



# There's nonlinear, and then there's nonlinear...

Priddis pulse probe data and large contrast AVO in the lab

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# Outline

**Two different kinds of nonlinearity**

**Nonlinearity type I:**

**Priddis pulse probe experiment -  
early results (with G. Margrave, M.  
Bertram & CREWES)**

**Nonlinearity type II:**

**Large contrast AVO in the lab (with  
F. Mahmoudian)**

# Two different types of nonlinearity

$$\left[ \nabla^2 - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2}{\partial t^2} \right] P(\mathbf{r}, t) = f(\mathbf{r}, t)$$

# Two different types of nonlinearity

$$\begin{aligned} \mathcal{L}P_1 &= f_1 \\ \mathcal{L}P_2 &= f_2 \end{aligned} \quad \rightarrow \quad \mathcal{L}(P_1 + P_2) = f_1 + f_2$$

$$\left[ \nabla^2 - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2}{\partial t^2} \right] P(\mathbf{r}, t) = f(\mathbf{r}, t)$$

...absent explicit linearization (e.g., Innanen CREWES 2010),

|   |
|---|
| <b>Type I</b> $\mathcal{L}(P_1 + P_2) \neq f_1 + f_2$ |
|---|

# Two different types of nonlinearity

Even given a linear wave equation

$P = P [c(\mathbf{r})]$  is nonlinear

$$\left[ \nabla^2 - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2}{\partial t^2} \right] P(\mathbf{r}, t) = f(\mathbf{r}, t)$$

**Type II**  $R = \frac{c_1 - c_0}{c_1 + c_0} = \frac{1}{4} (\delta c) + \frac{1}{8} (\delta c)^2 + \dots$

**Nonlinearity Type I:**  
**The Priddis pulse-probe  
experiment**

# **Type I: Priddis pulse-probe**

## **Wave nonlinearity**

**acoustic medical imaging  
physical modeling  
theory**

## **Seismic nonlinearity**

**Zhukov et al., 2007 (TLE)**

**Margrave et al., 2008 (CREWES)**

**Campman et al., 2012 (EAGE)**

# Type I: Priddis pulse-probe



## CREWES Field Experiment July 2012

### Objectives

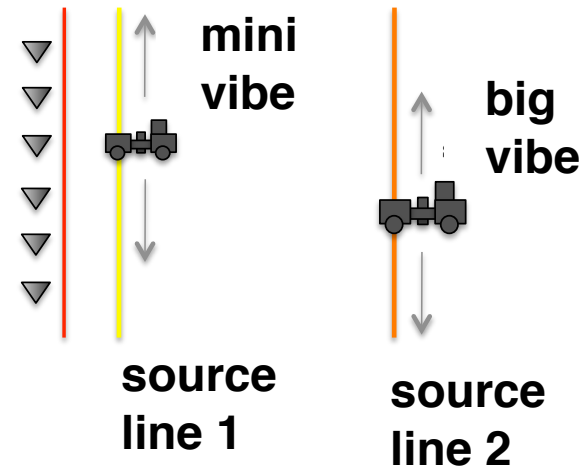
**Autoseis & array tests**  
**Dynamite charge size**  
**“Pulse probe” revisited**

### Partners

**Global Geophysical**  
**Geokinetics**  
**Outsource**  
**INOVA**

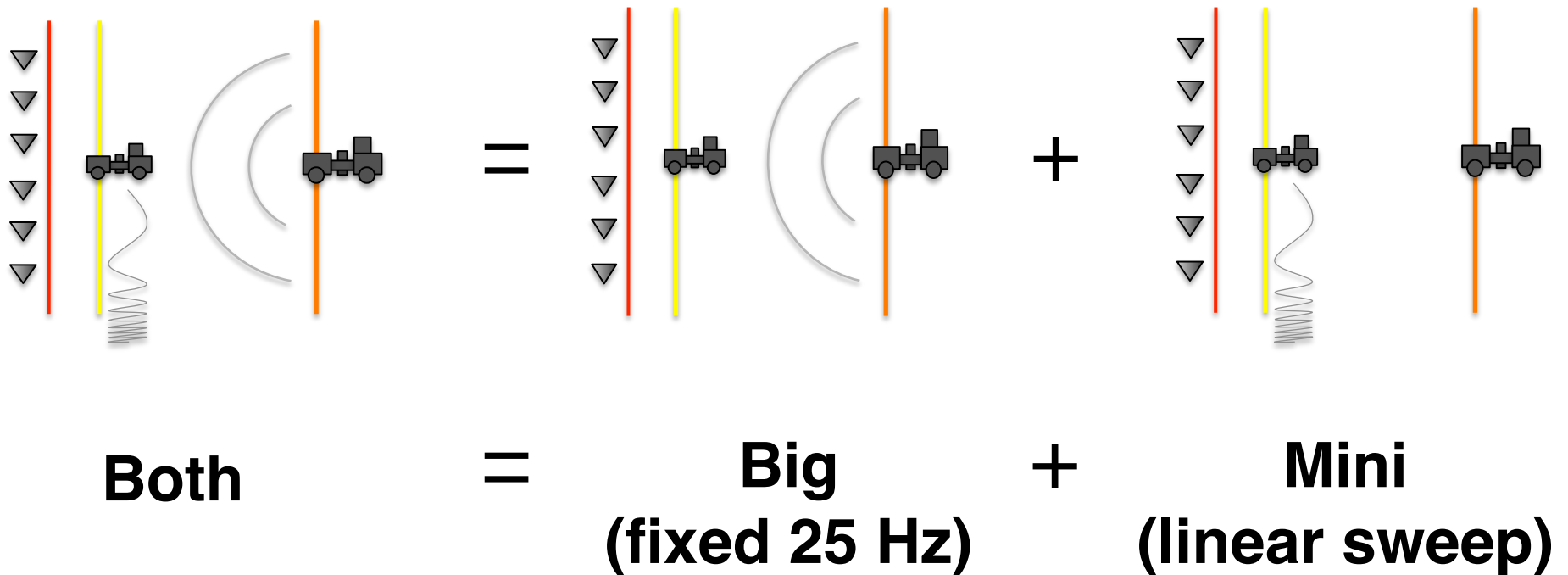


# Type I: Priddis pulse-probe



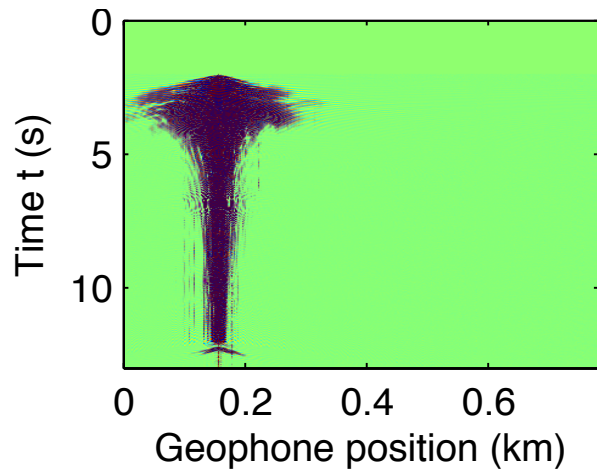
# Type I: Priddis pulse-probe

## Superposition

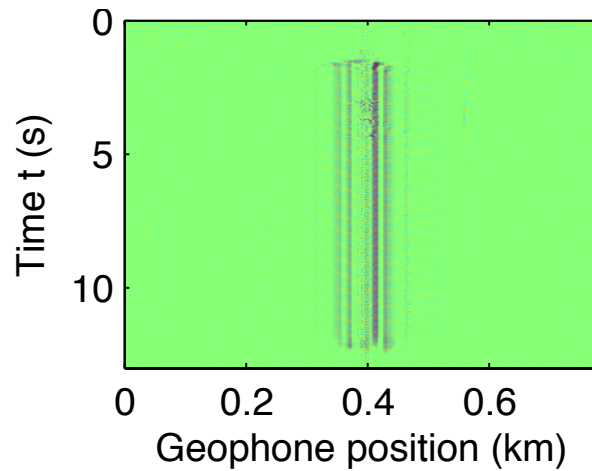


# Type I: Priddis pulse-probe

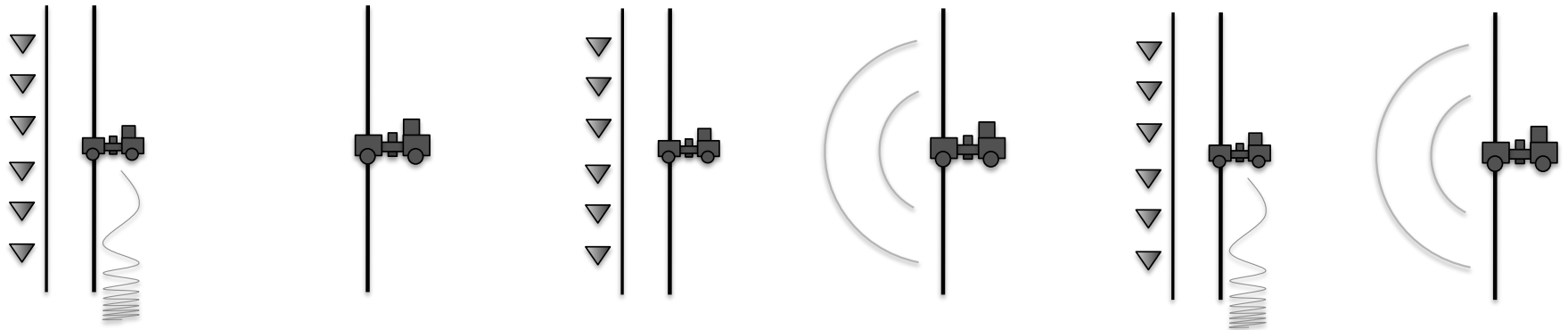
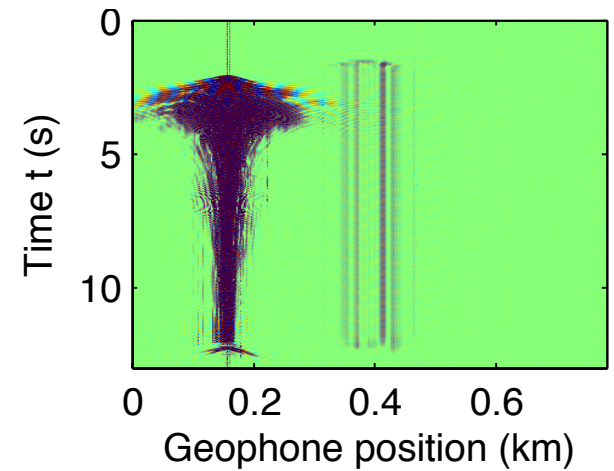
## Mini only



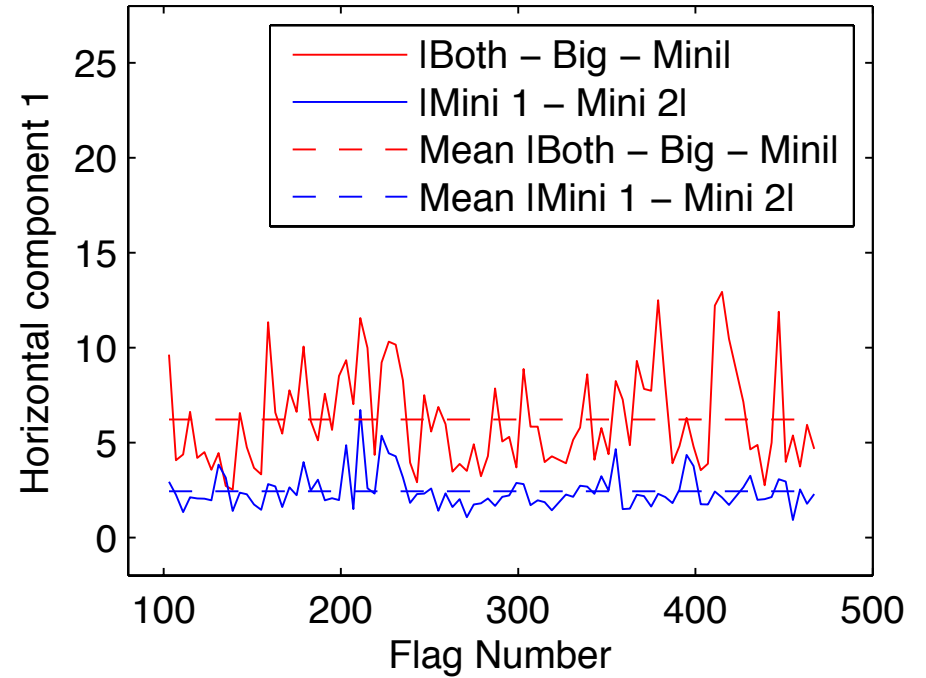
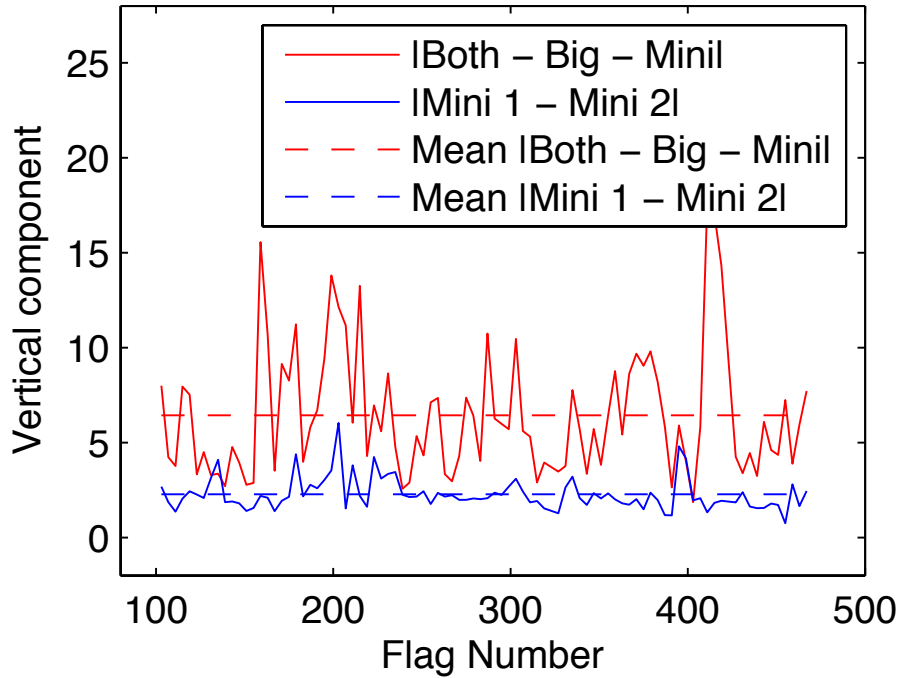
## Big only



## Both



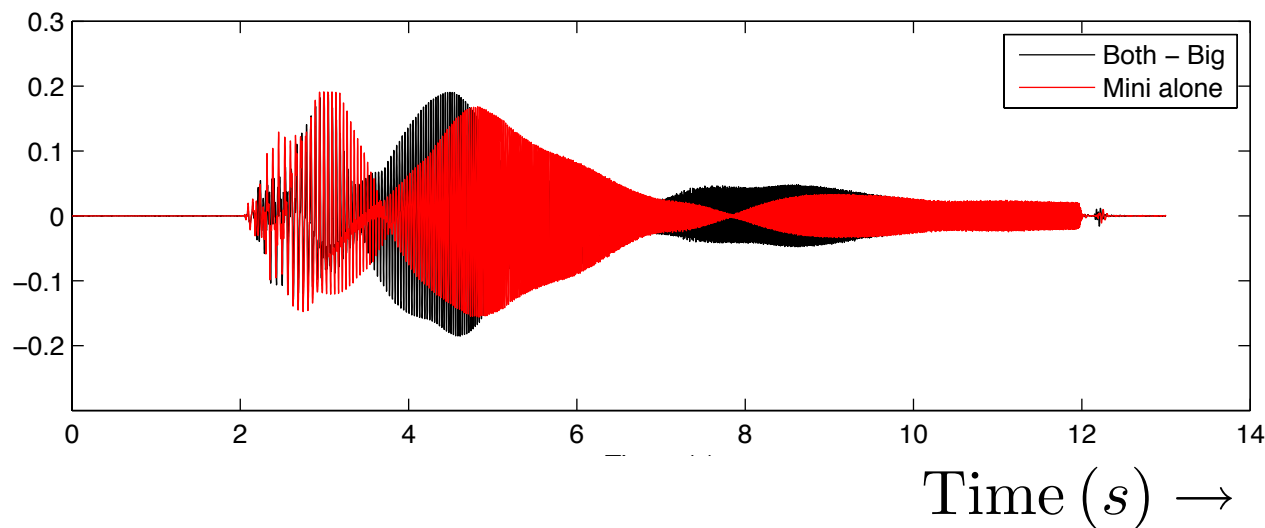
# Type I: Priddis pulse-probe



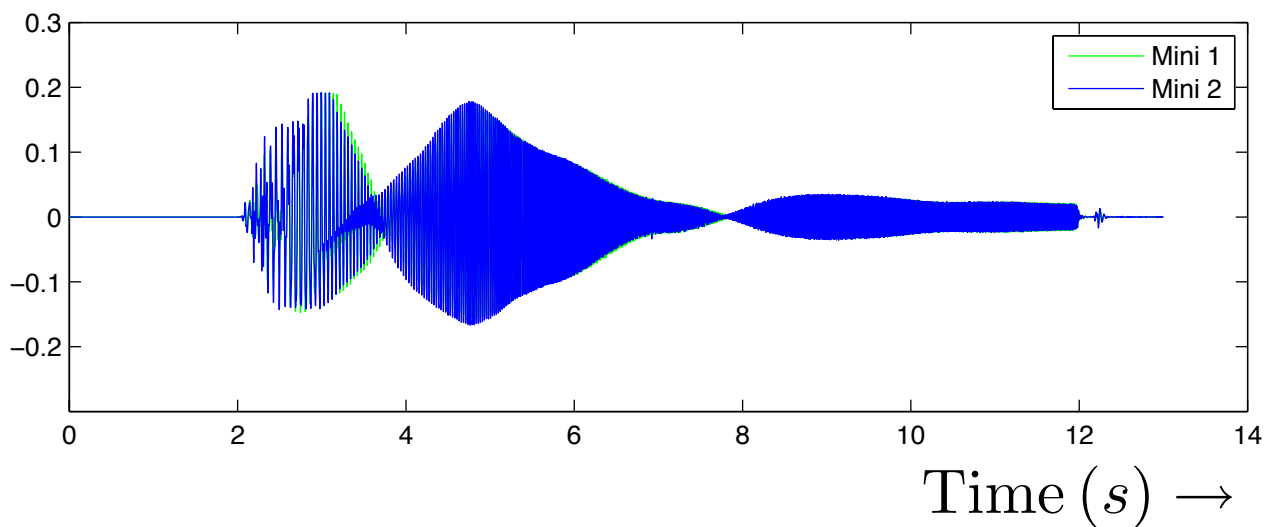
**Superposition**  
**Both - Big - Mini = 0**

# Type I: Priddis pulse-probe

**Mini alone**  
**Both - Big**

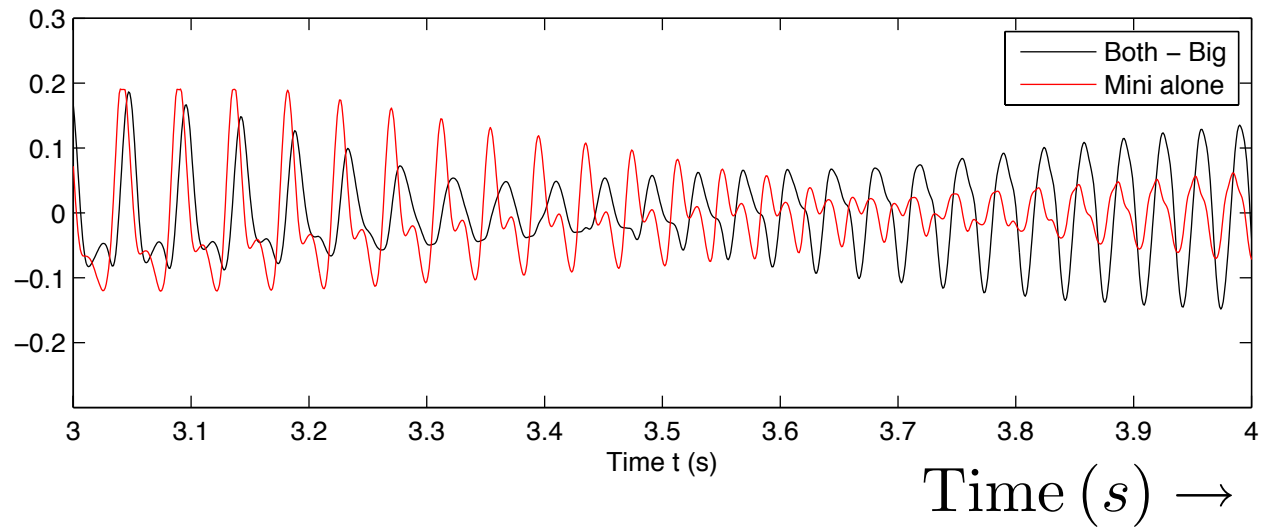


**Mini 1**  
**Mini 2**

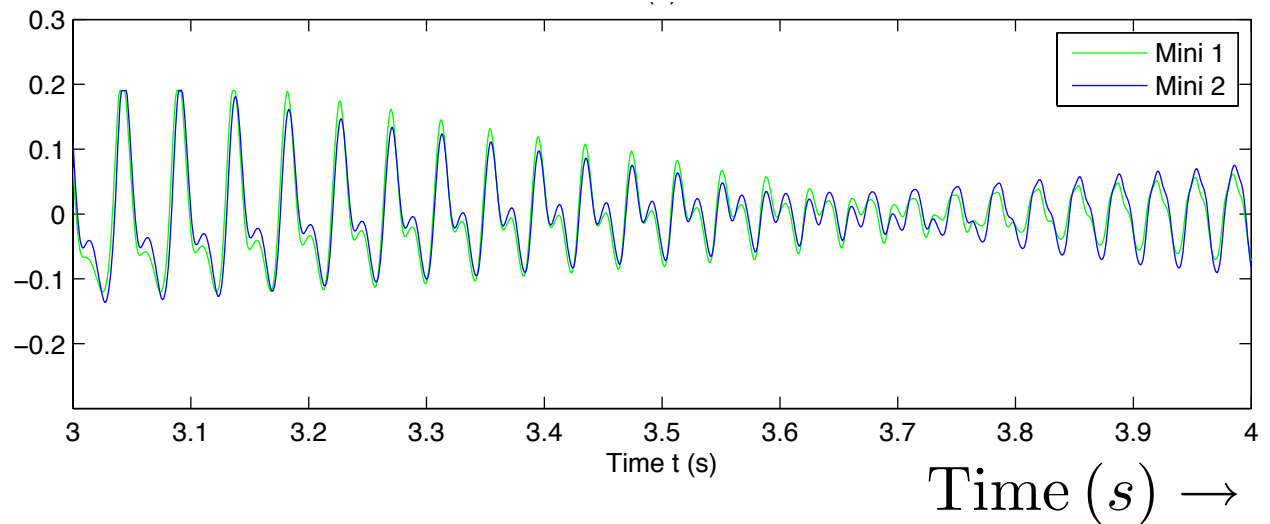


# Type I: Priddis pulse-probe

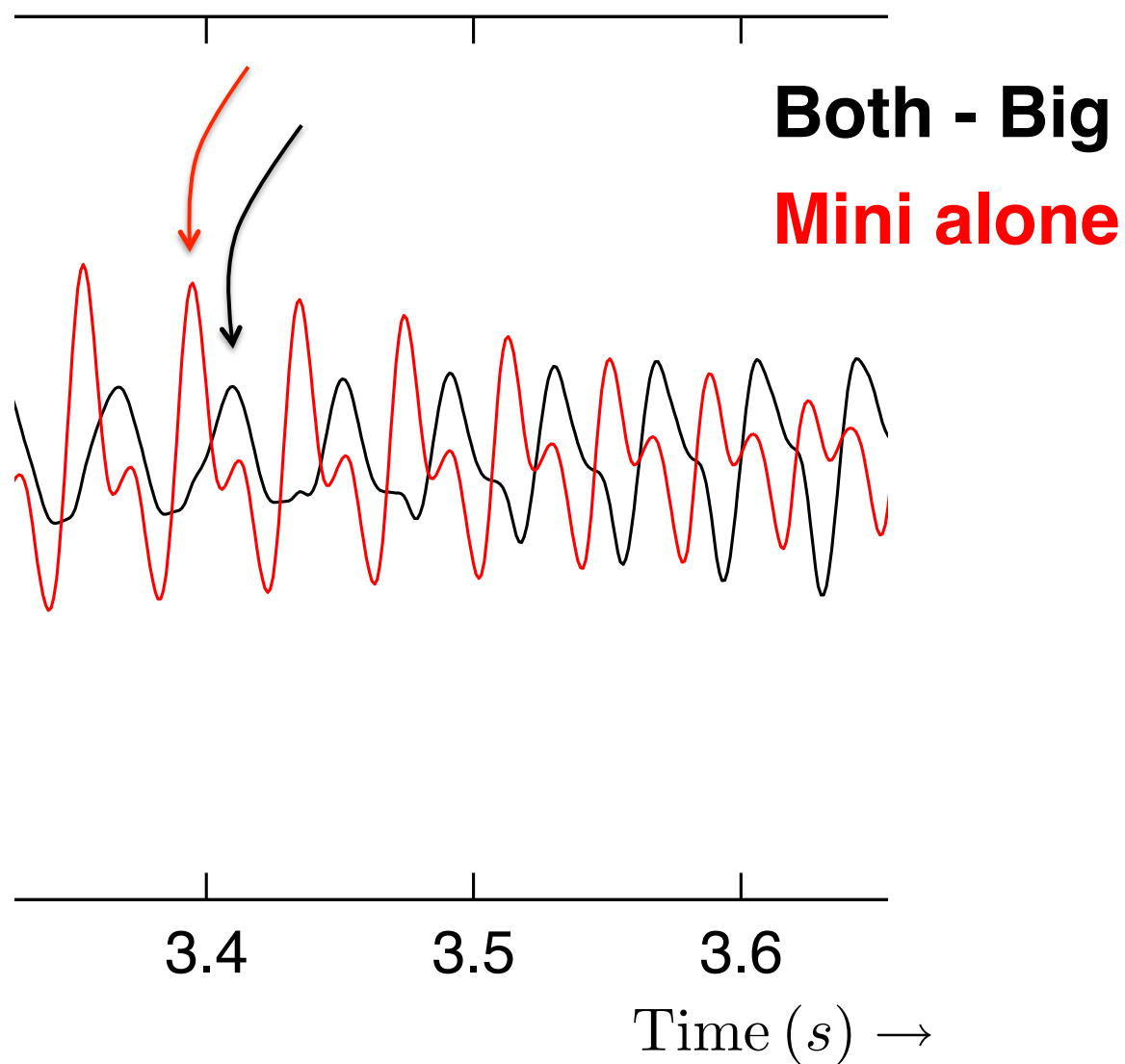
**Mini alone**  
**Both - Big**



**Mini 1**  
**Mini 2**



# Type I: Priddis pulse-probe

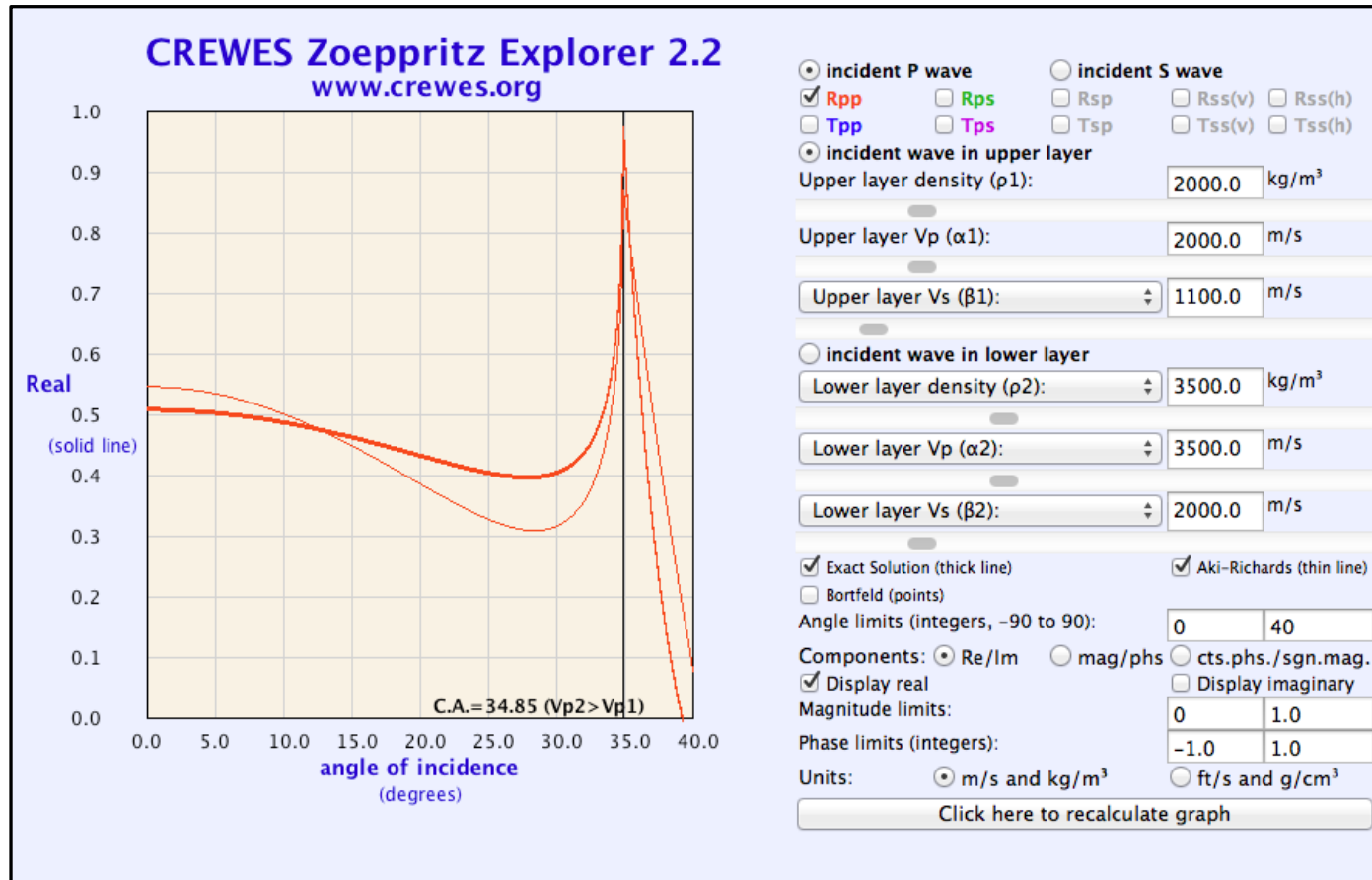


**Nonlinearity Type II:**

**Large contrast AVO in the lab**

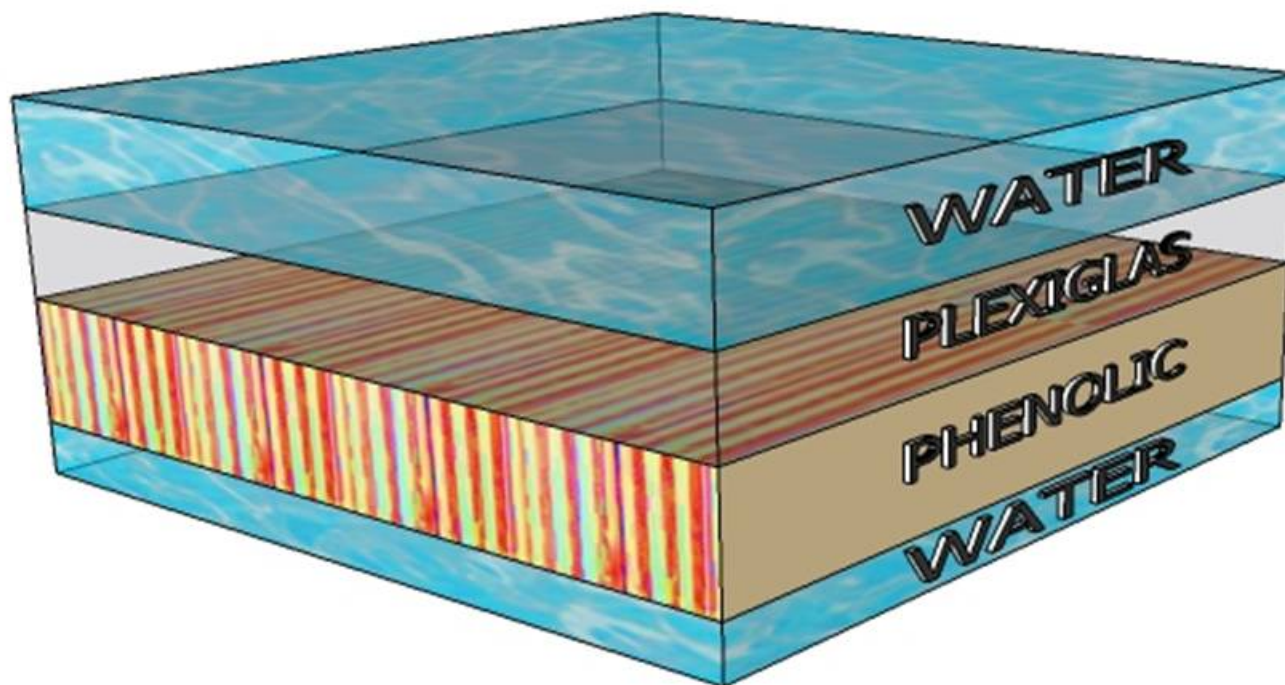


# Type II: large contrast AVO



$V_P$  contrast  $\approx 55\%$     $V_S$  contrast  $\approx 58\%$     $\rho$  contrast  $\approx 55\%$

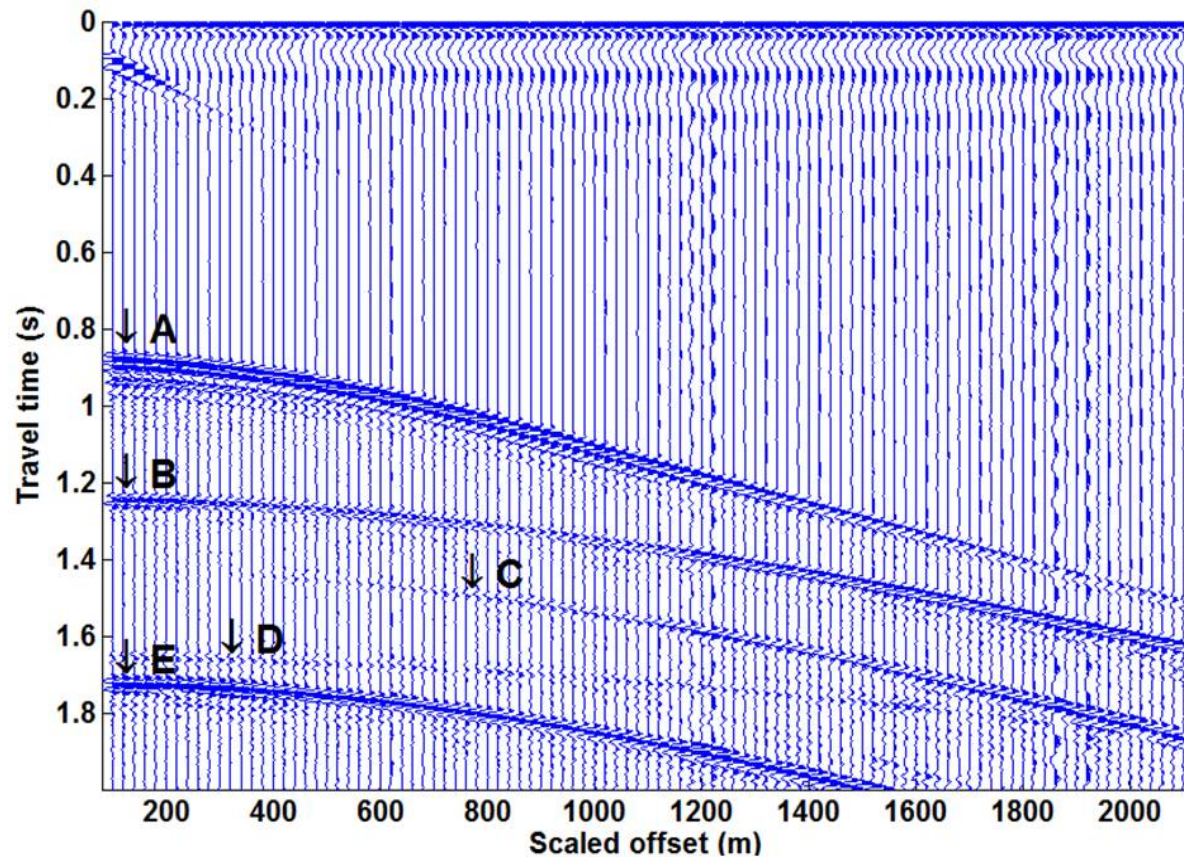
# Type II: large contrast AVO



**Physical model designed to study anisotropic AVO**

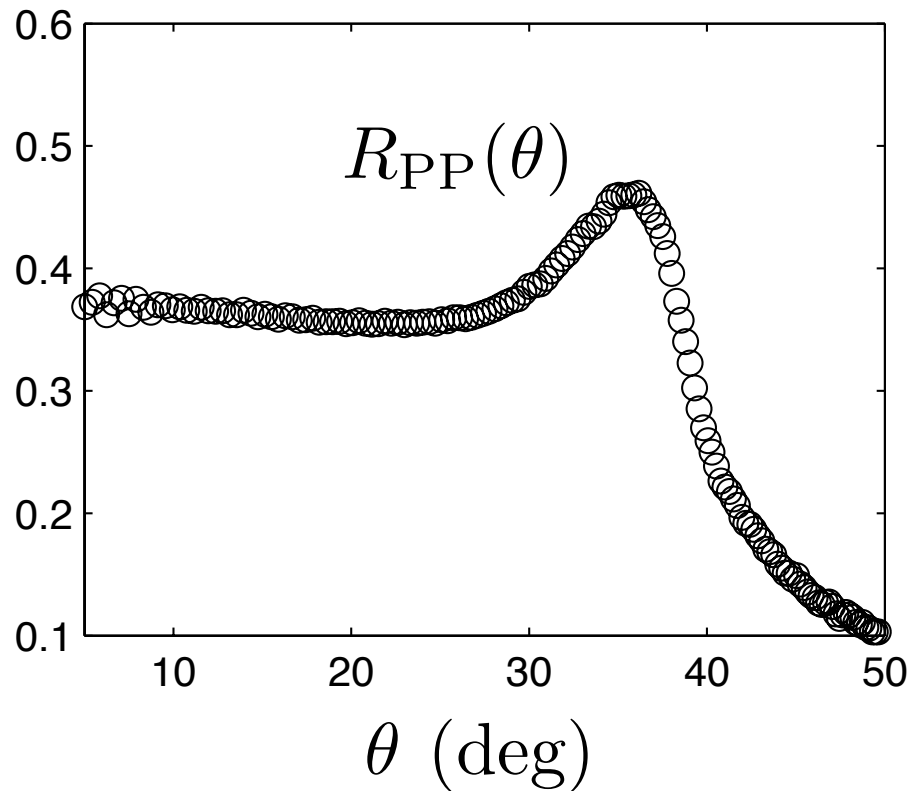
**See also Mahmoudian et al., CREWES 2012**

# Type II: large contrast AVO

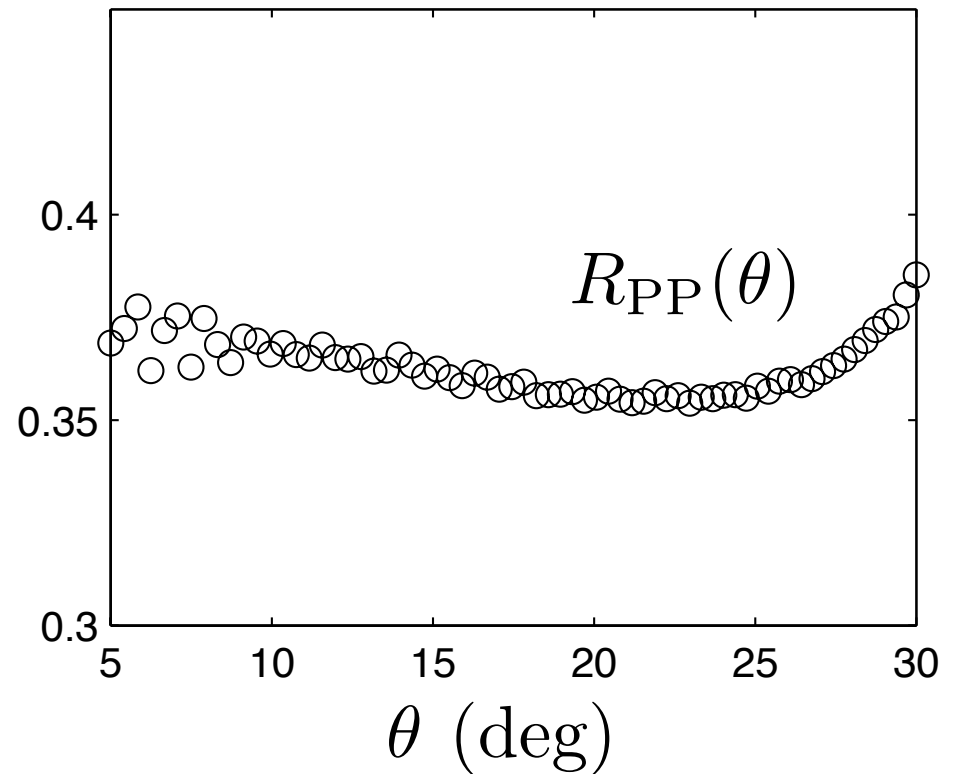


**Consider reflection (A) – water-plexiglas boundary**

# Type II: large contrast AVO



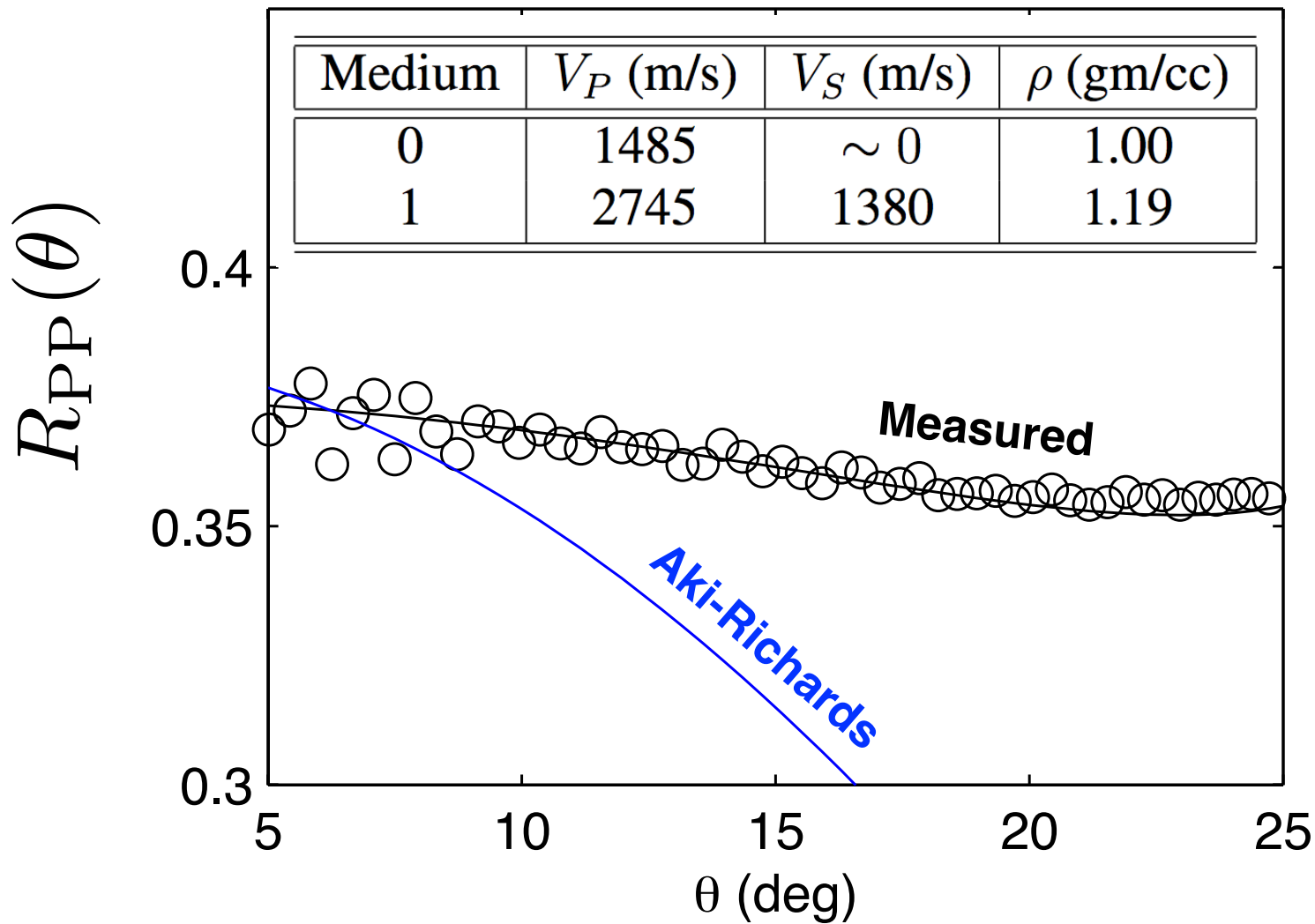
**Full angle range**



**Low angles**

**See also Mahmoudian et al., CREWES 2012**

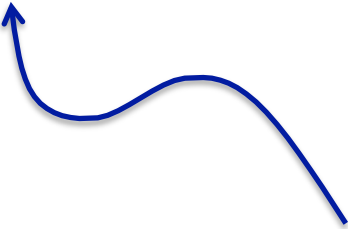
# Type II: large contrast AVO



# Type II: large contrast AVO

1<sup>st</sup> order ~ Aki-Richards

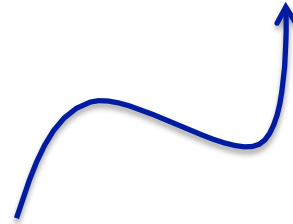
$$R_{PP}(\theta) = R_{PP}^{(1)}(\theta) + R_{PP}^{(2)}(\theta) + R_{PP}^{(3)}(\theta) + \dots,$$


$$R_{PP}^{(1)}(\theta) = \frac{1}{2} (1 - 3 \sin^2 \theta) \frac{\Delta V_P}{V_P} + 2 \sin^2 \theta \frac{\Delta V_S}{V_S} + \frac{1}{2} \frac{\Delta \rho}{\rho}$$

# Type II: large contrast AVO

**2<sup>nd</sup> order – no zero  
offset contribution**

$$R_{PP}(\theta) = R_{PP}^{(1)}(\theta) + R_{PP}^{(2)}(\theta) + R_{PP}^{(3)}(\theta) + \dots,$$




$$R_{PP}^{(2)}(\theta) = \frac{3}{2} \sin^2 \theta \left( \frac{\Delta V_P}{V_P} \right)^2 - 6 \sin^2 \theta \left( \frac{\Delta V_P}{V_P} \right) \left( \frac{\Delta V_S}{V_S} \right) + 5 \sin^2 \theta \left( \frac{\Delta V_S}{V_S} \right)^2$$

# Type II: large contrast AVO

3<sup>rd</sup> order

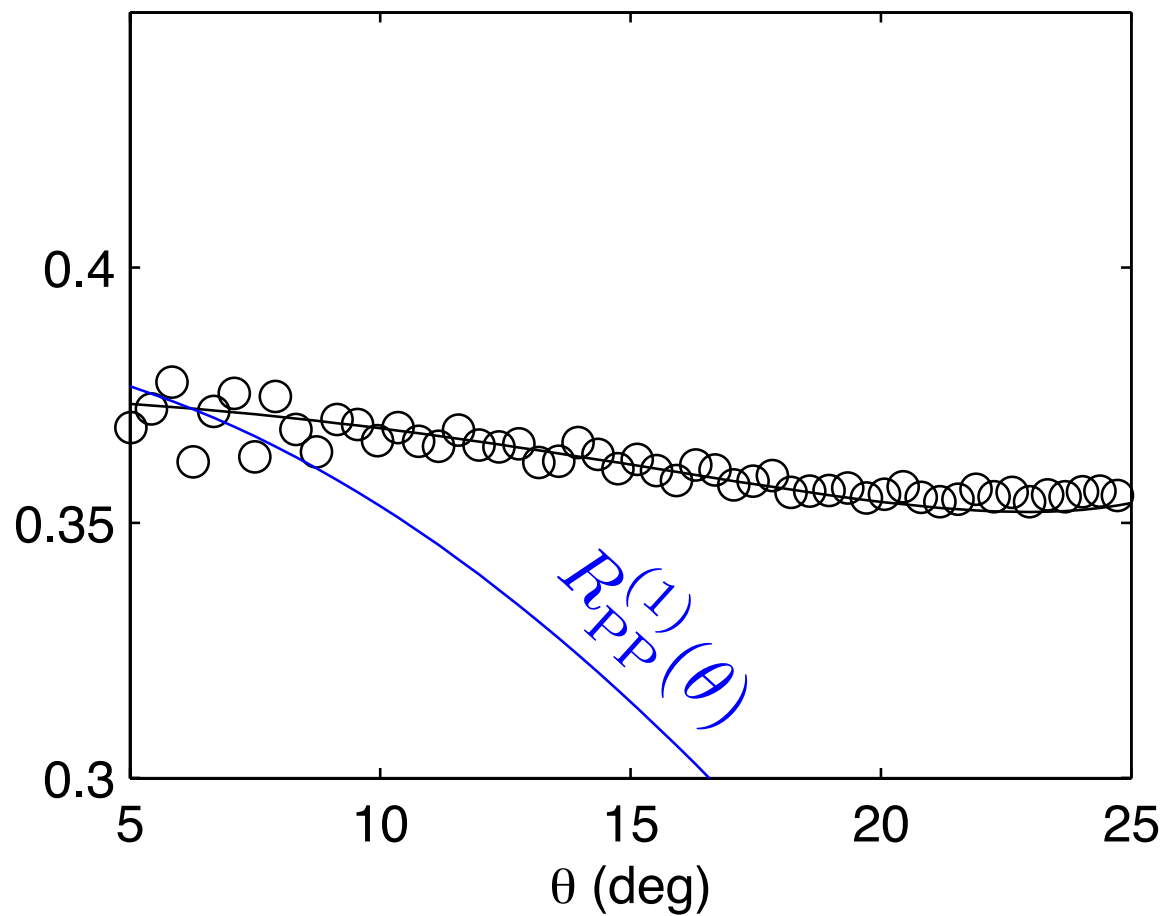
$$R_{PP}(\theta) = R_{PP}^{(1)}(\theta) + R_{PP}^{(2)}(\theta) + R_{PP}^{(3)}(\theta) + \dots,$$


$$R_{PP}^{(3)}(\theta) = -\frac{1}{8} \left( \frac{\Delta V_P}{V_P} \right)^2 \left( \frac{\Delta \rho}{\rho} \right) - \frac{1}{8} \left( \frac{\Delta \rho}{\rho} \right)^2 \left( \frac{\Delta V_P}{V_P} \right) \\ + \left[ \frac{13}{2} \left( \frac{\Delta V_S}{V_S} \right)^3 + \frac{1}{4} \left( \frac{\Delta V_P}{V_P} \right)^3 + \frac{3}{8} \left( \frac{\Delta V_P}{V_P} \right) \left( \frac{\Delta \rho}{\rho} \right)^2 \right. \\ \left. + \text{four terms} - \left( \frac{\Delta V_P}{V_P} \right) \left( \frac{\Delta V_S}{V_S} \right) \left( \frac{\Delta \rho}{\rho} \right) \right] \sin^2 \theta$$



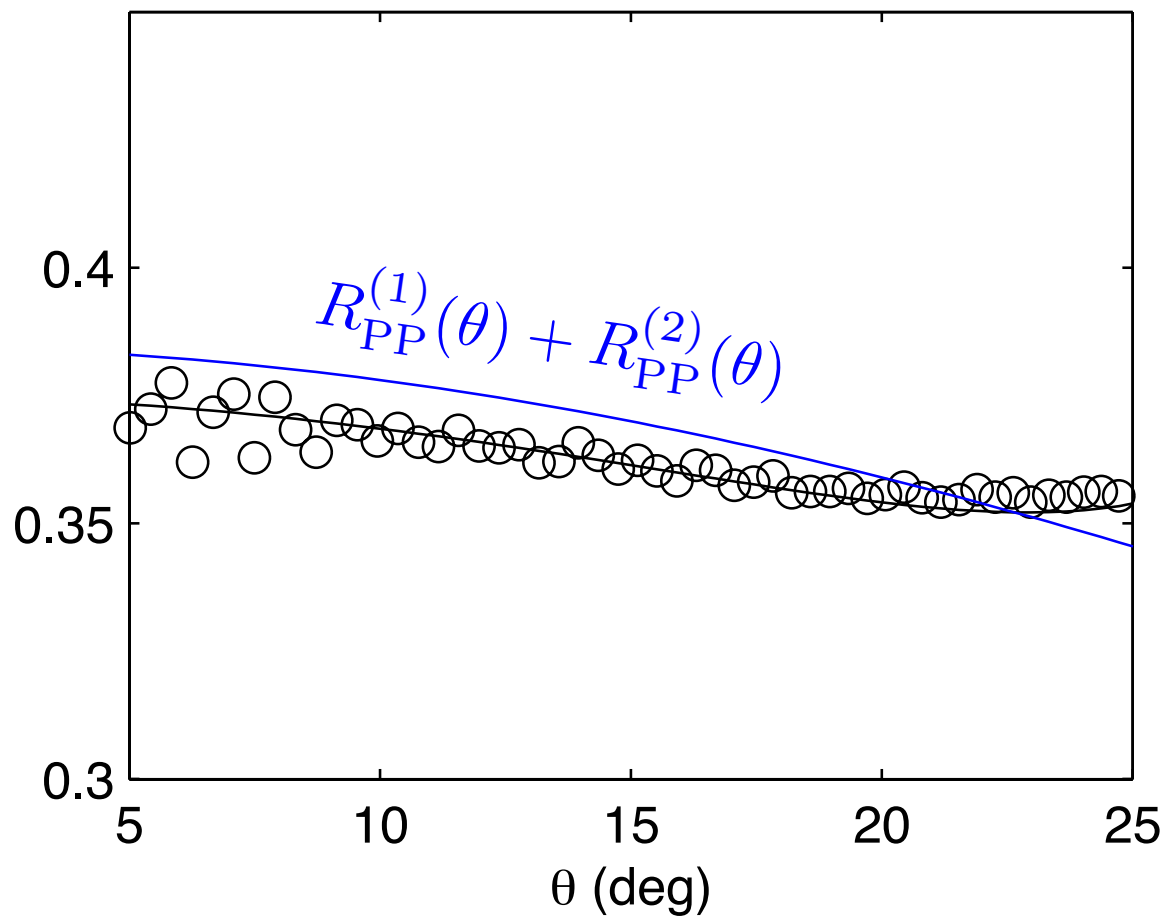
# Type II: large contrast AVO

$R_{PP}$  1<sup>st</sup> order



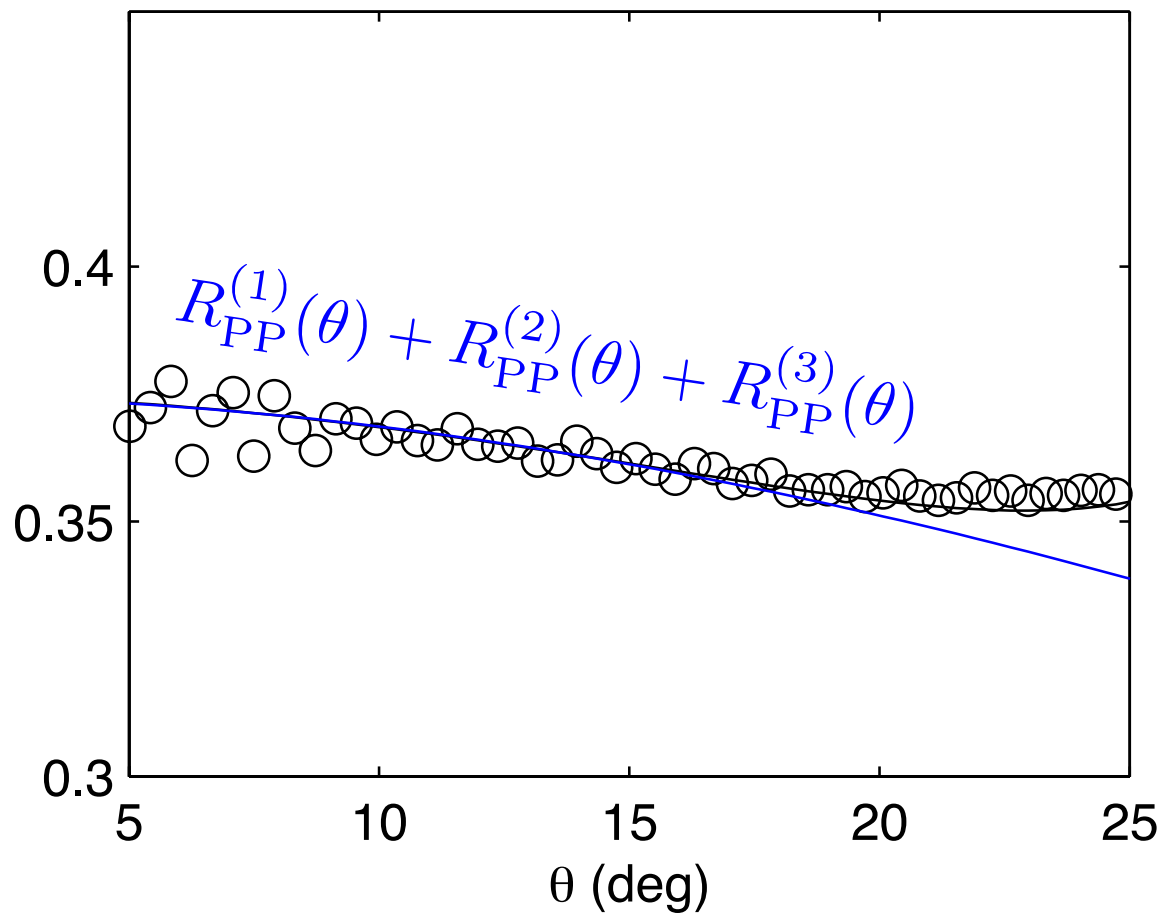
# Type II: large contrast AVO

$R_{PP}$  2<sup>nd</sup> order



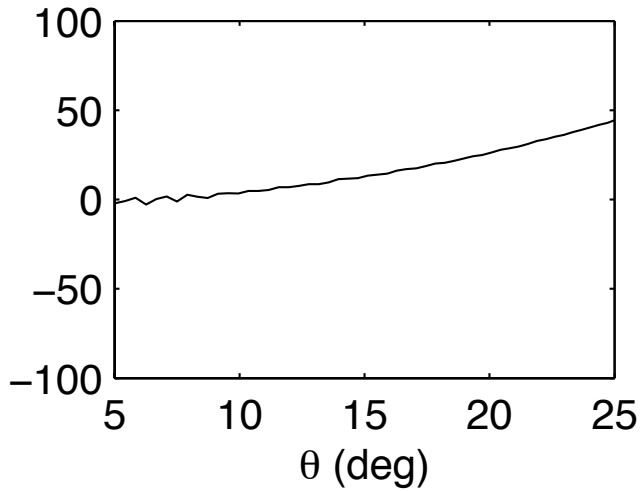
# Type II: large contrast AVO

$R_{PP}$  3<sup>rd</sup> order



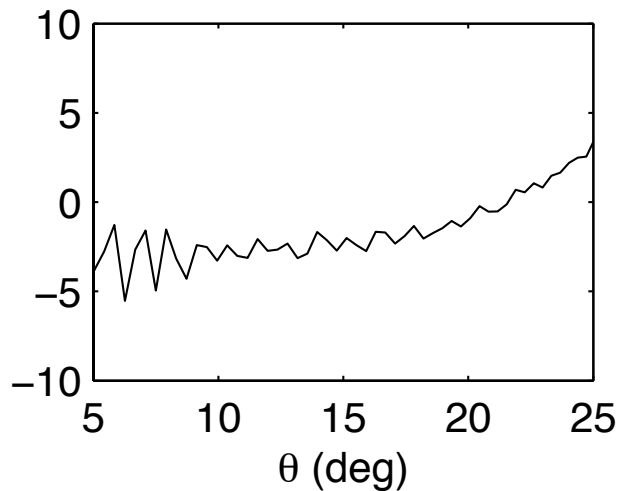
# Type II: large contrast AVO

**% Error 1<sup>st</sup> order**



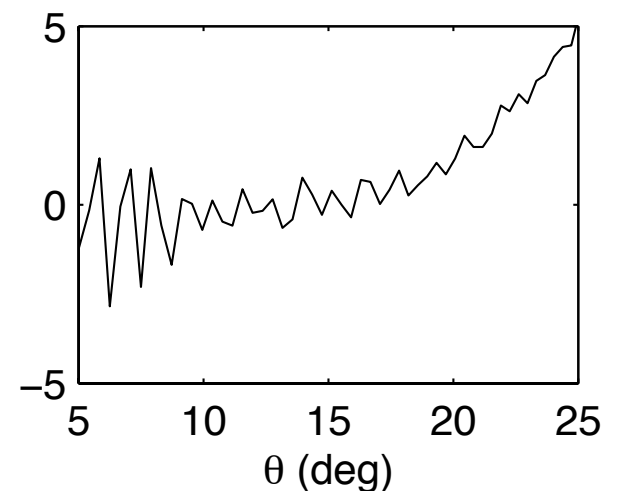
**Error  
Significant**

**% Error 2<sup>nd</sup> order**



**Trend  
Corrected**

**% Error 3<sup>rd</sup> order**



**Magnitudes  
Corrected**

# Conclusions

**Two types of NL – 1 definitely, 1 possibly present in exploration-monitoring seismology**

**Priddis pulse probe – does a “background” seismic wave field change seismic responses?**

**Unconfirmed (vibe feedback vs. NL)**

**Uses? Target differentiation**

**Nonlinear AVO in the lab – interpretable approximations for large contrast targets**

# Acknowledgments



**Brian Russell**