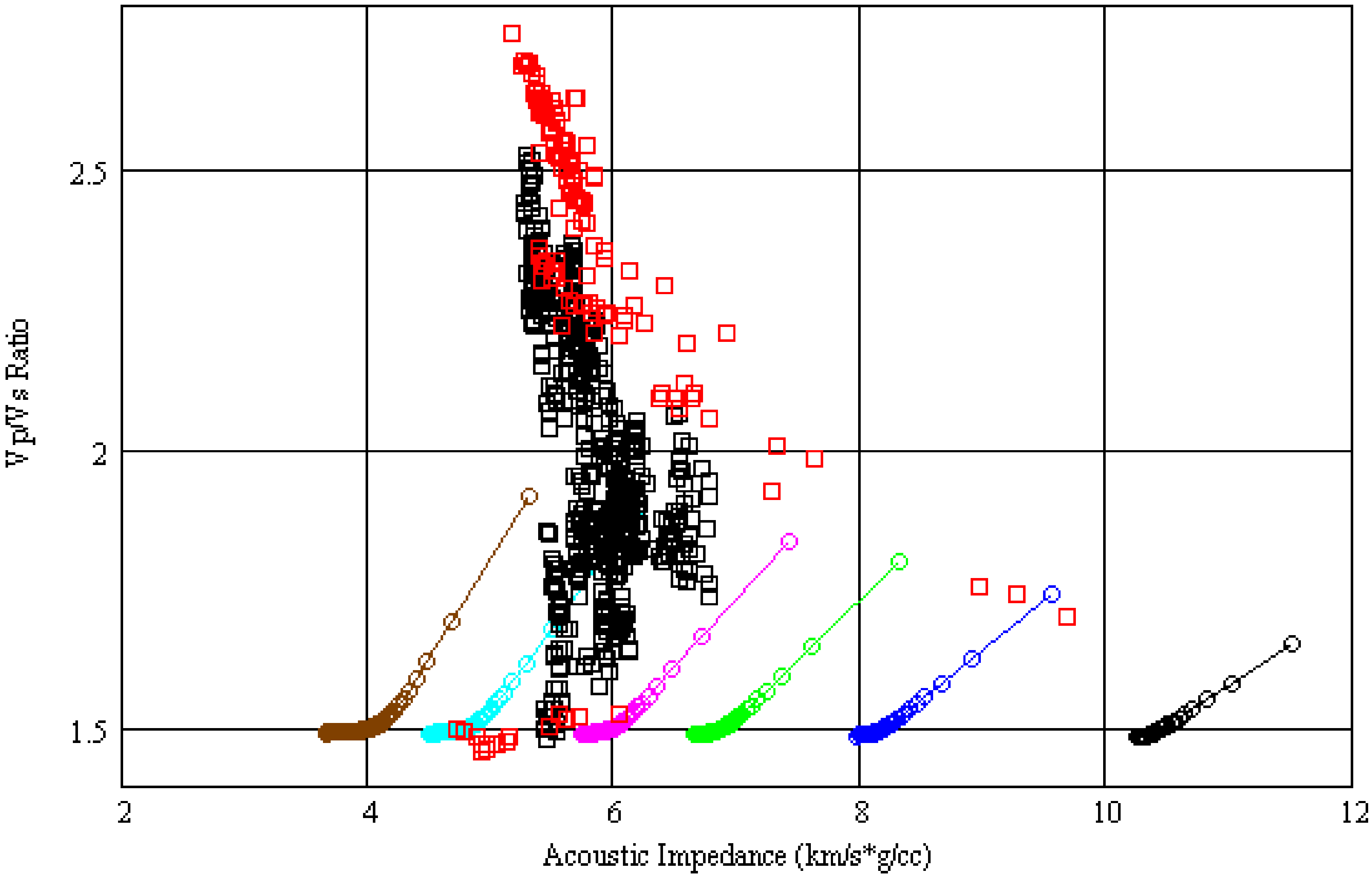



Next, we show the log and seismic data superimposed on the RPTs, where the log data has been integrated to time.

# Avseth/Ødegaard Rock Physics Template

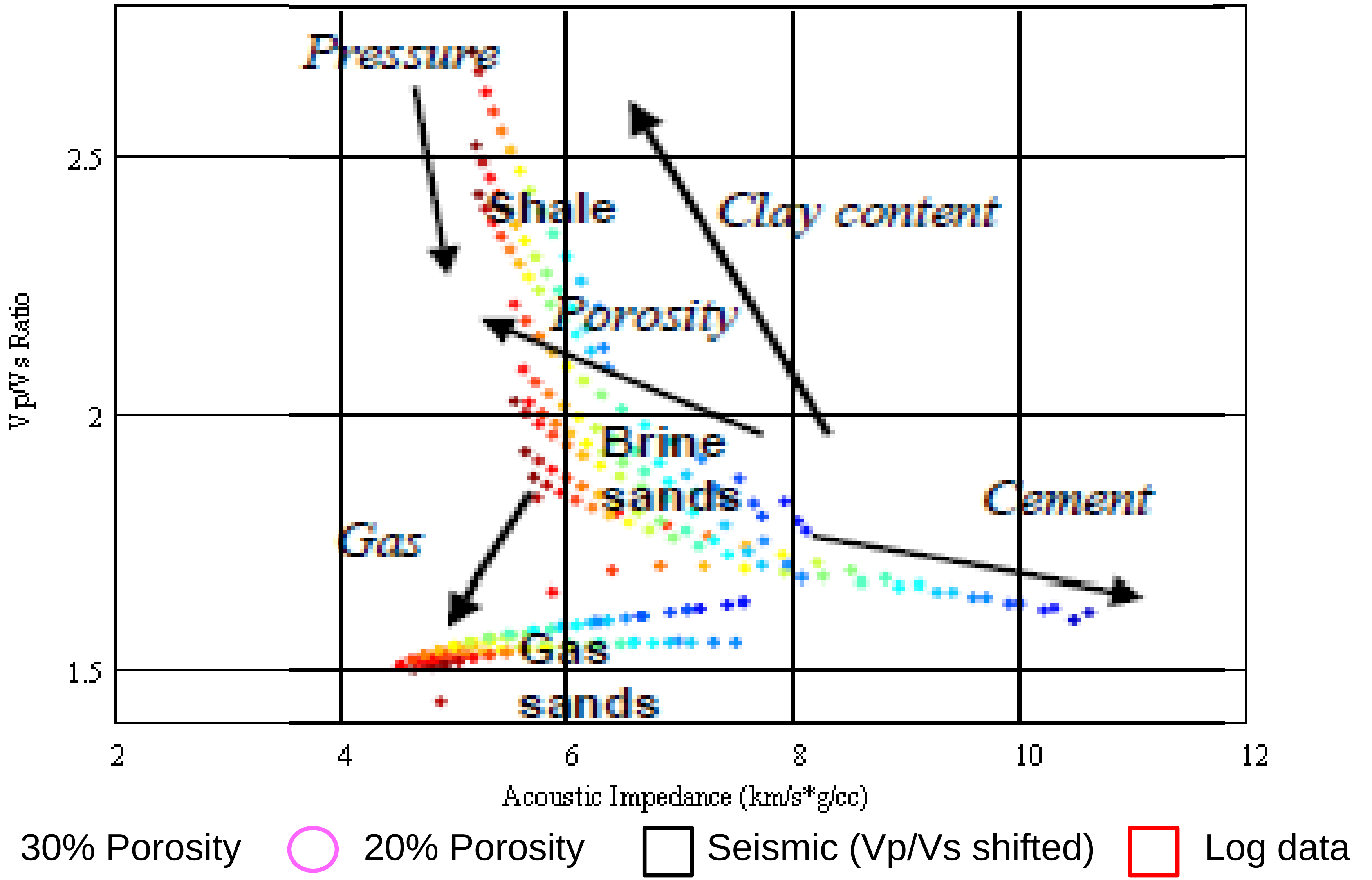


# New Rock Physics Template




30% Porosity 20% Porosity Seismic (Vp/Vs shifted) Log data

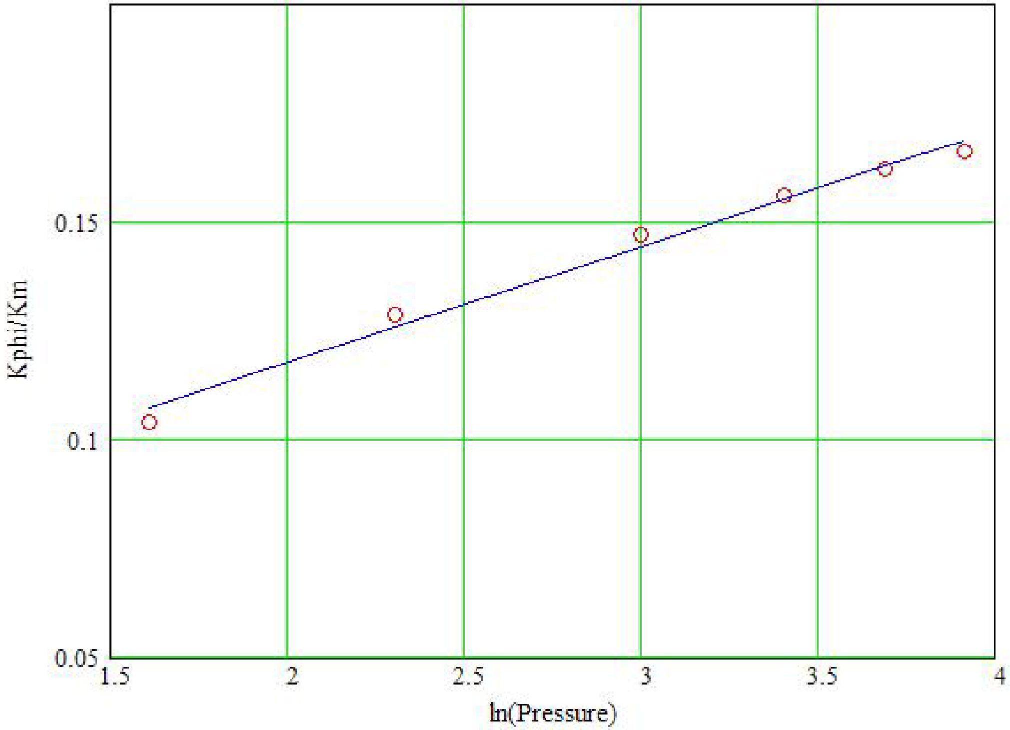
# New Rock Physics Template



# Pressure changes

A plot of $K_\phi/K_m$ vs log(pressure) for the Han dataset at different pressures, with the least-squares fit:

$$\frac{K_\phi}{K_m} = 0.065 + 0.027\ln(P)$$




From this, Russell and Smith (2007) derive a relationship between change in pore space stiffness and pressure, which can be used to alter pressure in the new template:

$$\Delta K_\phi = 0.027 K_m \Delta P / P$$

# Conclusions

- In this talk, we proposed a new approach to the computation of a rock physics template using pore space stiffness.
- We showed how pore space stiffness could be used to estimate the dry rock bulk modulus as a function of porosity, used a similar equation for shear modulus, and from this developed the new template.
- Comparing the new template to the Ødegaard and Avseth approach using lab, measured log and seismic data:
  - The template fits are both reasonable, and quite similar.
  - The pore space stiffness method gives a better fit to the Murphy et al. (1993) lab data.
  - Pressure changes can be modeled empirically.
  - The new method is based on the physics of the reservoir and passes Occam's razor: "All things being equal, simpler explanations are generally better than more complex ones".

# Acknowledgements

We wish to thank the CREWES sponsors and our colleagues at Hampson-Russell, CGGVeritas, and CREWES.

BR would like to especially thank Qing Li, who was the one who said to me: “Why not use the same equation for changing shear modulus as you do for bulk modulus”, which should have been obvious to me, but wasn’t!

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