Exact, linear and nonlinear poroelastic AVO modeling

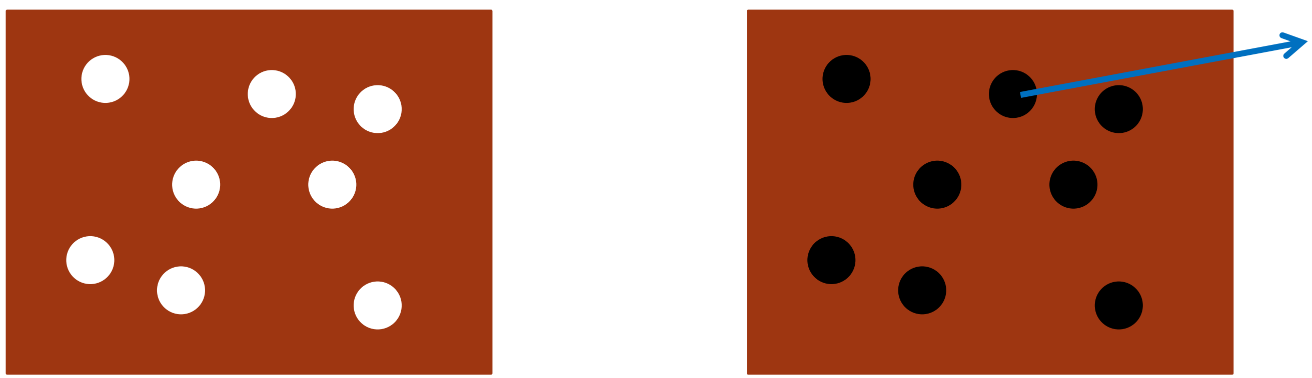
By: Steven Kim and Kris Innanen

Outline

- Review of poroelasticity
  - Biot (1941), Gassmann (1951)
  - Russell, Gray, and Hampson (2011)
- Review of AVO (full and linearized)
  - Zoeppritz
  - Aki and Richards
  - Russell and Gray
- Research objectives
  - Implementation of poroelasticity into full elastic equations
  - Extension – nonlinear AVO
- A different model of poroelastic reflections

Poroelasticity

- Biot (1941) and Gassmann (1951) devise relationships of elastic moduli to poroelastic moduli
  - Dry modulus (skeleton framework) to a saturated modulus (fluid filled)



Fluids
- Brine
- Gas

H_{dry} H_{sat}

$$H = \text{modulus}$$

Poroelasticity

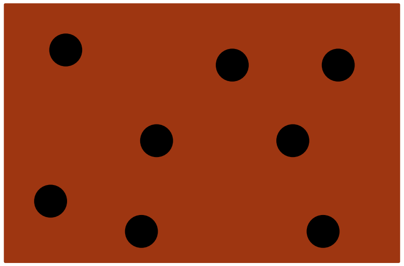
- Biot (1941) and Gassmann (1951) devise relationships of elastic moduli to poroelastic moduli

$$\mu_{\mathrm{sat}} = \mu_{\mathrm{dry}} \qquad \lambda_{\mathrm{sat}} = \lambda_{\mathrm{dry}} + f \qquad K_{\mathrm{sat}} = K_{\mathrm{dry}} + f$$

$$f = \alpha^2 M$$

$$\alpha = 1 - \frac{K_{\mathrm{dry}}}{K_{\mathrm{m}}} \qquad M = \left(\frac{\alpha - \phi}{K_{\mathrm{m}}} + \frac{\phi}{K_{\mathrm{fl}}} \right)^{-1}$$

$f = 0$



$f = f_{\mathrm{brine}}$

Poroelasticity

$$V_P = \sqrt{\frac{\lambda_{\text{sat}} + 2\mu_{\text{sat}}}{\rho_{\text{sat}}}} = \sqrt{\frac{K_{\text{sat}} + (4/3)\mu_{\text{sat}}}{\rho_{\text{sat}}}}$$

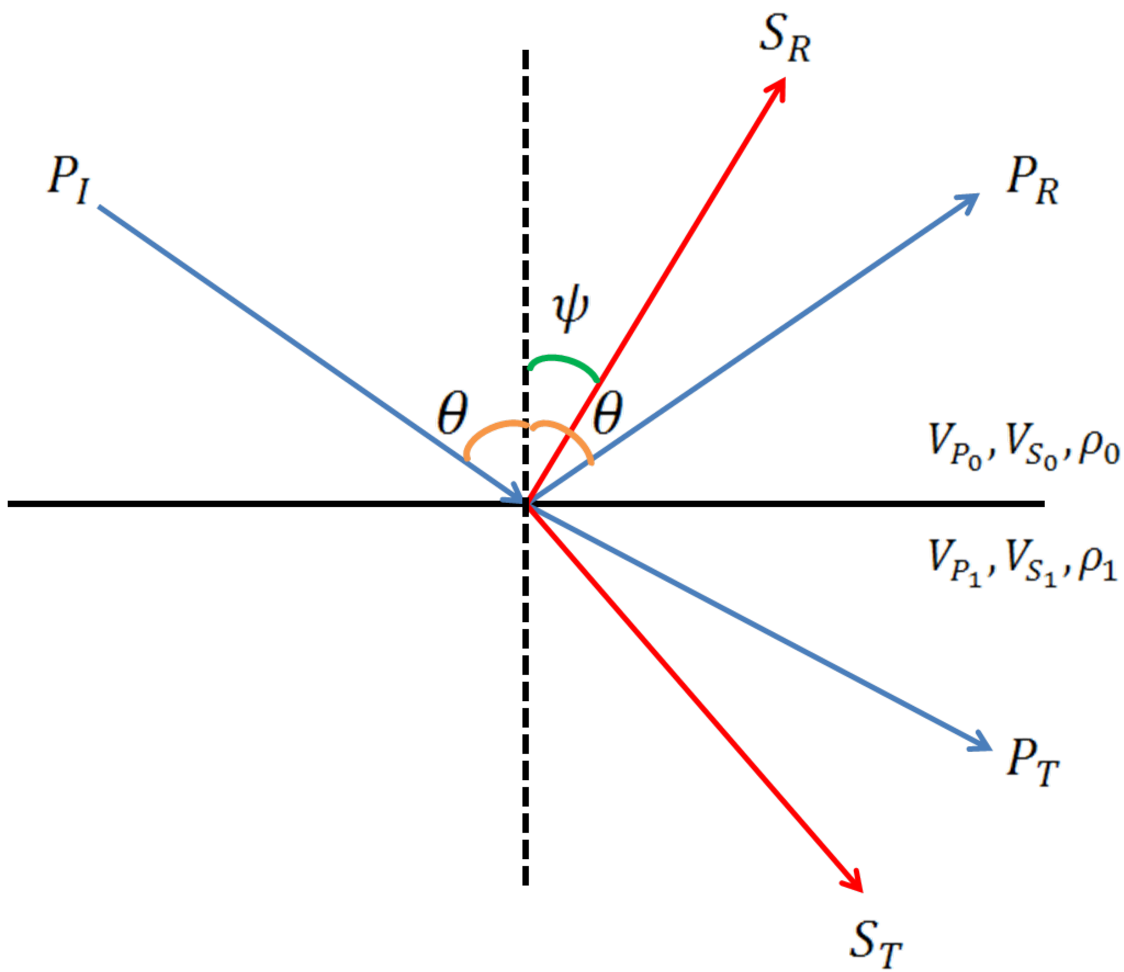
$$V_S = \sqrt{\frac{\mu_{\text{sat}}}{\rho_{\text{sat}}}}$$

$$\downarrow$$

$$(V_P)_{\text{sat}} = \sqrt{\frac{\lambda_{\text{dry}} + f + 2\mu_{\text{dry}}}{\rho_{\text{sat}}}} = \sqrt{\frac{K_{\text{dry}} + f + (4/3)\mu_{\text{dry}}}{\rho_{\text{sat}}}}$$

$$(V_S)_{\text{sat}} = \sqrt{\frac{\mu_{\text{dry}}}{\rho_{\text{sat}}}}$$

Incident P-wave model



<u>Exact</u>

- Zoeppritz

<u>Approx.</u>

- Aki and Richards
- Shuey
- Russell and Gray

Zoeppritz equations

- Keys (1989)

$$\begin{bmatrix} -X & -\sqrt{1-B^2X^2} & CX & -\sqrt{1-D^2X^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-C^2X^2} & DX \\ 2B^2X^2\sqrt{1-X^2} & B(1-2B^2X^2) & 2AD^2X\sqrt{1-C^2X^2} & -AD(1-2D^2X^2) \\ -(1-2B^2X^2) & 2B^2X\sqrt{1-B^2X^2} & AC(1-2D^2X^2) & 2AD^2X\sqrt{1-D^2X^2} \end{bmatrix} \begin{bmatrix} R_{\mathrm{PP}} \\ R_{\mathrm{PS}} \\ T_{\mathrm{PP}} \\ T_{\mathrm{PS}} \end{bmatrix} = \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2B^2X^2 \end{bmatrix}$$

$$A = \frac{\rho_1}{\rho_0} \quad B = \frac{V_{S_0}}{V_{P_0}} \quad C = \frac{V_{P_1}}{V_{P_0}} \quad D = \frac{V_{S_1}}{V_{P_0}} \quad X = \sin\theta$$

Linearized AVO

- Aki and Richards (2002)

$$R_{PP}^{(\mathrm{AR})}(\theta) \approx (1+\tan^2\theta)\frac{\Delta V_P}{2V_P} + \left(\frac{-8\sin^2\theta}{\gamma_{sat}^2}\right)\frac{\Delta V_S}{2V_S} + \left(1-\frac{4\sin^2\theta}{\gamma_{sat}^2}\right)\frac{\Delta\rho}{2\rho}$$

$$\gamma_{\mathrm{sat}}^2 = \left(\frac{V_{P_{\mathrm{avg}}}}{V_{S_{\mathrm{avg}}}}\right)_{\mathrm{sat}}^2 \qquad \gamma_{\mathrm{dry}}^2 = \left(\frac{V_{P_{\mathrm{avg}}}}{V_{S_{\mathrm{avg}}}}\right)_{\mathrm{dry}}^2$$

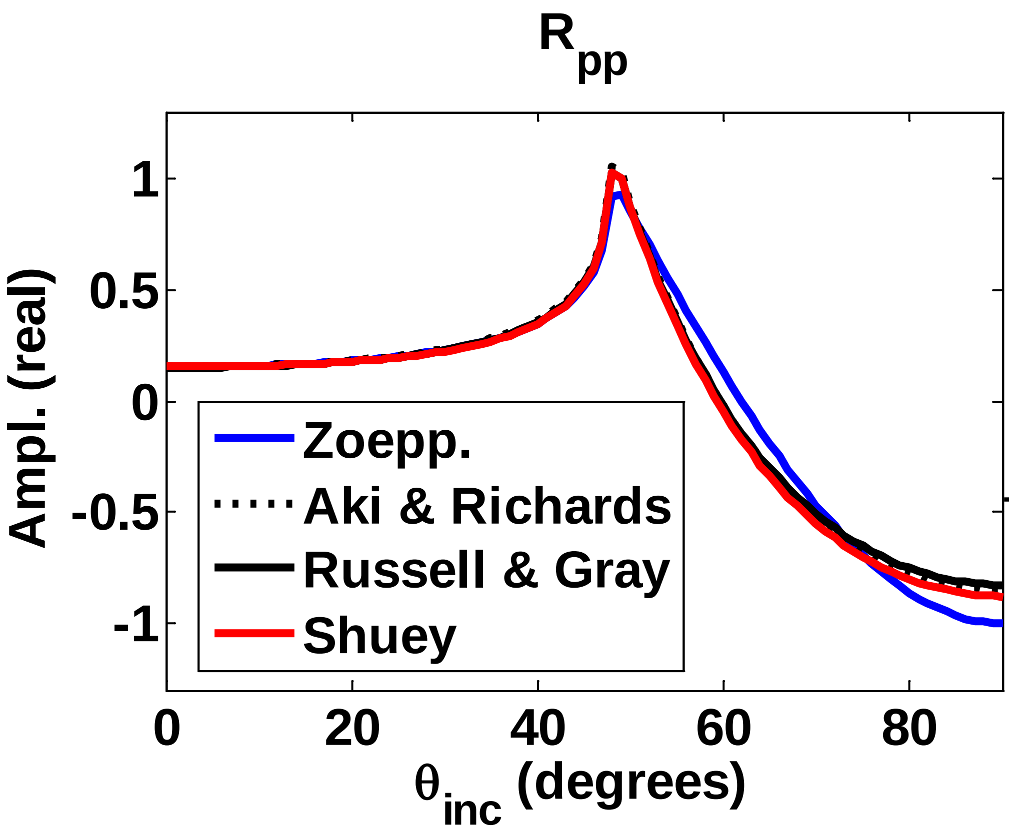
Linearized AVO and poroelasticity

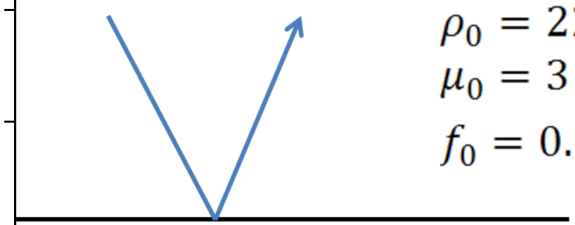
- Russell and Gray's AVO approximation (fluid-mu-rho equation)

$$R_{PP}^{(\mathrm{RG})}(\theta) \approx \left[\left(1-\frac{\gamma_{\mathrm{dry}}^2}{\gamma_{\mathrm{sat}}^2}\right)\frac{\sec^2\theta}{4}\right]\frac{\Delta f}{f}+\left[\frac{\gamma_{\mathrm{dry}}^2}{4\gamma_{\mathrm{sat}}^2}\sec^2\theta-\frac{2}{\gamma_{\mathrm{sat}}^2}\sin^2\theta\right]\frac{\Delta\mu}{\mu}+\left[\frac{1}{2}-\frac{\sec^2\theta}{4}\right]\frac{\Delta\rho}{\rho}$$

$$\gamma_{\mathrm{sat}}^2=\left(\frac{V_{P_{\mathrm{avg}}}}{V_{S_{\mathrm{avg}}}}\right)_{\mathrm{sat}}^2 \qquad \gamma_{\mathrm{dry}}^2=\left(\frac{V_{P_{\mathrm{avg}}}}{V_{S_{\mathrm{avg}}}}\right)_{\mathrm{dry}}^2$$

Example poroelastic AVO curve





$$\rho_0 = 2235\ kg/m^3$$
$$\mu_0 = 3\ GPa$$
$$f_0 = 0.659\ GPa$$

$$\rho_1 = 2263\ kg/m^3$$
$$\mu_1 = 3\ GPa$$
$$f_1 = 7.02\ GPa$$

Research objectives

- Adapt poroelastic reflection modeling into the exact elastic equations
- Produce series expansions of Rpp in orders of $\Delta f / f$, $\Delta\mu/\mu$, and $\Delta\rho/\rho$ (reflectivity series) and a_f, a_μ, a_ρ (perturbation series)
- Compare with the Russell and Gray Approximation
- Observe the role of nonlinear terms
  - Numerical accuracy
  - Geophysical interpretability

Poroelastic reflector modelling

- What is the poroelastic AVO problem?

$$a_f = 1 - \frac{f_0}{f_1} \qquad a_\mu = 1 - \frac{\mu_0}{\mu_1} \qquad a_\rho = 1 - \frac{\rho_0}{\rho_1}$$

$$A = \frac{\rho_1}{\rho_0} = \left(1 - a_\rho\right)^{-1} \qquad B = \frac{V_{S_0}}{V_{P_0}} = \gamma_{\text{sat}}^{-1}$$

$$C = \frac{V_{P_1}}{V_{P_0}} = \left\{\left(1 - a_\rho\right)\left[\left(\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\left(1 - a_\mu\right)^{-1} + \left(1 - \frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\left(1 - a_f\right)^{-1}\right]\right\}^{1/2}$$

$$D = \frac{V_{S_1}}{V_{P_0}} = \gamma_{\text{sat}}^{-1}\left[\left(1 - a_\mu\right)^{-1}\left(1 - a_\rho\right)\right]^{1/2}$$

Zoeppritz equations

- These elastic constants which now contain poroelastic elements can be replaced into the Zoeppritz equations
- Using Cramer's rule, we can solve for linear and non-linear expressions of Rpp or Rps

$$R_{PP}(\theta) = R_{PP}^{(1)}(\theta) + R_{PP}^{(2)}(\theta) + R_{PP}^{(3)}(\theta) + \ldots$$

$$R_{PS}(\theta) = R_{PS}^{(1)}(\theta) + R_{PS}^{(2)}(\theta) + R_{PS}^{(3)}(\theta) + \ldots$$

Linearized poroelastic AVO

$$R_{PP}^{(1)}(\theta)=\left[\left(1-\frac{\gamma_{\mathrm{dry}}^{2}}{\gamma_{\mathrm{sat}}^{2}}\right)\frac{1+\sin^{2}\theta}{4}\right]a_{f}+\left[\frac{\gamma_{\mathrm{dry}}^{2}}{4\gamma_{\mathrm{sat}}^{2}}\left(1+\sin^{2}\theta\right)-\frac{2}{\gamma_{\mathrm{sat}}^{2}}\sin^{2}\theta\right]a_{\mu}+\left[\frac{1}{2}-\frac{\left(1+\sin^{2}\theta\right)}{4}\right]a_{\rho}$$

Linearized poroelastic AVO

Derived from Zoeppritz

$$R_{PP}^{(1)}(\theta) = \left[\left(1-\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\frac{1+\sin^2\theta}{4}\right]a_f + \left[\frac{\gamma_{\text{dry}}^2}{4\gamma_{\text{sat}}^2}\left(1+\sin^2\theta\right) - \frac{2}{\gamma_{\text{sat}}^2}\sin^2\theta\right]a_\mu + \left[\frac{1}{2} - \frac{\left(1+\sin^2\theta\right)}{4}\right]a_\rho$$

Derived from Aki-Richards (Russell and Gray)

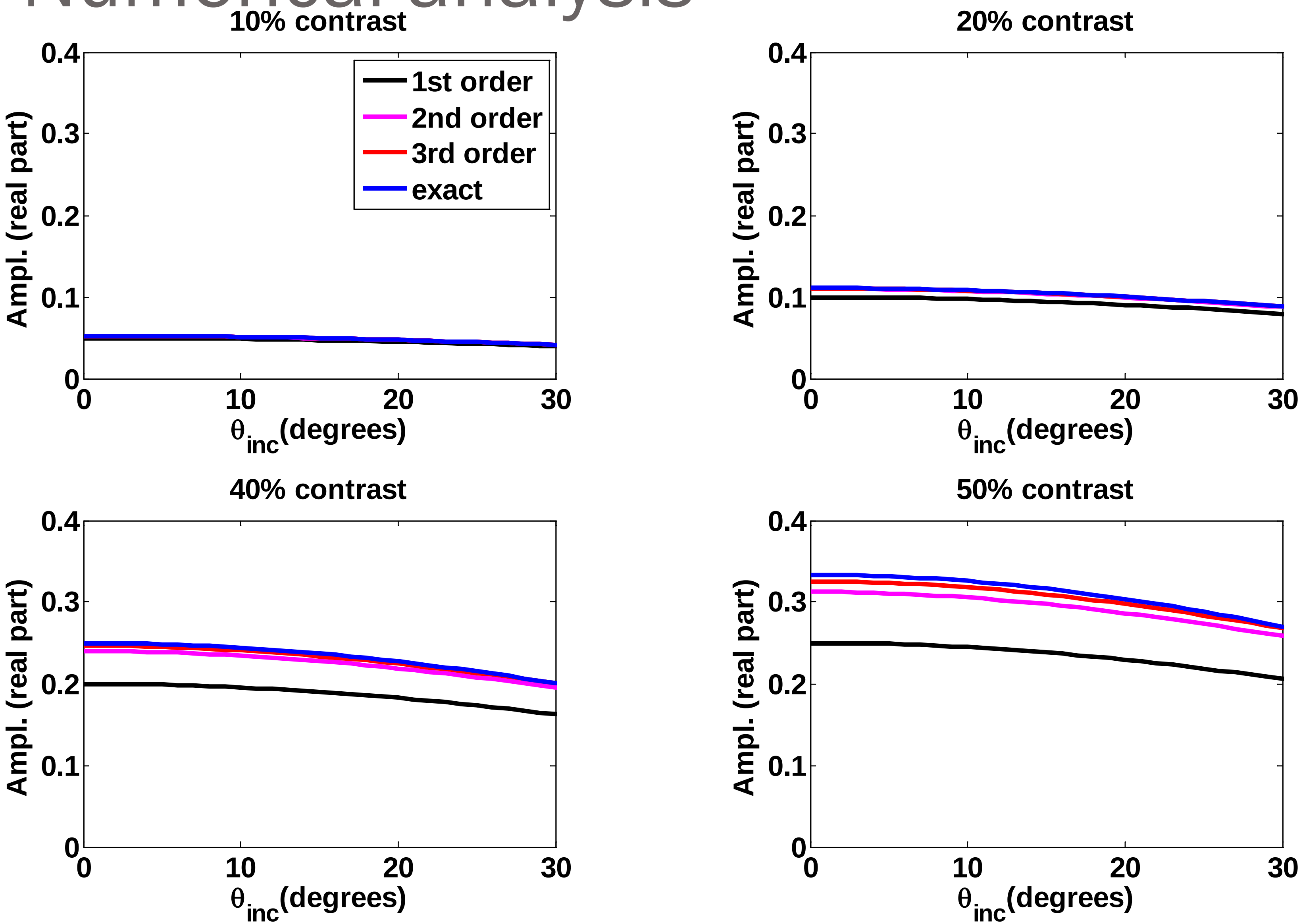
$$R_{PP}^{(\text{RG})}(\theta) \approx \left[\left(1-\frac{\gamma_{\text{dry}}^2}{\gamma_{\text{sat}}^2}\right)\frac{\sec^2\theta}{4}\right]\frac{\Delta f}{f} + \left[\frac{\gamma_{\text{dry}}^2}{4\gamma_{\text{sat}}^2}\sec^2\theta - \frac{2}{\gamma_{\text{sat}}^2}\sin^2\theta\right]\frac{\Delta\mu}{\mu} + \left[\frac{1}{2} - \frac{\sec^2\theta}{4}\right]\frac{\Delta\rho}{\rho}$$

Non-Linear poroelastic AVO

$$R^{(2)}(\theta) \approx W_{\Delta_1}\frac{\Delta f}{f} + W_{\Delta_2}\frac{\Delta\mu}{\mu} + W_{\Delta_3}\frac{\Delta\rho}{\rho} + W_{\Delta_4}\left(\frac{\Delta f}{f}\right)^2 + W_{\Delta_5}\left(\frac{\Delta\mu}{\mu}\right)^2 + W_{\Delta_6}\left(\frac{\Delta\rho}{\rho}\right)^2 + W_{\Delta_7}\frac{\Delta f}{f}\frac{\Delta\mu}{\mu} + W_{\Delta_8}\frac{\Delta f}{f}\frac{\Delta\rho}{\rho} + W_{\Delta_9}\frac{\Delta\mu}{\mu}\frac{\Delta\rho}{\rho}$$

$$R^{(3)}(\theta) \approx W_{\Delta_1}\frac{\Delta f}{f} + W_{\Delta_2}\frac{\Delta\mu}{\mu} + W_{\Delta_3}\frac{\Delta\rho}{\rho} + W_{\Delta_4}\left(\frac{\Delta f}{f}\right)^2 + W_{\Delta_5}\left(\frac{\Delta\mu}{\mu}\right)^2 + W_{\Delta_6}\left(\frac{\Delta\rho}{\rho}\right)^2 + W_{\Delta_7}\frac{\Delta f}{f}\frac{\Delta\mu}{\mu} + W_{\Delta_8}\frac{\Delta f}{f}\frac{\Delta\rho}{\rho} + W_{\Delta_9}\frac{\Delta\mu}{\mu}\frac{\Delta\rho}{\rho} + W_{\Delta_{10}}\left(\frac{\Delta f}{f}\right)^3 + W_{\Delta_{11}}\left(\frac{\Delta\mu}{\mu}\right)^3 + W_{\Delta_{12}}\left(\frac{\Delta\rho}{\rho}\right)^3 + W_{\Delta_{13}}\left(\frac{\Delta f}{f}\right)^2\frac{\Delta\mu}{\mu} + W_{\Delta_{14}}\left(\frac{\Delta f}{f}\right)^2\frac{\Delta\rho}{\rho} + W_{\Delta_{15}}\left(\frac{\Delta\mu}{\mu}\right)^2\frac{\Delta f}{f} + W_{\Delta_{16}}\left(\frac{\Delta\mu}{\mu}\right)^2\frac{\Delta\rho}{\rho} + W_{\Delta_{17}}\left(\frac{\Delta\rho}{\rho}\right)^2\frac{\Delta f}{f} + W_{\Delta_{18}}\left(\frac{\Delta\rho}{\rho}\right)^2\frac{\Delta\mu}{\mu} + W_{\Delta_{19}}\frac{\Delta f}{f}\frac{\Delta\mu}{\mu}\frac{\Delta\rho}{\rho}$$

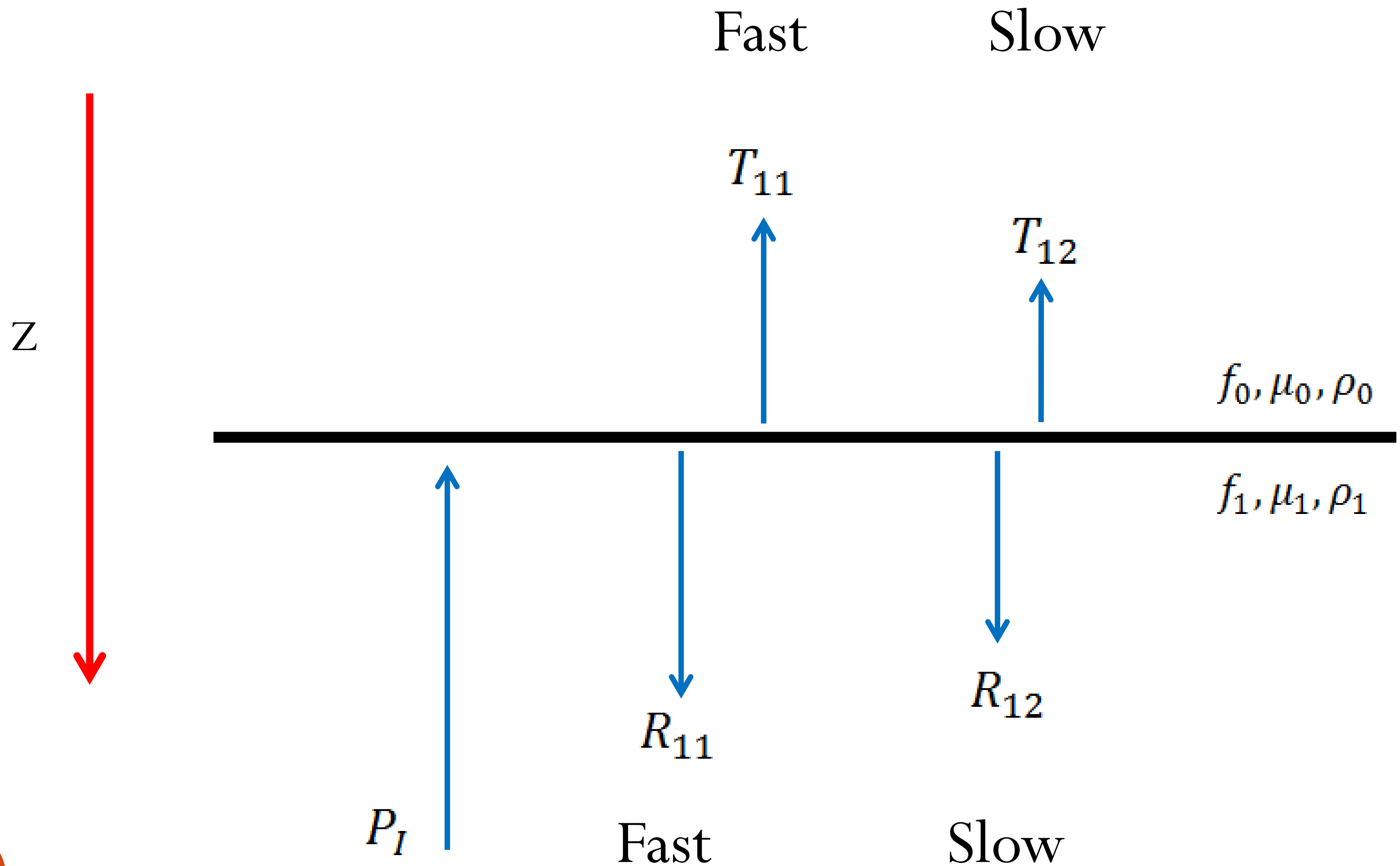
Numerical analysis



A different perspective

- Gurevich et al. (2002) studied fluid-saturated porous media for normal incidence reflection and transmission coefficients
  - Frequency dependent
  - Show relative amplitude displacement for fast and slow P-waves
  - Bound to Biot's critical frequency (experiments below 0.1Mhz)
  - For effective elastic conditions
- Objectives
  - Express in terms similar to Russell et al. (2011)
  - Differences/Similarities (Russell-Gray and Gurevich)
  - Expand and analyze

A different perspective



A different perspective

- For a fast P-wave…

$$R_{11}(\omega) = \frac{\rho_1 v_1 - (1 - X)\rho_0 v_0}{\rho_1 v_1 + (1 + X)\rho_0 v_0}$$

A different perspective

$$R_{11}(\omega)=\frac{\rho_1 v_1-\left(1-\dfrac{\left(K_{1\mathrm{dry}}+\frac{4}{3}\mu_1+f_1\right)(k_1)_{\mathrm{fast}}\left(\dfrac{\alpha_0 M_0}{K_{0\mathrm{dry}}+\frac{4}{3}\mu_0+f_0}-\dfrac{\alpha_1 M_1}{K_{1\mathrm{dry}}+\frac{4}{3}\mu_1+f_1}\right)^2}{\sqrt{\dfrac{i\omega\eta_0 N_0}{\kappa_0}}+\sqrt{\dfrac{i\omega\eta_1 N_1}{\kappa_1}}}\right)\rho_0 v_0}{\rho_1 v_1+\left(1+\dfrac{\left(K_{1\mathrm{dry}}+\frac{4}{3}\mu_1+f_1\right)(k_1)_{\mathrm{fast}}\left(\dfrac{\alpha_0 M_0}{K_{0\mathrm{dry}}+\frac{4}{3}\mu_0+f_0}-\dfrac{\alpha_1 M_1}{K_{1\mathrm{dry}}+\frac{4}{3}\mu_1+f_1}\right)^2}{\sqrt{\dfrac{i\omega\eta_0 N_0}{\kappa_0}}+\sqrt{\dfrac{i\omega\eta_1 N_1}{\kappa_1}}}\right)\rho_0 v_0}$$

Ongoing research

- Linear and non-linear inversion
- Seeking suitable field data for testing
- Incorporate into SYNGRAM as synthetic tool

Acknowledgements

- University Research Award provided by Imperial Oil Limited
- CREWES Sponsors
- CREWES faculty, staff, and students

Questions?