The roles of baseline Earth elastic properties and time-lapse changes in determining difference AVO

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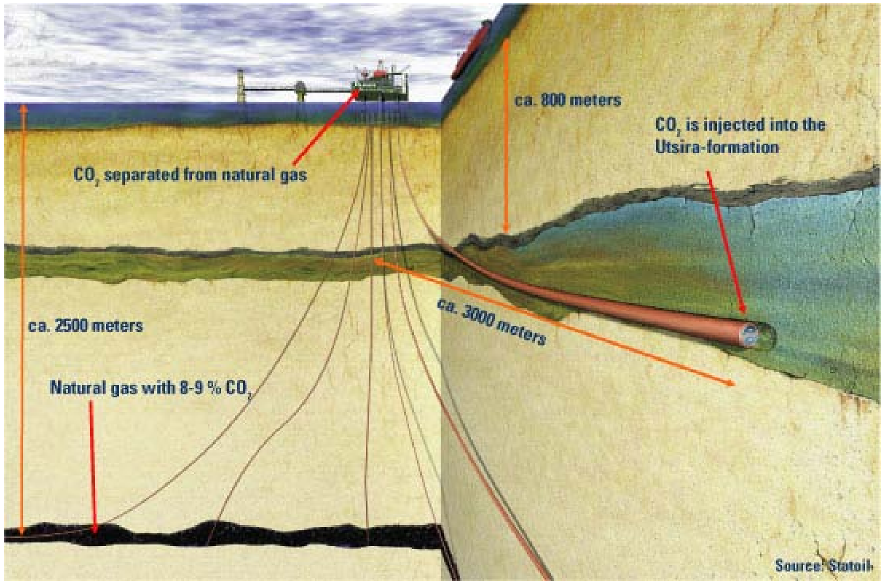
Outline

- Introduction and review
- A framework for time-lapse AVO
- Validation of results with a physical model
- Summary
- Conclusions
- Future work
- Acknowledgments

Time-lapse

- Monitoring changes in reservoir: production, EOR
- Repeated seismic surveys over calendar time
- The baseline and monitor survey
- Changes in seismic parameters



CO_2 storage in the Sleipner gas field (image courtesy of StatoilHydro)

AVO : Amplitude Versus Offset

Baseline and time-lapse changes

Baseline

$$\Delta V_{Pb} = V_{Pb} - V_{P_0}$$

$$\Delta V_{Sb} = V_{Sb} - V_{S_0}$$

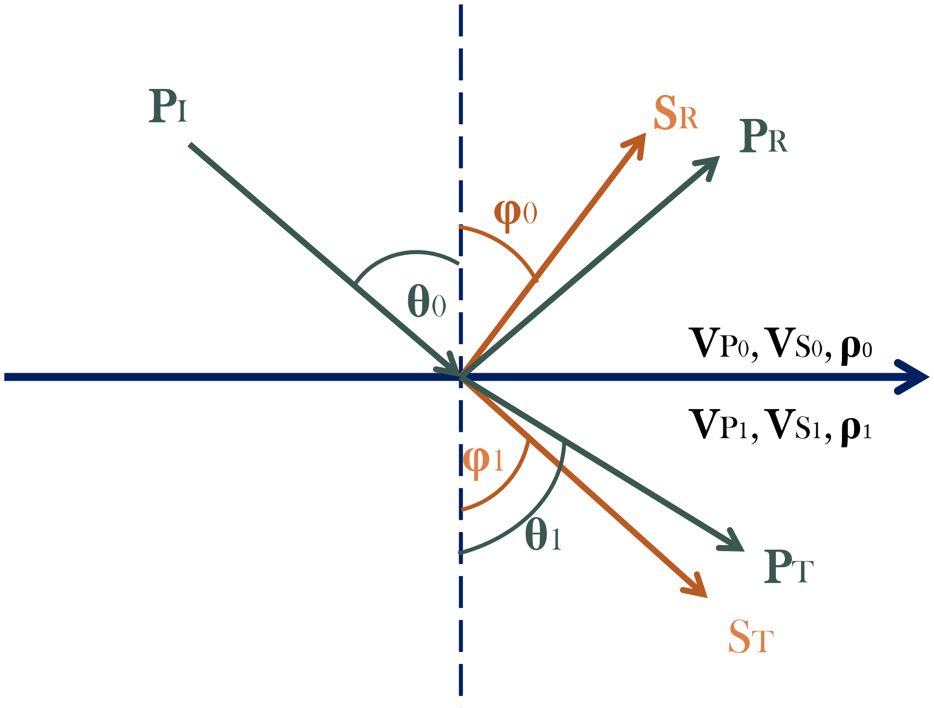
$$\Delta \rho_b = \rho_b - \rho_0$$

Time lapse

$$\delta V_P = V_{Pm} - V_{P_b}$$

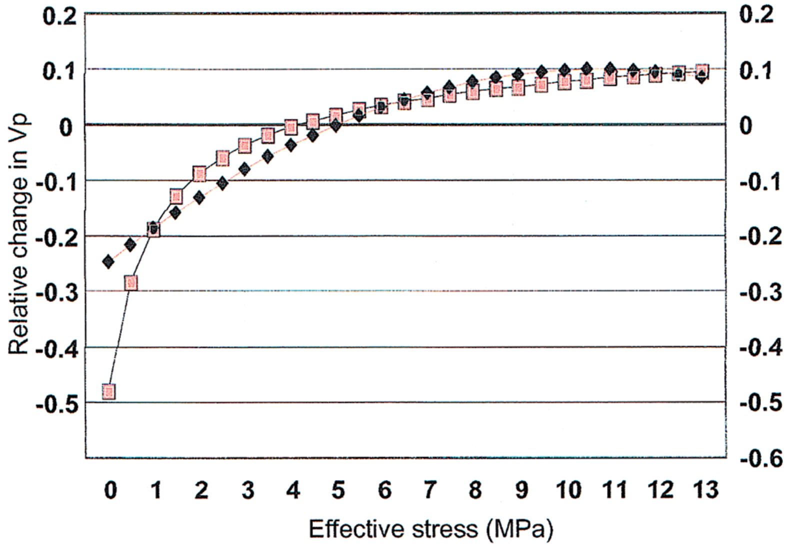
$$\delta V_S = V_{Sm} - V_{S_b}$$

$$\delta \rho = \rho_m - \rho_b$$



Time lapse changes (Landrø, 2001)

$$\Delta R_{PP}(\theta) \approx \frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)-2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho}+2\frac{\delta V_S}{V_S}\right)\sin^2\theta+\frac{\delta V_P}{2V_P}\tan^2\theta$$



Relative change in V_P versus pressure changes (Landrø et al., 2001)

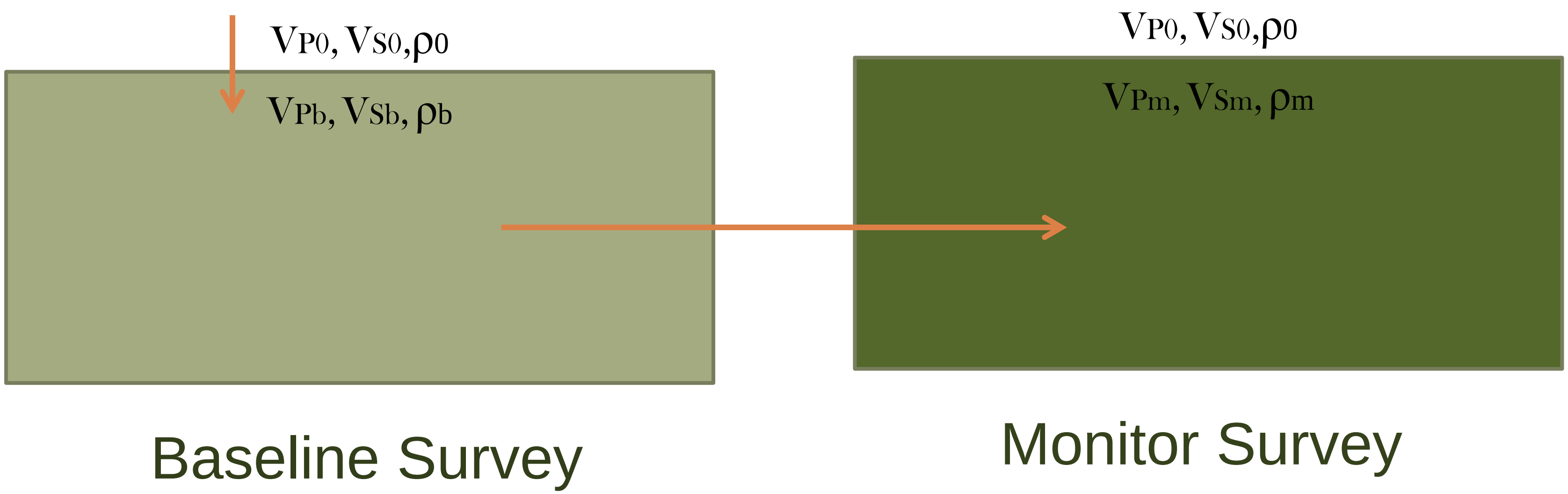
- Large time lapse changes in V_P are possible
- Linearized equation is inaccurate for large contrast

A general framework for time-lapse AVO

- Deriving $\Delta R_{PP}(\theta)$ from Zoeppritz equations
  - Linear or Aki-Richards approximation
  - Nonlinear correction

- Examine linear and nonlinear terms for:
  - Agreement with Landrø at small contrast
  - Behaviour of approximations at large contrast

A time-lapse problem



Baseline perturbation $\frac{\Delta V_P}{V_P} = 2\frac{V_{Pb} - V_{P0}}{V_{Pb} + V_{P0}}, \quad \frac{\Delta V_S}{V_S} = 2\frac{V_{Sb} - V_{S0}}{V_{Sb} + V_{S0}}, \quad \frac{\Delta \rho}{\rho} = 2\frac{\rho_b - \rho_0}{\rho_b + \rho_0}$

Time-lapse perturbation $\frac{\delta V_P}{V_P} = 2\frac{V_{Pm} - V_{Pb}}{V_{Pm} + V_{Pb}}, \quad \frac{\delta V_S}{V_S} = 2\frac{V_{Sm} - V_{Sb}}{V_{Sm} + V_{Sb}}, \quad \frac{\delta \rho}{\rho} = 2\frac{\rho_m - \rho_b}{\rho_m + \rho_b}$

Time-lapse AVO

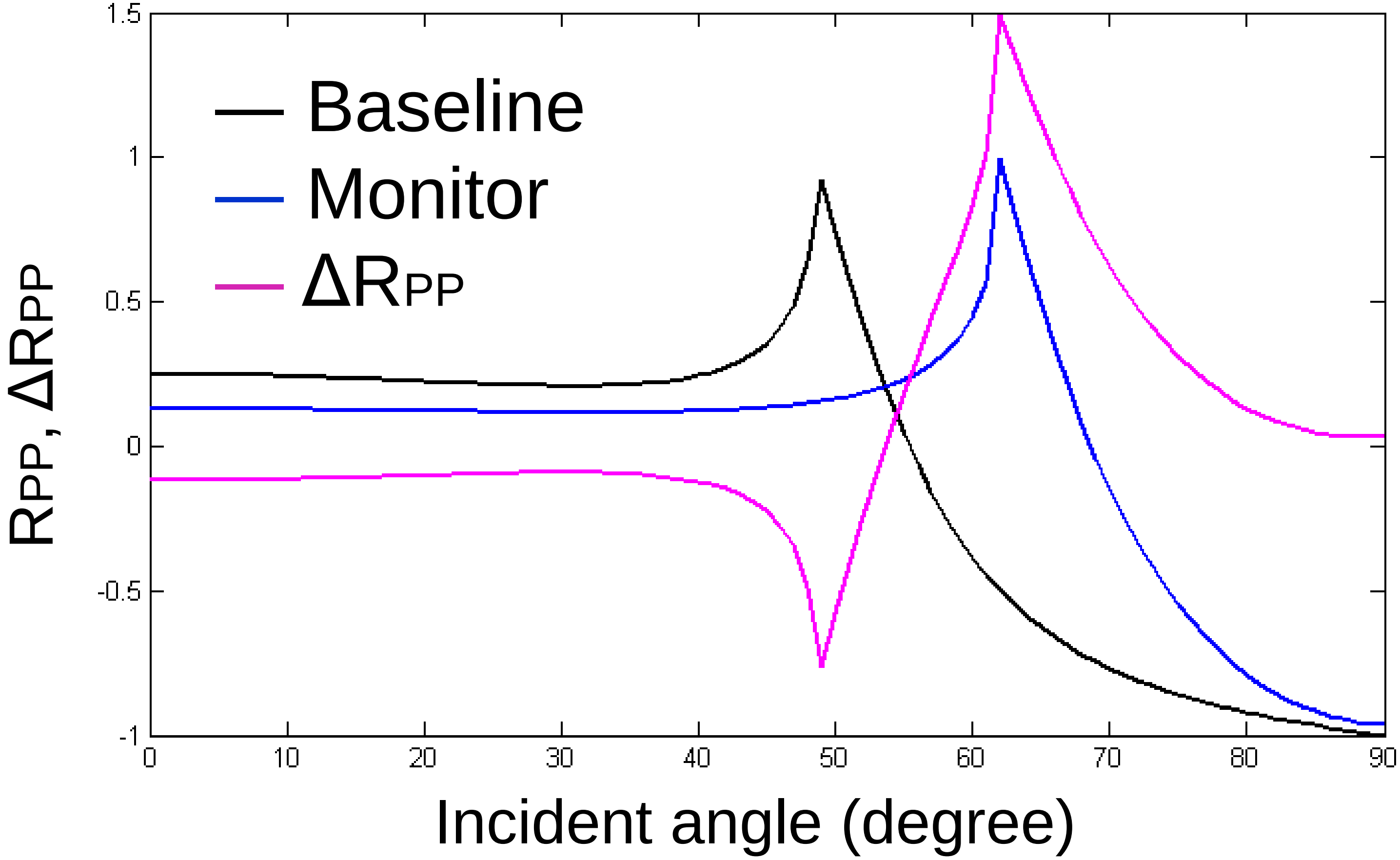
- Zoeppritz equations for baseline and monitoring targets:

$$P_{BL}\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_{BL} \qquad R_{PP}^{BL}(\theta) = \frac{\det(P_P)}{\det(P)} \qquad P_M\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_M \qquad R_{PP}^{M}(\theta) = \frac{\det(P_P)}{\det(P)}$$

- Expand $\Delta R_{PP}(\theta)$ in orders of perturbation parameters

$$\Delta R_{PP}(\theta) = R_{PP}^{M}(\theta) - R_{PP}^{BL}(\theta)$$

R_{PP} for the Baseline and Monitor survey and ΔR_{PP}



More details in modeled data (Jabbari and Innanen, 2012)

Examining linear and nonlinear terms

$$\Delta R_{PP}(\theta) = \Delta R_{PP}^{(1)}(\theta) + \Delta R_{PP}^{(2)}(\theta) + \Delta R_{PP}^{(3)}(\theta) + \ldots$$

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{2}\left(\frac{\delta\rho}{\rho} + \frac{\delta V_P}{V_P}\right) - 2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho} + 2\frac{\delta V_S}{V_S}\right)\sin^2\theta + \frac{\delta V_P}{2V_P}\sin^2\theta$$

$$\Delta R_{PP}^{(2)}(\theta) = \Gamma_{\delta VP}\left(\frac{\delta V_P}{V_P}\right)^2 + \Gamma_{\delta VS}\left(\frac{\delta V_S}{V_S}\right)^2 + \Gamma_{\delta\rho}\left(\frac{\delta\rho}{\rho}\right)^2 + \Gamma_{\delta\rho\delta VS}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)$$
$$+\Gamma_{\delta VP\Delta VP}\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta V_P}{V_P}\right) + \Gamma_{\delta\rho\Delta VS}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta V_S}{V_S}\right) + \Gamma_{\delta\rho\Delta\rho}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta\rho}{\rho}\right)$$
$$+\Gamma_{\delta VS\Delta\rho}\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right) + \Gamma_{\delta VS\Delta VS}\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta V_S}{V_S}\right)$$

Agreement of linear term in ΔR_{PP} with Landrø's work

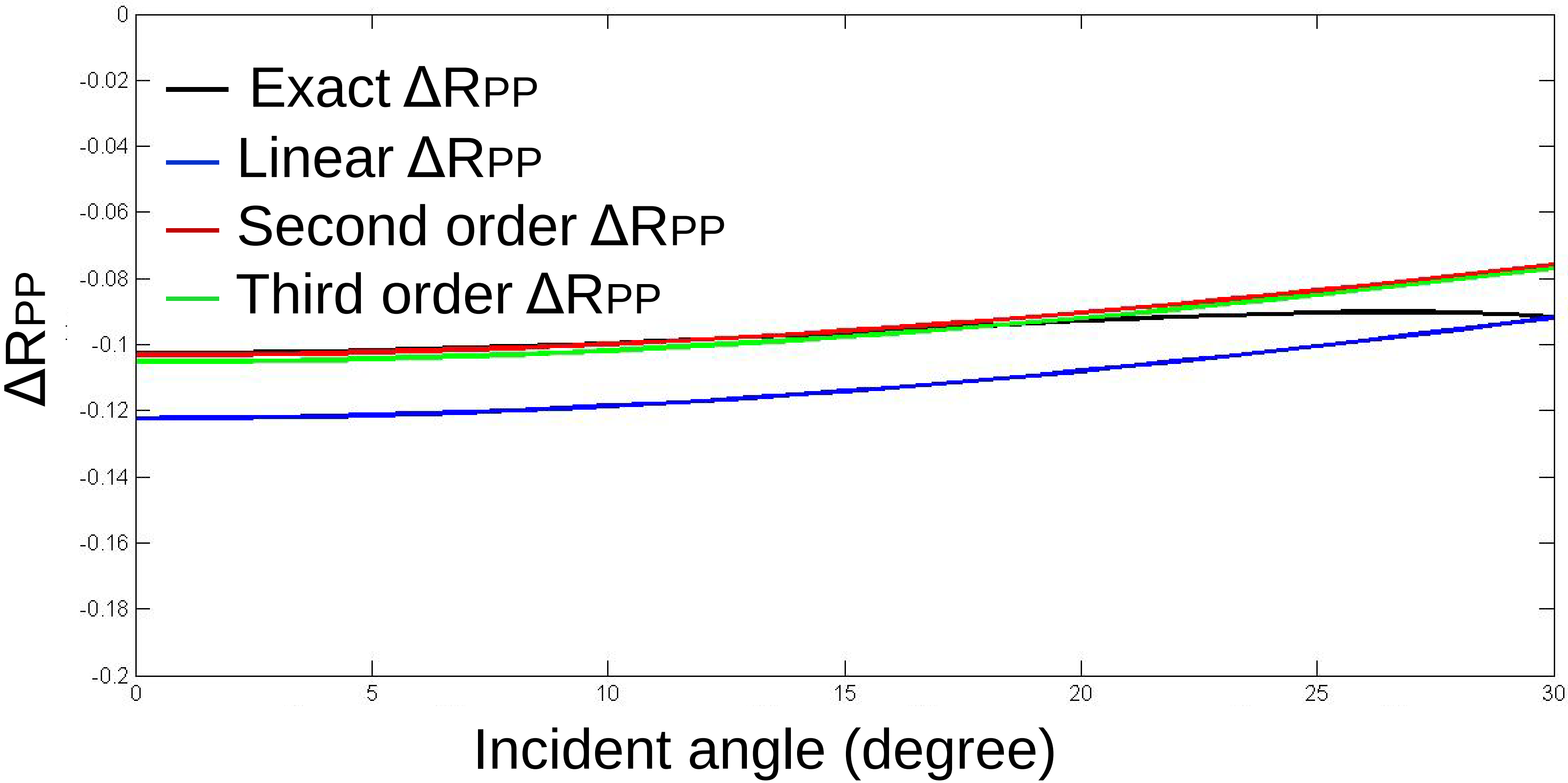
- Our first order term

$$\Delta R_{PP}^{(1)}(\theta)=\frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)-2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho}+2\frac{\delta V_S}{V_S}\right)\sin^2\theta+\frac{\delta V_P}{2V_P}\sin^2\theta$$

- Landrø's approximation

$$\Delta R_{PP}(\theta)\approx\frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)-2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho}+2\frac{\delta V_S}{V_S}\right)\sin^2\theta+\frac{\delta V_P}{2V_P}\tan^2\theta$$

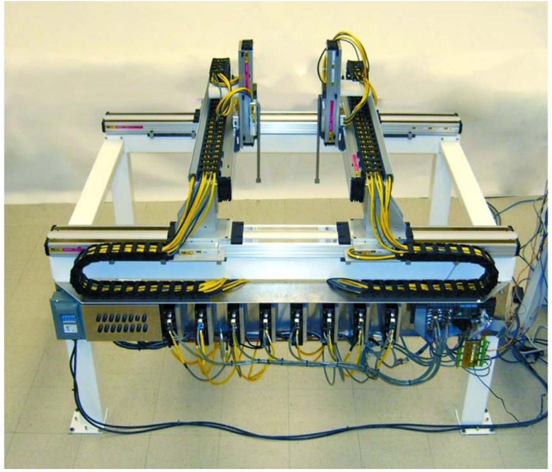
ΔR_{PP} for the exact, linear, second and third order approximation

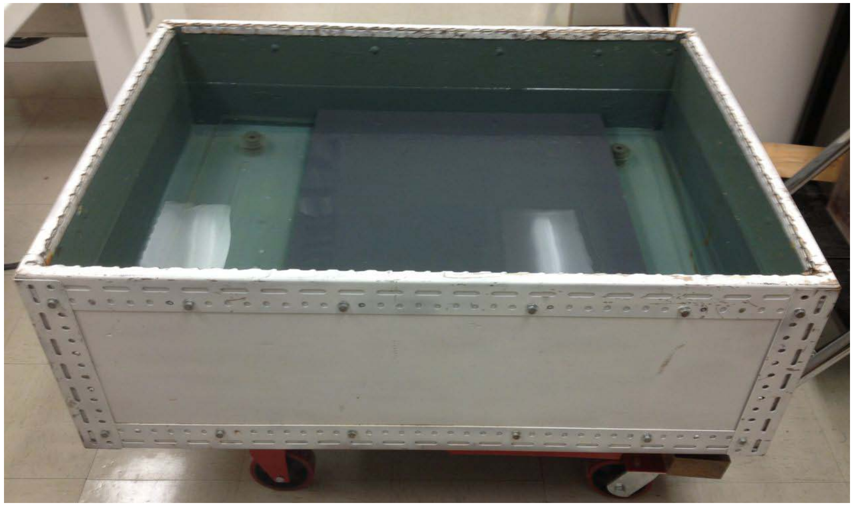


More details in modeled data (Jabbari and Innanen, 2012)

Physical modelling

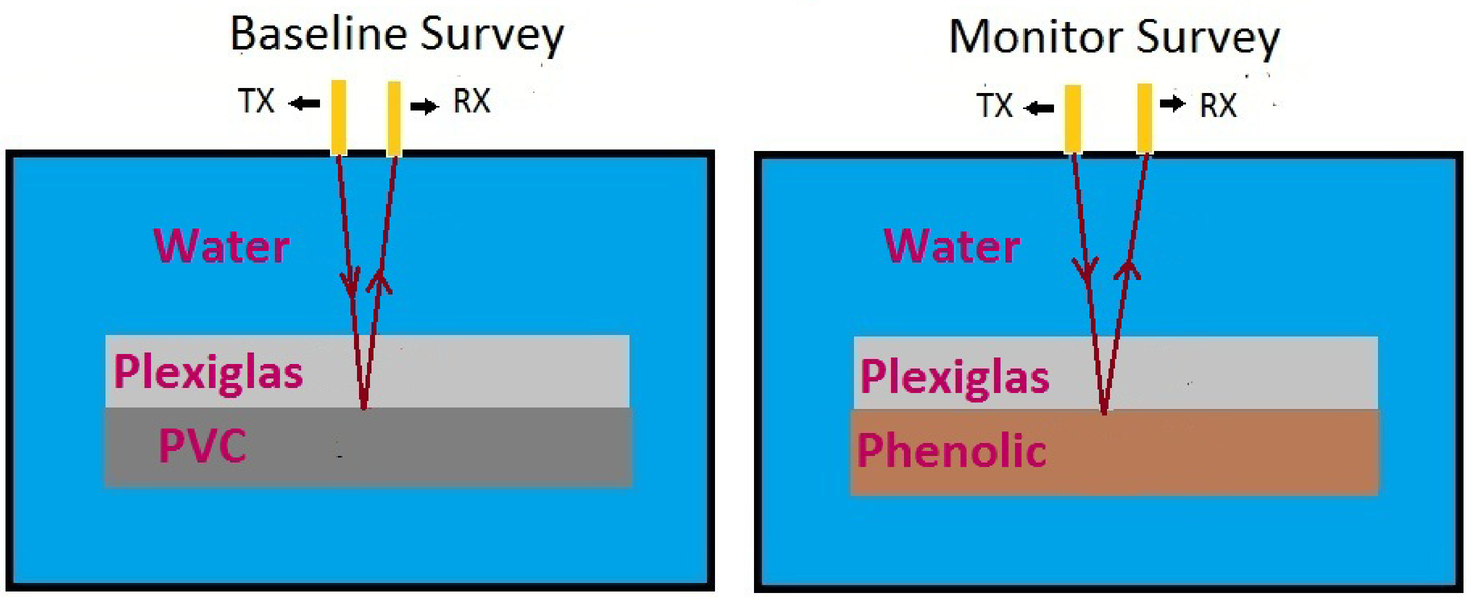
- Array of transducers for sources and detectors
- The model contains water and different material blocks
- Common midpoint (CMP) gathers are recorded on SEG-Y files
- Scaling factor of 10,000:
  - **1 mm** $\longrightarrow$ **10 m**
  - **1 MHz** $\longrightarrow$ **100 Hz**





The University of Calgary Seismic Physical Modelling Facility (J. Wong and Lawton, 2009)

Modeling a time-lapse problem

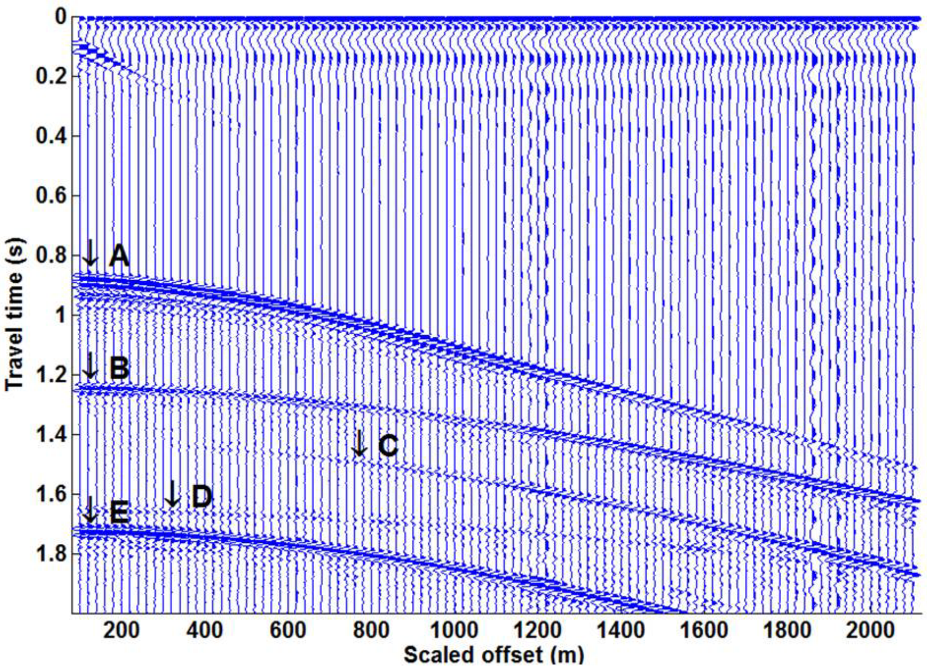


More details in modeled data (Jabbari and Innanen, 2012)

CMP (common mid point) gather

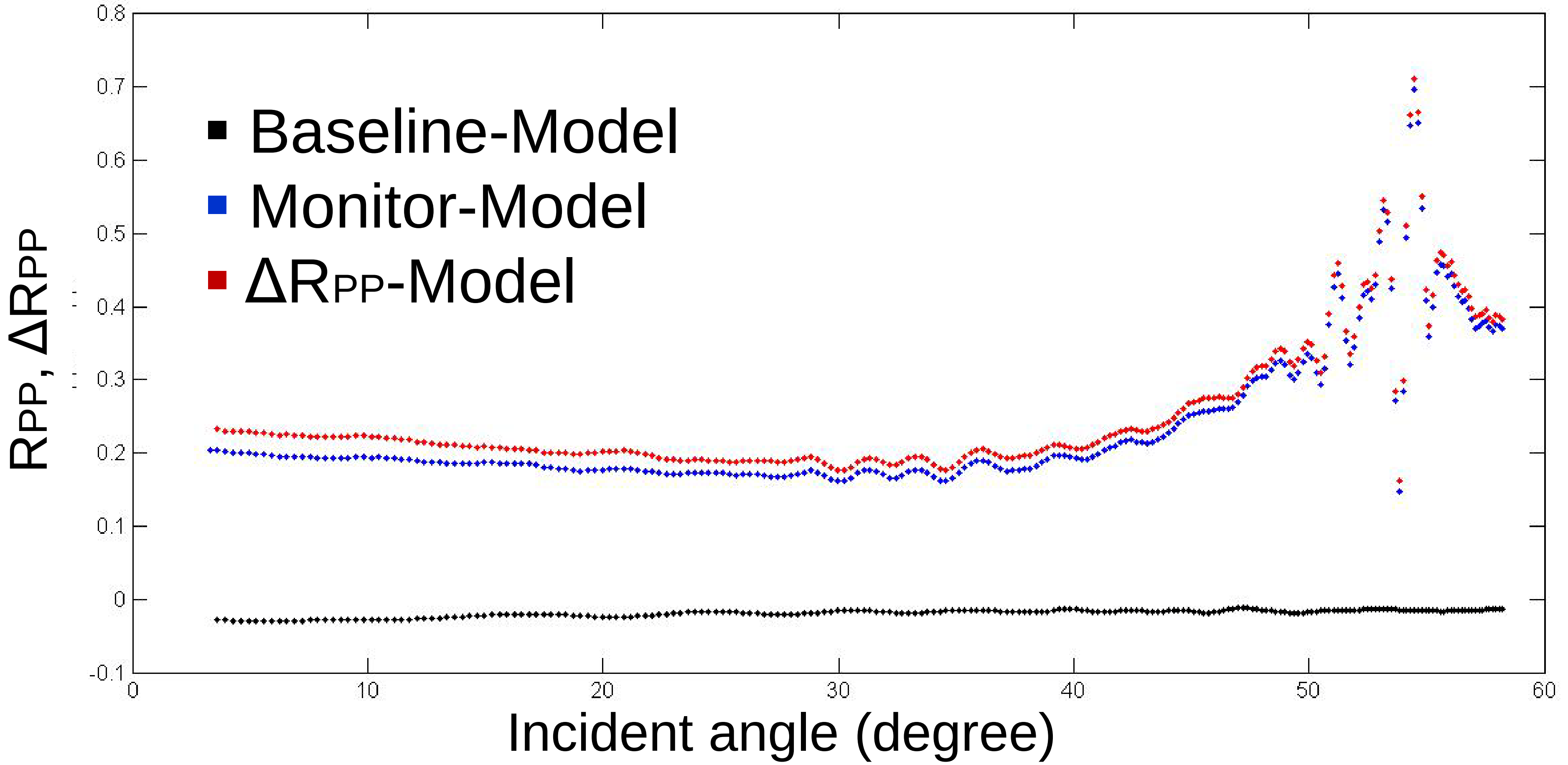
Corrections (Mahmoudian et al. 2012)

- Geometrical spreading
- Emergence angle
- Free surface
- Transmission loss
- Source/receiver directivity



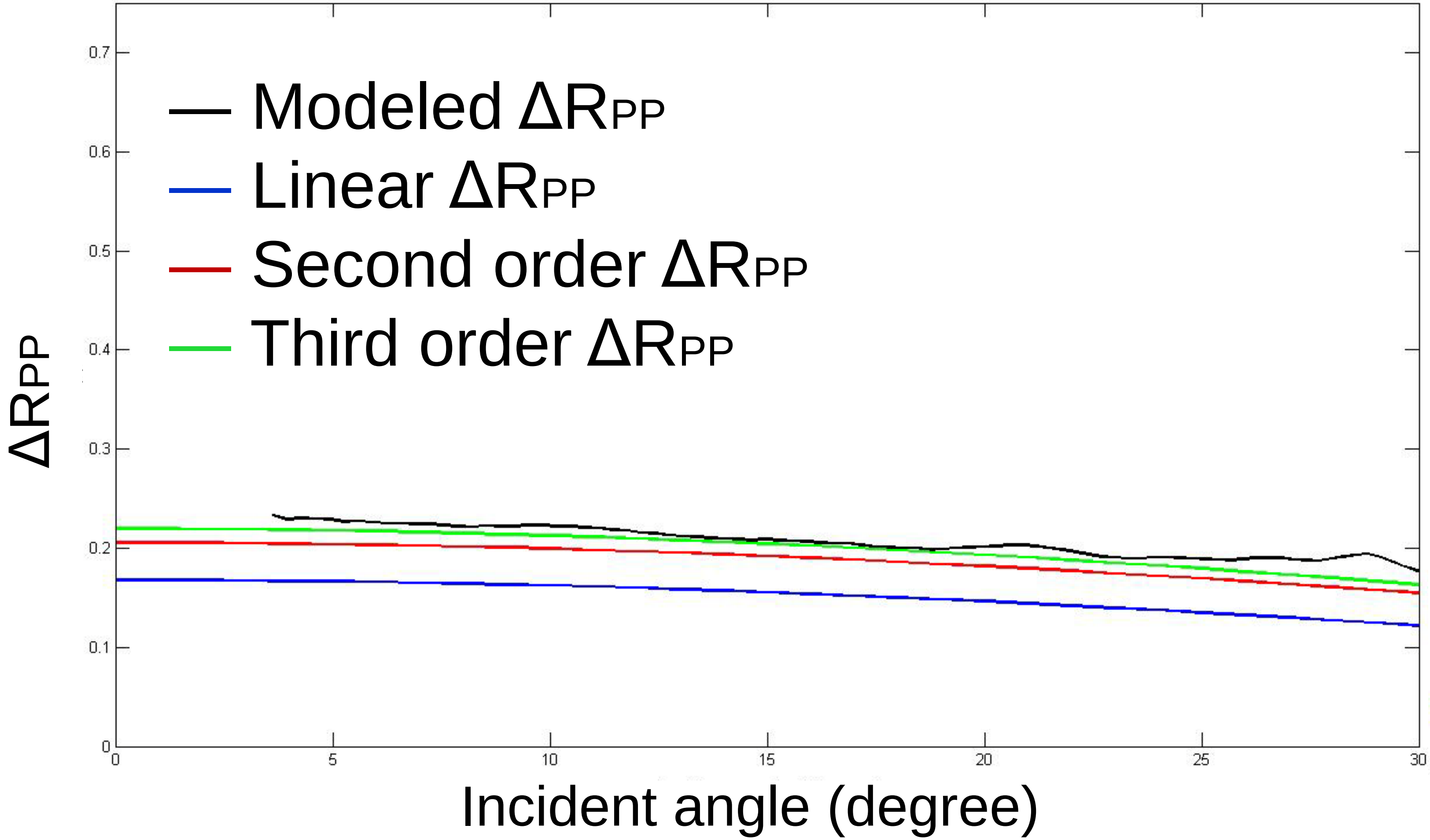
CMP gather along the plexiglas- phenolic interface in monitor survey model

Time-lapse difference data in the physical model



More details in modeled data (Jabbari and Innanen, 2012)

ΔRPP for the model, linear, second and third order approximation



More details in modeled data (Jabbari and Innanen, 2012)

Summary

- A framework for linear and non linear time-lapse AVO analysis is formulated.
- Agreement of linear term in ΔR_{PP} with Landrø's work.
- Higher order approximations made corrections in ΔR_{PP}.
- Physical model validated the importance of low order interpretable nonlinear corrections.

Conclusions

- In plausible large contrast time-lapse scenarios Landrø's approximation requires correction.
- Physical modeling study validates nonlinear framework with real data in controlled settings.

Future work

- Further numerical, analytical examination of ΔR_{PP}, ΔR_{PS}, ΔR_{SS}
- Validation of time-lapse AVO formula using physical modeling data
- Modeling of inversion of field data example

Acknowledgments

- Dr Joe Wong
- Faranak Mahmoudian
- CREWES Students and Staffs
- CREWES Sponsors

Questions

More details of the time-lapse model

Material	V_P (m/sec)	V_S (m/sec)	ρ(g/cm^3)
Water	1480	-	1.00
Pexiglas	2745	1380	1.19
PVC	2370	1122	1.13
Phenolic	3500	1700	1.39

HTI Medium

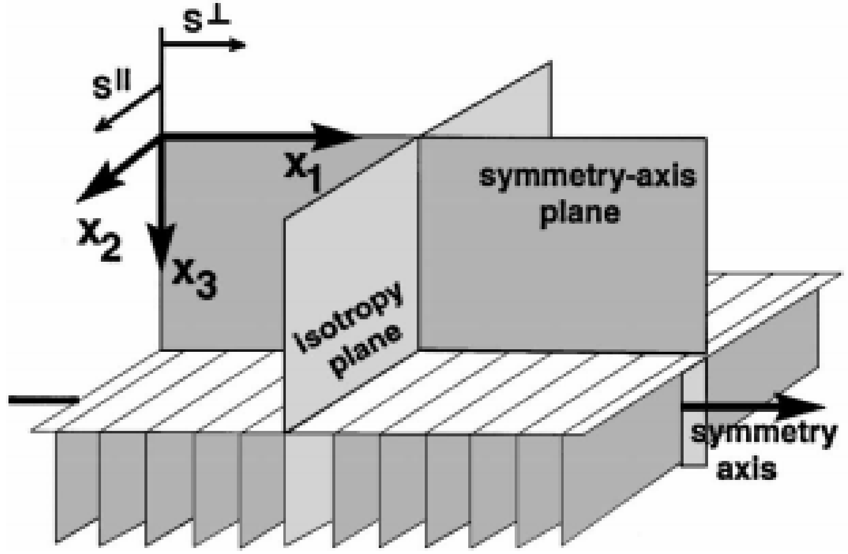


FIG. 1. Sketch of the transversely isotropic model with a horizontal symmetry axis caused by a system of parallel vertical cracks. HTI media contain two vertical planes of mirror symmetry defined by the crack orientation (after Rüger, 1997).

Ghost Raypath (Mahmoudian et al. 2012)

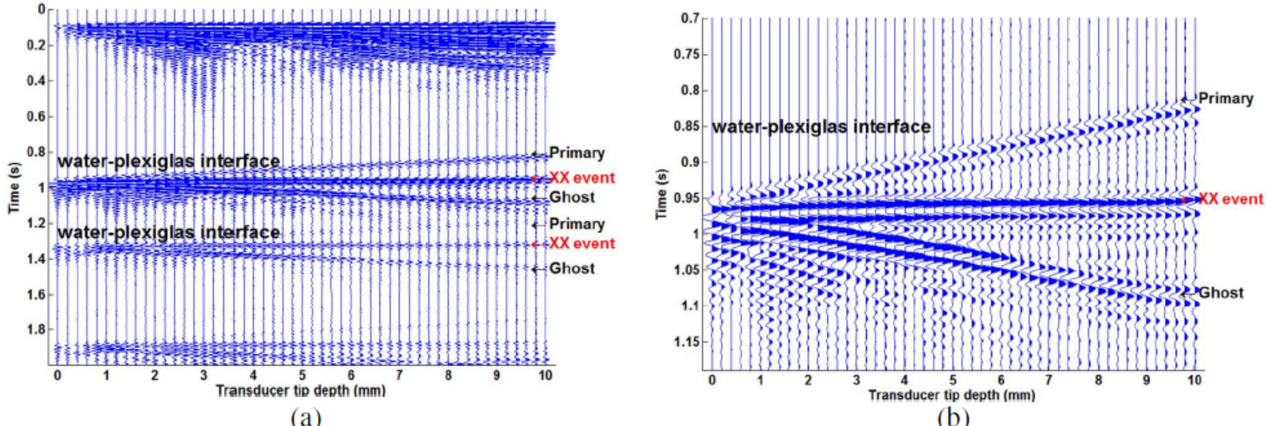


FIG. 5. (a) Data acquired at a single source-receiver offset of 10mm with different transducer depths in water. b) Expanded time scale to show detail of the reflections from the top of the plexiglas.

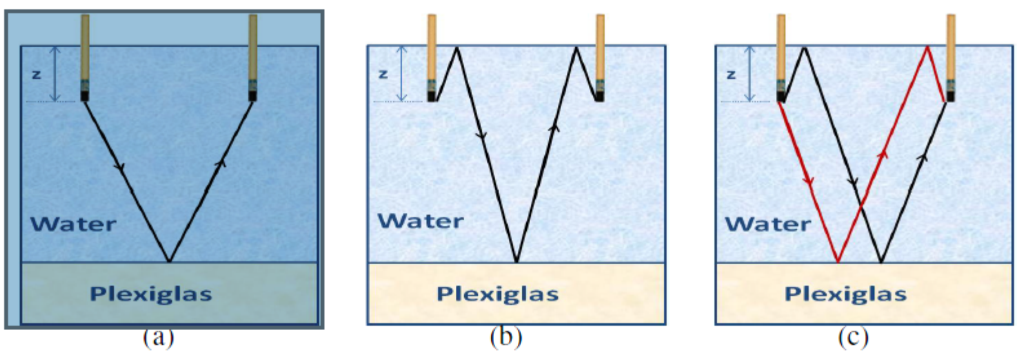


FIG. 6. (a) Primary raypath. (b) Ghost raypath. (c) Asymmetric raypaths, two single-leg source and receiver ghosts, identified as "XX-event" in Figure 5.

Source/receiver directivity (Mahmoudian et al. 2012)

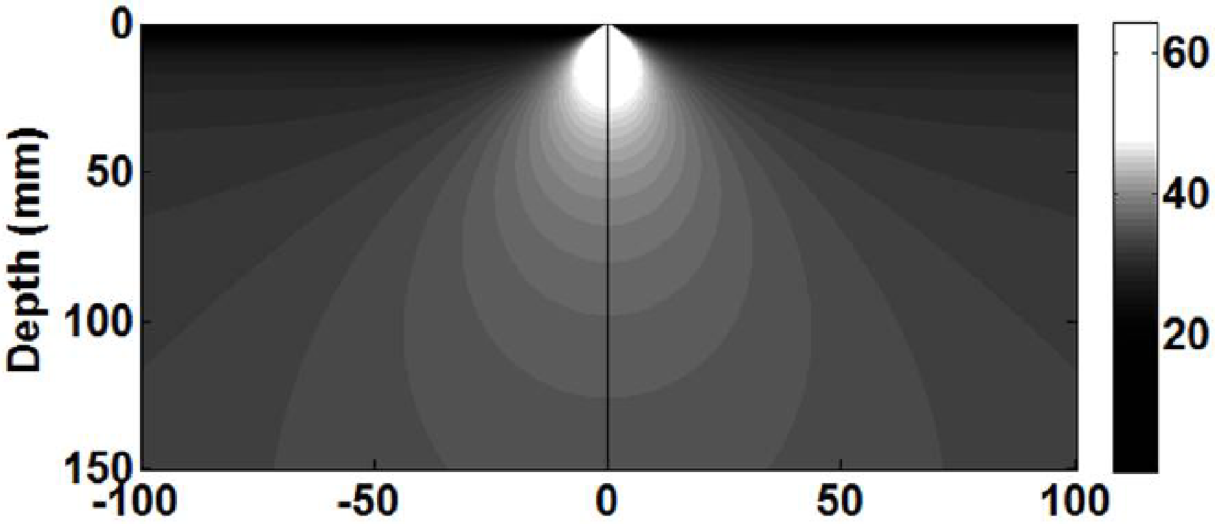


FIG. 8. The calculated pressure field for a circular transducer of a diameter of 12mm as a function of depth and angle for a frequency of 200 kHZ (after Buddensiek et al. (2009)).

Zoeppritz matrix- Elastic parameters

$$P\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_P \qquad R_{PP}(\theta_0) = \frac{\det(P_P)}{\det(P)} \qquad b_P \equiv \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2(BX)^2 \end{bmatrix}$$

$$X = Sin(\theta_0),\quad A \equiv \frac{\rho_1}{\rho_0},\quad B \equiv \frac{V_{S_0}}{V_{P_0}},\quad C \equiv \frac{V_{P_1}}{V_{P_0}},\quad D \equiv \frac{V_{S_1}}{V_{P_0}},\quad E \equiv \frac{V_{P_1}}{V_{S_0}},\quad F \equiv \frac{V_{S_1}}{V_{S_0}}.$$

$$P \equiv \begin{bmatrix} -X & -\sqrt{1-(BX)^2} & CX & \sqrt{1-(DX)^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-(CX)^2} & -DX \\ 2B^2X\sqrt{1-X^2} & B\left(1-2(BX)^2\right) & 2AD^2X\sqrt{1-(CX)^2} & AD\left(1-2(DX)^2\right) \\ -\left(1-2(BX)^2\right) & 2B^2X\sqrt{1-(BX)^2} & AC\left(1-2(DX)^2\right) & -2AD^2X\sqrt{1-(DX)^2} \end{bmatrix}$$

Second order correction (Jabbari and Innanen, 2012)

$$\Gamma_{\delta V_P} = \frac{1}{2} + \sin^2(\theta_0)$$

$$\Gamma_{\delta V_S} = 4\left(\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - 2\left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right)$$

$$\Gamma_{\delta\rho} = \left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0) - \frac{1}{4}\left(\frac{V_{S_0}}{V_{P_0}}\right) \sin^2(\theta_0) + \frac{1}{4}$$

$$\Gamma_{\delta\rho\delta V_S} = 2\left(2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right)$$

$$\Gamma_{\Delta V_S \delta V_S} = 8\left(\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right)$$

$$\Gamma_{\Delta V_P \delta V_P} = \sin^2(\theta_0)$$

$$\Gamma_{\Delta\rho\delta V_S} = 2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)$$

$$\Gamma_{\Delta V_S \delta\rho} = 2\left(2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \left(\frac{V_{S_0}}{V_{P_0}}\right)^2 \sin^2(\theta_0)\right)$$

$$\Gamma_{\Delta\rho\delta\rho} = 2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2(\theta_0) - \frac{1}{2}\left(\frac{V_{S_0}}{V_{P_0}}\right) \sin^2(\theta_0)$$

Third order correction (Jabbari and Innanen, 2012)

$$
\begin{aligned}
\Delta R^{(3)}_{PP}(\theta_0) ={}& 8\left(\frac{15}{64}X^2+\frac{5}{64}\right)\left(\frac{\delta V_P}{V_P}\right)^3 + 8\left(\frac{7}{4}B^3X^2-2B^2X^2\right)\left(\frac{\delta V_S}{V_S}\right)^3 \\
&+\left(\frac{1}{2}B^3X^2-\frac{3}{8}BX^2+\frac{1}{8}\right)\left(\frac{\delta\rho}{\rho}\right)^3 + 4\left(\frac{1}{2}B^2X^2-B^3X^2\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right) \\
&+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)+ \\
&\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)]+ \\
&4\left(2B^3X^2-\frac{1}{2}B^2X^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&+4\left(\frac{1}{4}B^2X^2-\frac{1}{8}X^2-\frac{1}{16}\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&+2\left(B^2X^2-B^3X^2-\frac{1}{4}BX^2-\frac{1}{8}X^2-\frac{1}{8}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right) \\
&-8\left(B^3X^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&+2\left(B^2X^2\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)] \\
&+2\left(\frac{3}{2}B^2X^2-\frac{1}{2}B^3X^2-\frac{1}{8}BX^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta\rho}{\rho}\right)+\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)] \\
&\left(B^2X^2\right)[\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)^2+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_P}{V_P}\right)^2+\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta V_P}{V_P}\right)^2] \\
&-4\left(B^3X^2\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)^2+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta V_S}{V_S}\right)^2+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta V_S}{V_S}\right)^2] \\
&+8\left(\frac{13}{4}B^3X^2-2B^2X^2\right)[\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta V_S}{V_S}\right)^2+\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta V_S}{V_S}\right)^2] \\
&+2\left(\frac{3}{4}B^3X^2+\frac{1}{4}B^2X^2-\frac{1}{16}BX^2\right)[\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)^2+\left(\frac{\Delta V_S}{V_S}\right)\left(\frac{\delta\rho}{\rho}\right)^2 \\
&+\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right)^2]+4\left(2B^3X^2-\frac{3}{4}B^2X^2\right)[\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)^2+\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta V_S}{V_S}\right)^2 \\
&+\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)^2]+8\left(\frac{13}{64}X^2-\frac{1}{64}\right)[\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta V_P}{V_P}\right)^2+\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta V_P}{V_P}\right)^2] \\
&+4\left(\frac{1}{8}B^2X^2-\frac{1}{16}X^2-\frac{1}{32}\right)[\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta V_P}{V_P}\right)^2+\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)^2+\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta V_P}{V_P}\right)^2] \\
&+2\left(\frac{1}{2}B^2X^2-\frac{1}{2}B^3X^2-\frac{1}{8}BX^2-\frac{1}{16}X^2-\frac{1}{16}\right)[\left(\frac{\Delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)^2+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\delta\rho}{\rho}\right)^2 \\
&+\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta\rho}{\rho}\right)^2]+\left(2B^2X^2-\frac{1}{2}B^3X^2-\frac{5}{8}BX^2-\frac{1}{8}\right)[\left(\frac{\Delta\rho}{\rho}\right)\left(\frac{\delta\rho}{\rho}\right)^2 \\
&+\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta\rho}{\rho}\right)^2]
\end{aligned}
\tag{27}
$$