A Perspective on Full-Waveform Inversion

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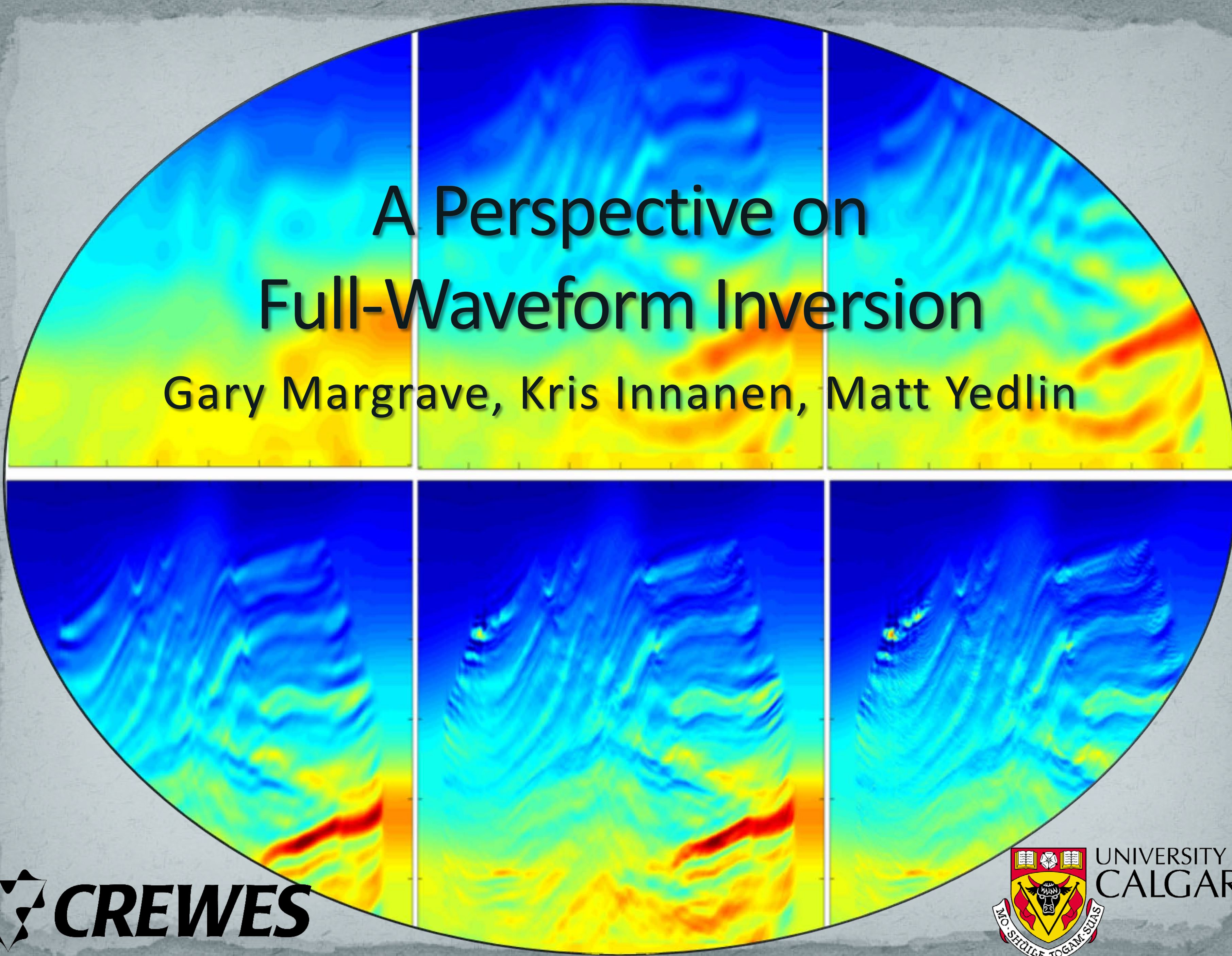

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Outline

Standard methodology (SM)

Full-waveform inversion (FWI)

Imaging conditions

Examining the FWI gradient

Well Validation versus Data Validation

Iterative Modelling Migration and Inversion (IMMI)

Conclusions

Purpose

- Identify commonalties between FWI and SM.
- Identify weaknesses of FWI and which elements of SM might help.
- Find a practical way forward.

The Standard Methodology (SM)

1. Compensation for attenuation (anelasticity)
2. Estimation and removal of source waveform (deconvolution)
3. Spatial focussing (prestack depth migration)
4. Impedance inversion

Clearly there are many more important processes (statics, noise reduction, model building, etc), but this abstraction targets those steps with special relevance to FWI.

The Standard Methodology (SM)

- Step1 is required to create a quasi-stationary dataset.
- Step 2 estimates the source wavelet in a way not usually done in FWI.
- Steps 1-3 create a *Reflectivity Image* which has no direct parallel in FWI.
- Step 4 converts the reflectivity image into an *Impedance Image*, which is also the main product of FWI.

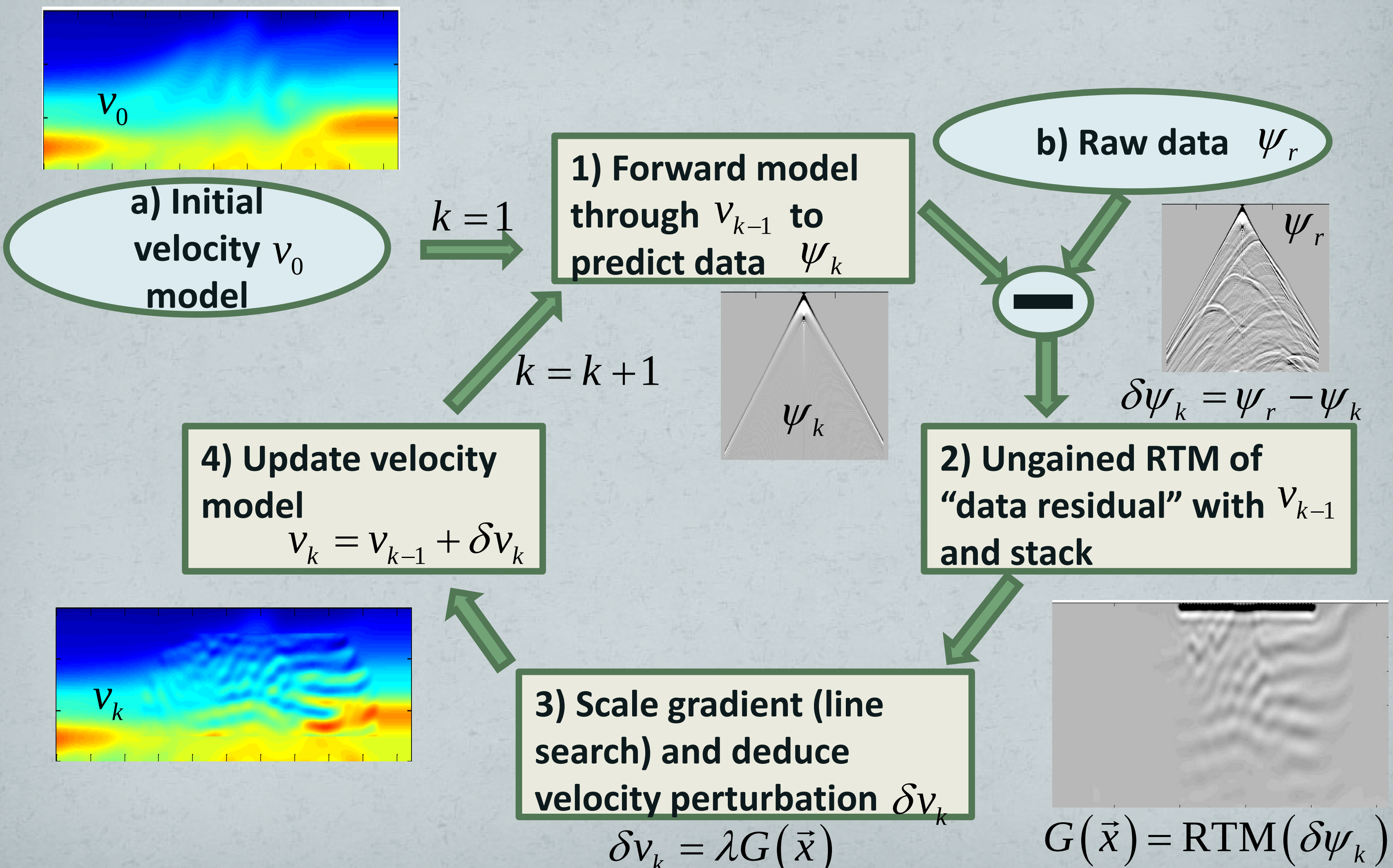
The Standard Methodology (SM)

- Impedance estimation involves tying to wells or *Well Validation*.
- Low-frequency information needed in step 4 is usually obtained from well control, which is not commonly done in FWI.
- The process is almost never iterated in that the impedance model is never used to predict data or re-migrate.

Full-waveform Inversion (FWI)

- Method is driven by minimization of the *data residual*, which is the difference between real and synthetic data.
- The final *Earth Model* is that which best predicts the recorded data, so this is *Data Validation.*
- Low-frequency information is assumed to come from the data.
- Generally implemented as an iteration involving:
  (1) modelling, (2) gradient estimation, and
  (3) model update.

Full-waveform Inversion (FWI) (Steepest descent)



Full-waveform Inversion (FWI)

- Step 2: Gradient estimation is an un-gained RTM of the data residual.
- Step 3: Estimation of the impedance perturbation is accomplished either

$\delta v_k = \lambda G(\vec{x})$ Steepest descent (most common)

or

$\delta v_k = H^{-1} G(\vec{x})$ Newton or Gauss-Newton (computationally extreme)

Comparison

	SM	FWI
Wavelet estimation	Deconvolution	Inversion loop
Reflectivity	PSDM	Not usually created
Imaging algorithm	PSDM	Un-gained RTM
Synthetic data	Inside PSDM	Data residual and in gradient
Impedance estimation	Impedance inversion	Scaled gradient is impedance update
Iteration	Not iterated	Iterated to minimal data difference
Initial model	Migration velocity model is complex	Initial model is smooth
Validation	Well validation	Data validation

Imaging conditions
Claerbout (1971)

Imaging conditions define what is calculated in PSDM

$$\hat{R}_d(\vec{x},\omega) = \frac{U(\vec{x},\omega)}{D(\vec{x},\omega)}$$ Deconvolution imaging condition

$U(\vec{x},\omega)$ “Upgoing” field (real data)

$D(\vec{x},\omega)$ “Downgoing” field (synthetic data)

U and D have similar meanings in PSDM and in FWI Gradient. FWI makes additional use of D to compute data residual.

Imaging conditions
Claerbout (1971)

$\hat{R}_c(\vec{x},\omega) = U\left(\vec{x},\omega\right) D^*\left(\vec{x},\omega\right)$ Correlation imaging condition

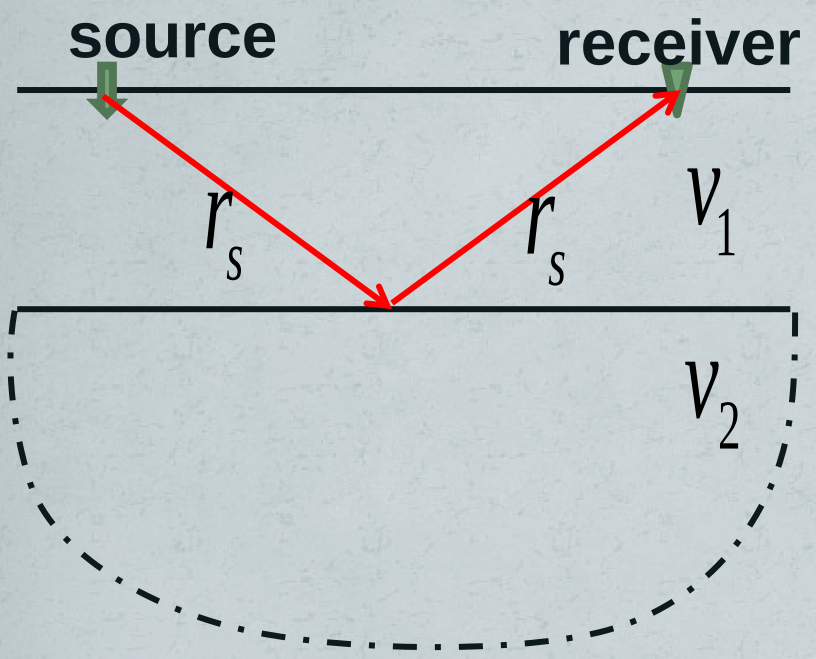
Assumes data have been gain corrected before PSDM. Using this imaging condition without gain is a “mistake”.

$$\hat{R}_{ds}(\vec{x},\omega) = \frac{U\left(\vec{x},\omega\right) D^*\left(\vec{x},\omega\right)}{\left\|D\left(\vec{x},\omega\right)\right\|^2 + \mu D_{\max}}$$

Stabilized deconvolution imaging condition

$$\hat{R}_{ds}(\vec{x},\omega) \approx \begin{cases} \hat{R}_d(\vec{x},\omega), \left\|D\left(\vec{x},\omega\right)\right\|^2 \gg \mu D_{\max} \\ \hat{R}_c(\vec{x},\omega), \left\|D\left(\vec{x},\omega\right)\right\|^2 \ll \mu D_{\max} \end{cases}$$

Imaging conditions Example



Specular P-P reflection:

$$\boxed{U\left(\text{receiver}\right) = R_T \frac{W_T\left(\omega\right)}{8\pi r_s} e^{i2kr_s}}$$

R_T True reflection coefficient

$W_T\left(\omega\right)$ True wavelet

$k = \dfrac{\omega}{v_1}$ Wavenumber

Imaging conditions Example

Downward continue data to reflector:

$$\boxed{U\left(\mathrm{refl}\right) = R_T \frac{W_T\left(\omega\right)}{4\pi r_s} e^{ik\,r_s}}$$

Synthetic shot model at reflector:

$$\boxed{D\left(\mathrm{refl}\right) = \frac{W\left(\omega\right)}{4\pi r_s} e^{ik\,r_s}}$$

$W\left(\omega\right)$ Wavelet estimate

Imaging conditions
Example

Deconvolution imaging condition:

$$R_d = \frac{U(\text{refl})}{D(\text{refl})} = \frac{\frac{R_T W_T(\omega)}{4\pi r_s} e^{ikr_s}}{\frac{W(\omega)}{4\pi r_s} e^{ikr_s}} \boxed{= R_T \frac{W_T(\omega)}{W(\omega)}}$$

Correlation imaging condition (no gain):

$$R_{c-u} = U(\text{refl}) D^*(\text{refl}) \boxed{= \frac{R_T}{r_s^2} \frac{W_T(\omega) W^*(\omega)}{(4\pi)^2}}$$

Correlation imaging condition (gained):

$$R_{c-g} = U_g(\text{refl}) D^*(\text{refl}) \boxed{= R_T \frac{W_T(\omega) W^*(\omega)}{(4\pi)^2}}$$

FWI Gradient

$$G_k\left(x,z,\omega\right)=\omega^2\sum_{s,r}\underbrace{\hat{\psi}_{s,k}(x,z,\omega)\delta\hat{\psi}^*_{r(s),k}(x,z,\omega)}_{\text{correlation}}$$

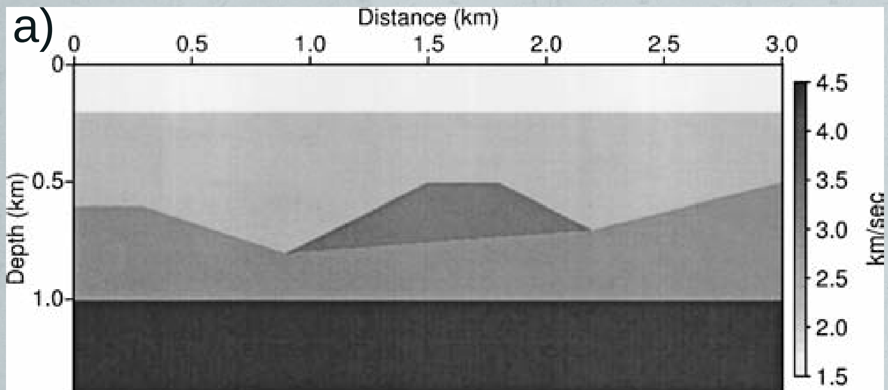
k	Iteration number
$\hat{\psi}_{s,k}(x,z,\omega)$	Synthetic shot record calculated at all points in the subsurface, essentially D
$\delta\hat{\psi}^*_{r(s),k}(x,z,\omega)$	Data residual (U – D) back propagated (downward continued) to all points in the subsurface.

The FWI gradient is an RTM with an ungained correlation imaging condition.

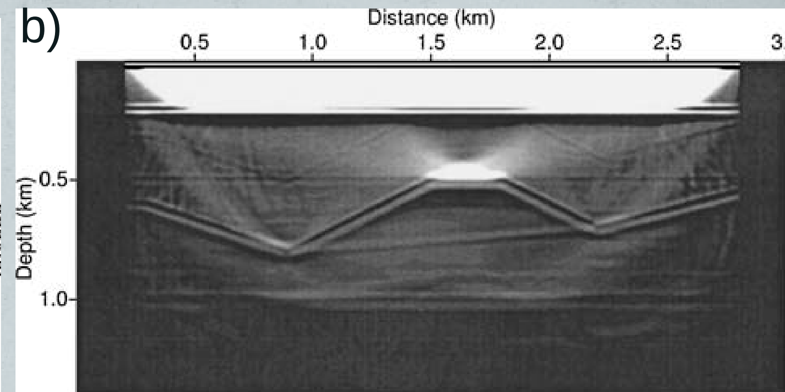
FWI Gradient

- FWI gradient is a very poorly scaled migration (Gray 1997, Shin et al. 2001).
- We show that $G \sim r^{-2}$ $(3\mathrm{D})$ so it fades very quickly.
- Thus a descent method that estimates an model update by $\delta v = \lambda G$ will converge very slowly if at all.
- Shin et al. and others show that the first order effect of the inverse Hessian is a gain correction. So $\delta v = H^{-1} G$ will converge much faster but H^{-1} is very computationally intensive.

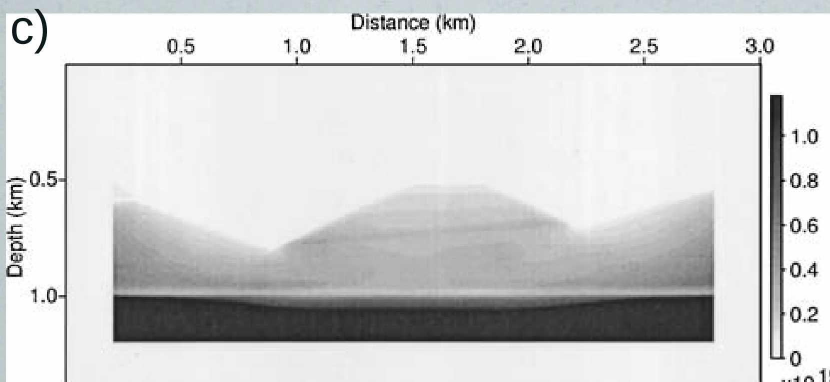
FWI Gradient
Example from Shin et al. (2001)



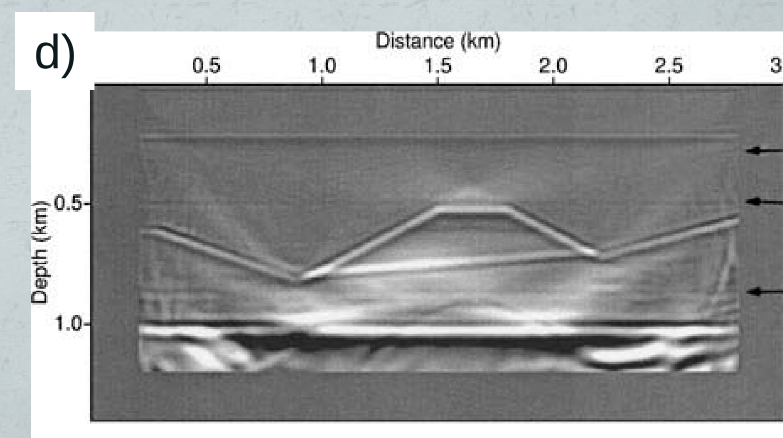
Actual Model



Gradient = G



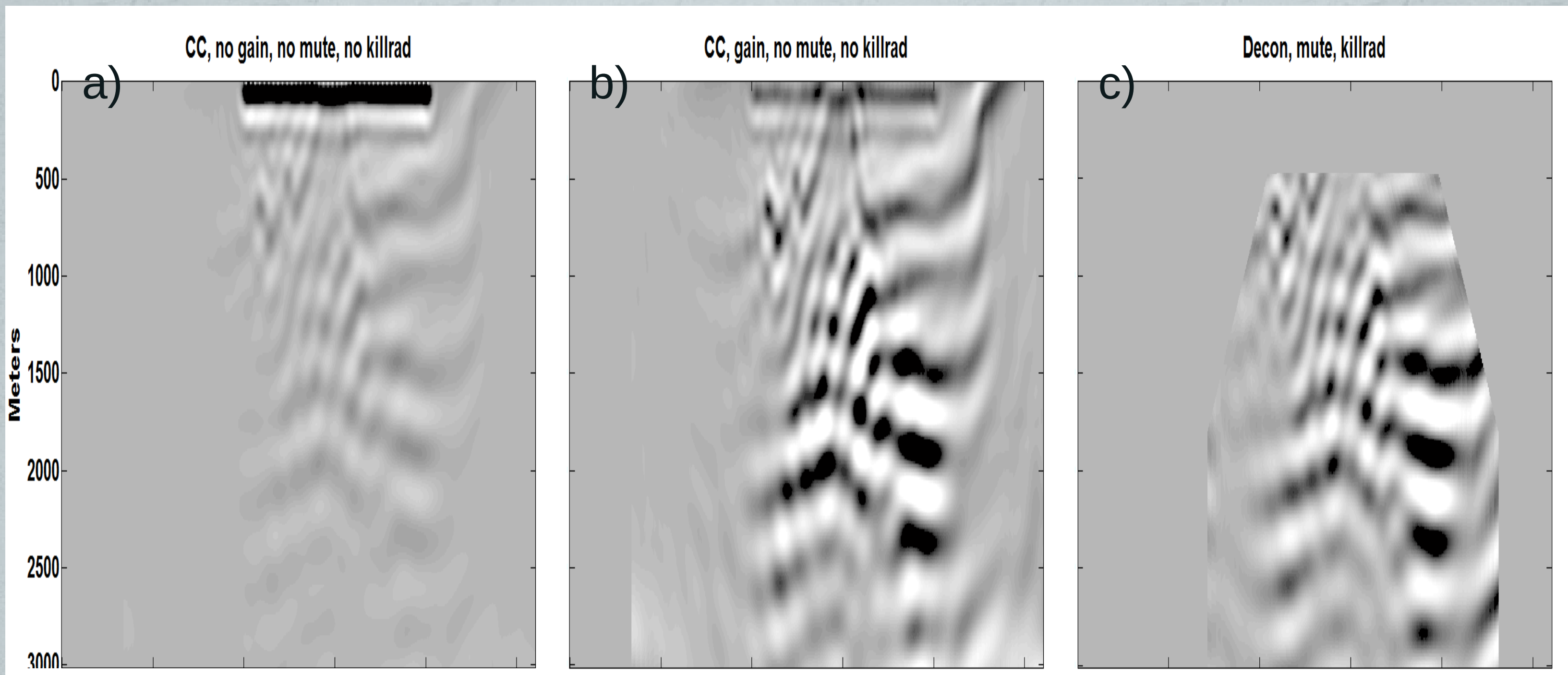
Diagonal part of H^{-1}



$H^{-1}G$

Effects similar to H^{-1} are obtained simply via the deconvolution imaging condition

Example from Margrave et al. (2010)



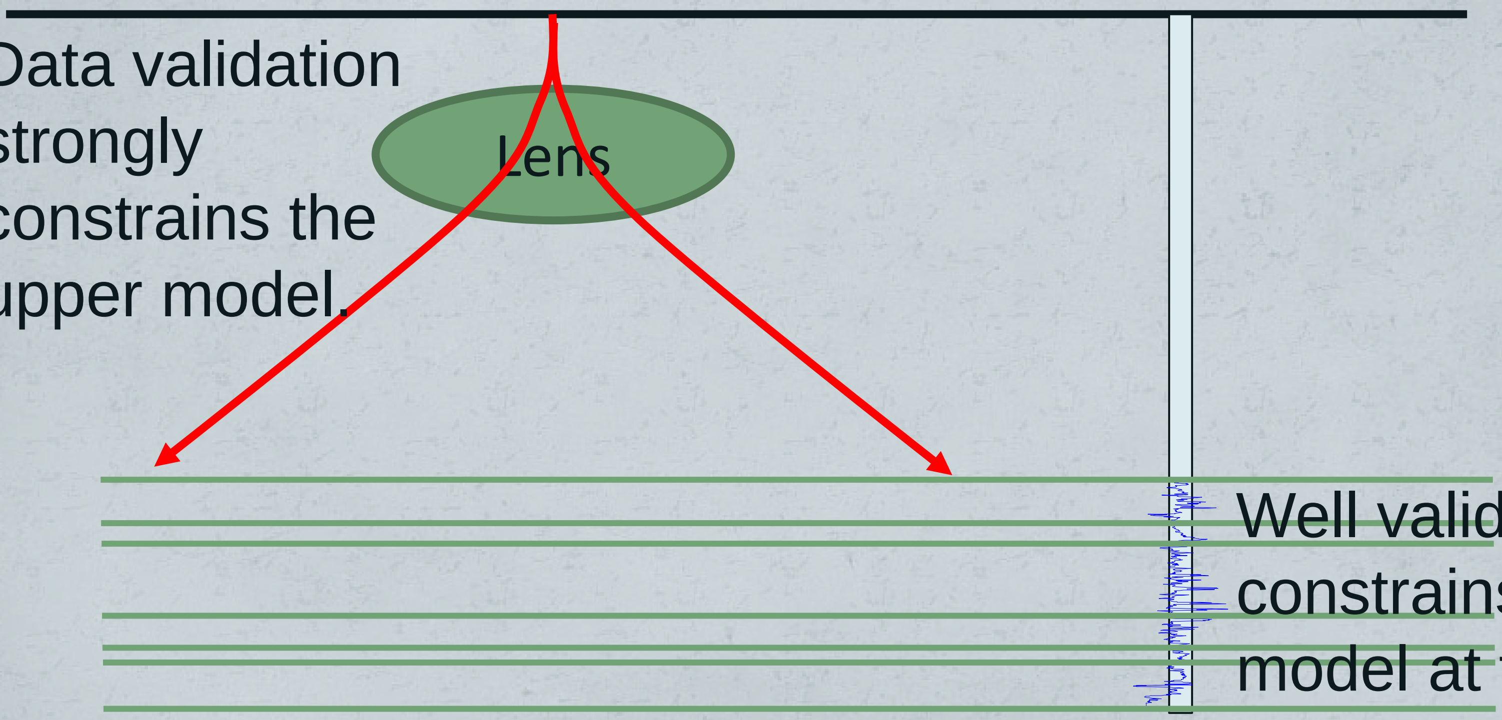
PSDM Correlation without gain

PSDM Correlation with gain

PSDM Deconvolution with mute

Well Validation versus Data Validation

Data validation strongly constrains the upper model.



Well validation constrains the model at target

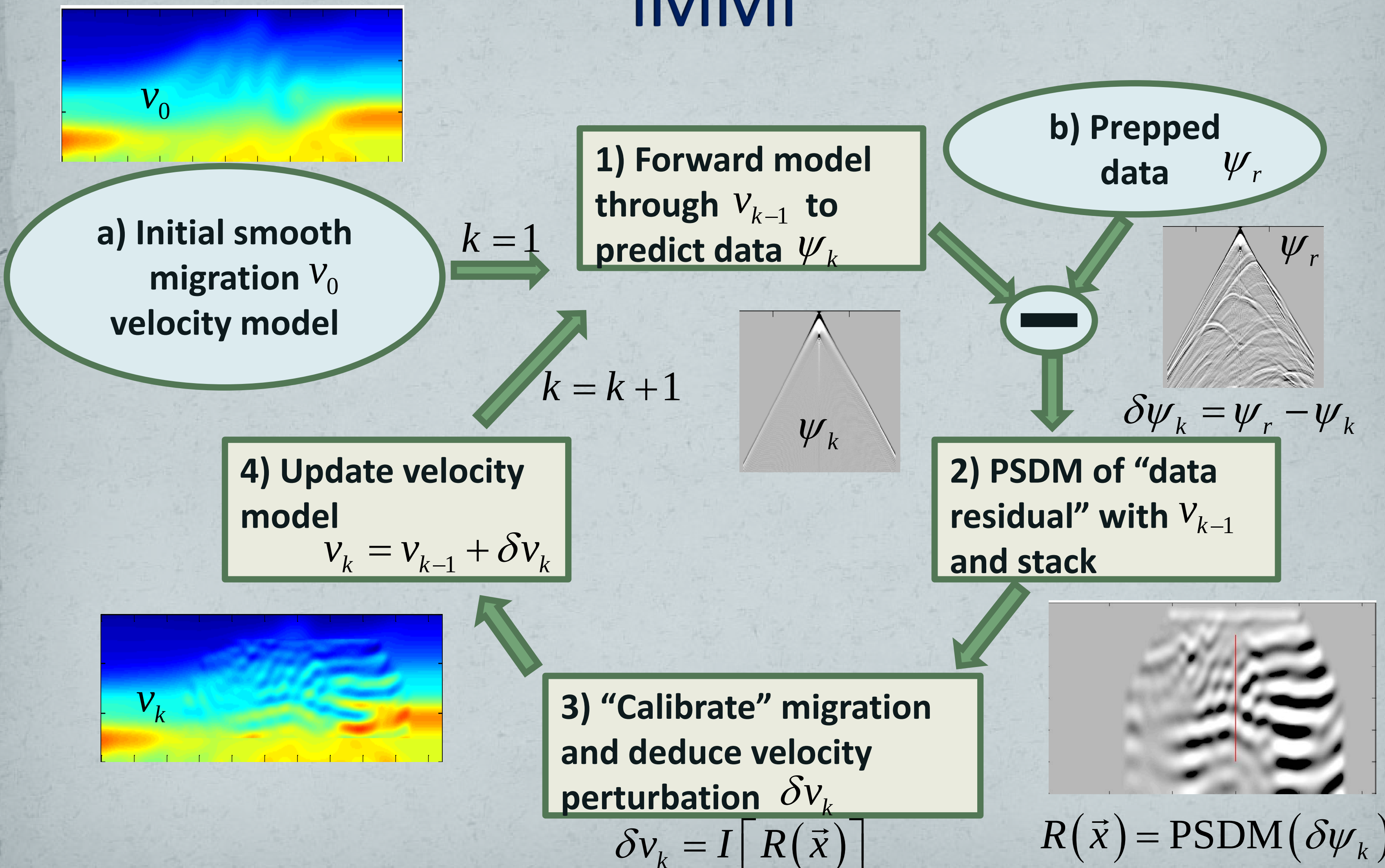
Iterative Modelling Migration and Inversion IMMI

Proposal: We can follow the spirit of FWI using processes from SM such as PSDM and Impedance Inversion.

Possible benefits:

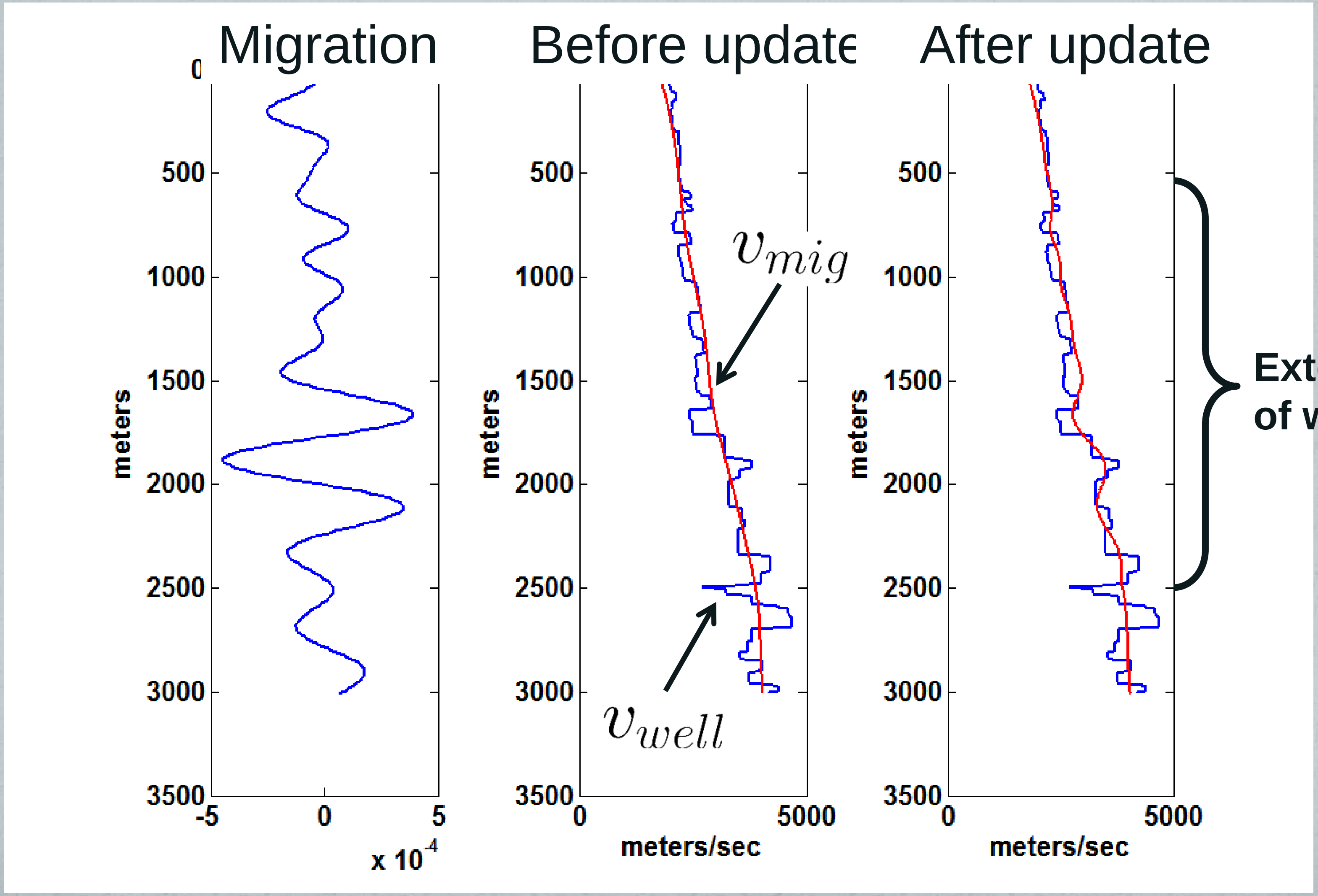
- Use existing software (e.g. PSDM)
- Use well control
- Faster convergence
- Use both well and data validation

Iterative Modelling Migration and Inversion IMMI

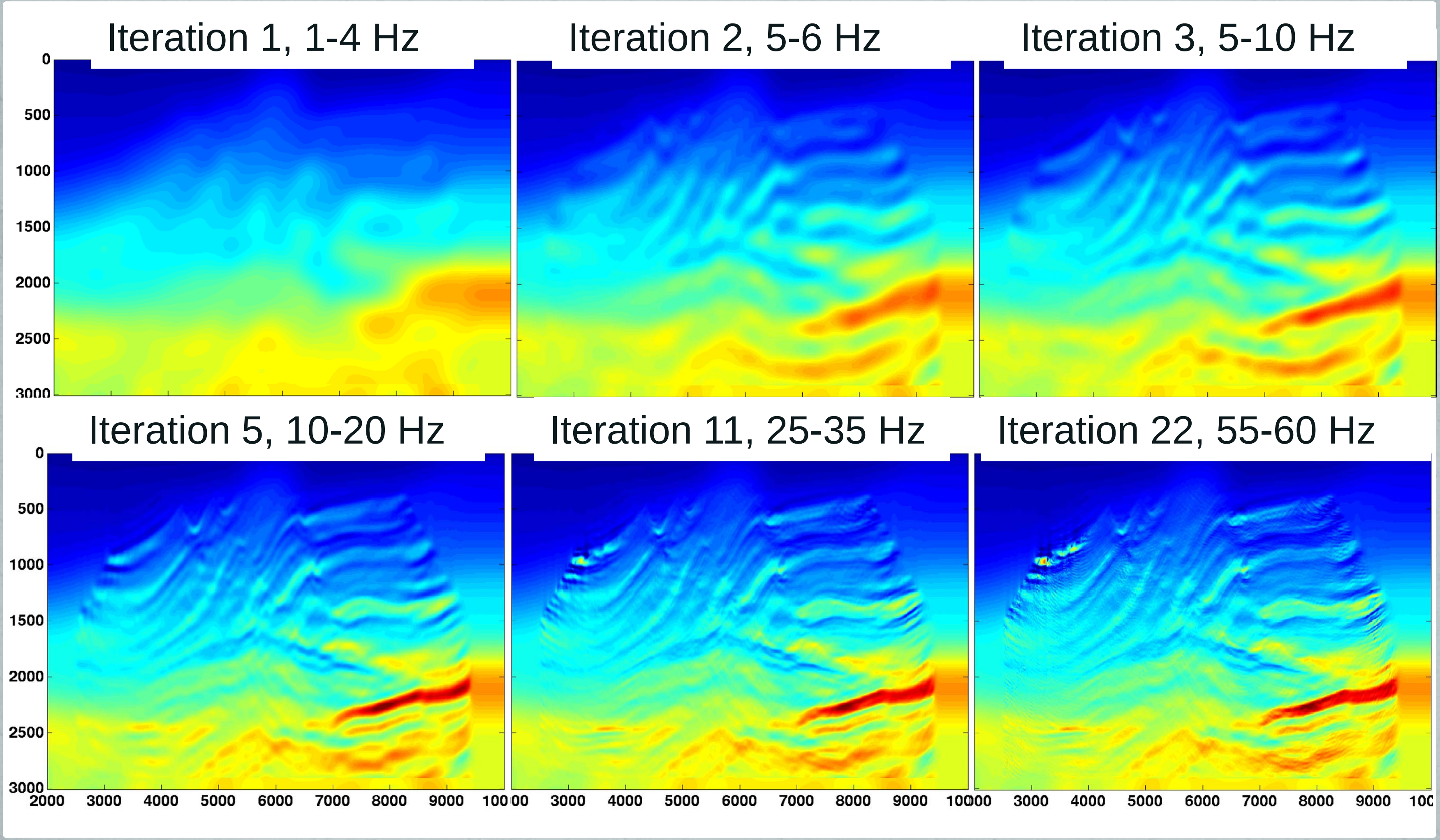


Calibration at the well

First iteration 0-5 Hz. (Margrave et al. 2010)



IMMI Example (Margrave et al., 2010)



An Impedance Imaging Condition

A PSDM can directly estimate impedance instead of reflectivity by using

$$R_k = \frac{I_{k+1} - I_k}{I_{k+1} + I_k}$$ Definition of reflection coefficient

$$\Delta I_k = 2 I_k R_k$$ Impedance imaging condition

$$I_{k+1} = I_k + \Delta I_k = I_k \left(1 + 2R_k\right)$$ Updated impedance for next iteration

Conclusions

•SM produces a reflectivity image that is converted to impedance with well control. Rarely iterated.

• FWI iteratively updates an impedance model until the match between real and synthetic data is acceptable.

• Both methods required low frequencies for the same reason.

• FWI gradient is a poorly scaled migration, which explains slow convergence for simple descent.

• A migration with a deconvolution imaging condition is similar to applying an inverse Hessian to gradient.

• IMMI is an attractive middle ground that can utilize strengths of both methods.

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