



# Internal multiple prediction in the continuous wavelet transform domain

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# Outline

Quick mention of AVO and FWI

Review of Hernandez thesis results (2012)

Internal multiple prediction on noisy land data

Continuous wavelet transform maxima

Continuous wavelet transforms

Processing with CWT maxima

Internal multiple prediction in the CWTM domain

# AVO and FWI

CREWES research priority (Margrave et al., 2013)

Support & pre-condition iterative seismic inversion with existing standard methodologies

$$\delta \mathbf{s} = -\mathbf{H}^{-1} \mathbf{g}$$

update



amplitude  
correction  
?



amplitude-  
compromised  
migration



# AVO and FWI

FWI for short offset (pre-critical) reflection data?


Will involve AVO info & updates in 3 parameters

$$\delta \mathbf{s} = -\mathbf{H}^{-1} \mathbf{g}$$

update  
in  $V_P$ ,  
 $V_S$ ,  $\rho$



amplitude  
correction  
?



amplitude-  
compromised  
migration



# AVO and FWI

A combined discrete-continuous formulation

determinant  $\swarrow$  gradients  $\downarrow$

$$\begin{bmatrix} \delta s_\kappa(\mathbf{r}) \\ \delta s_\rho(\mathbf{r}) \end{bmatrix} = \int d\mathbf{r}' \mathcal{H}_2^{-1}(\mathbf{r}, \mathbf{r}') \int d\mathbf{r}'' \begin{bmatrix} -H_{\rho\rho}(\mathbf{r}', \mathbf{r}'') & H_{\rho\kappa}(\mathbf{r}', \mathbf{r}'') \\ H_{\kappa\rho}(\mathbf{r}', \mathbf{r}'') & -H_{\kappa\kappa}(\mathbf{r}', \mathbf{r}'') \end{bmatrix} \begin{bmatrix} g_\kappa(\mathbf{r}'') \\ g_\rho(\mathbf{r}'') \end{bmatrix}$$

Hessian functions

$$\mathcal{H}_2(\mathbf{r}, \mathbf{r}') = \int d\mathbf{r}'' [H_{\kappa\kappa}(\mathbf{r}, \mathbf{r}'')H_{\rho\rho}(\mathbf{r}'', \mathbf{r}') - H_{\rho\kappa}(\mathbf{r}, \mathbf{r}'')H_{\kappa\rho}(\mathbf{r}'', \mathbf{r}')]$$

# AVO and FWI

Allowing a range of approximate Newton steps

$$\begin{bmatrix} \delta s_\kappa(\mathbf{r}) \\ \delta s_\rho(\mathbf{r}) \end{bmatrix} \approx \underbrace{-\frac{1}{\Gamma_{\kappa\kappa}(\mathbf{r})\Gamma_{\rho\rho}(\mathbf{r}) - \Gamma_{\rho\kappa}(\mathbf{r})\Gamma_{\kappa\rho}(\mathbf{r})}}_{\text{angle-dependence suppression}} \begin{bmatrix} \Gamma_{\rho\rho}(\mathbf{r}) & -\Gamma_{\rho\kappa}(\mathbf{r}) \\ -\Gamma_{\kappa\rho}(\mathbf{r}) & \Gamma_{\kappa\kappa}(\mathbf{r}) \end{bmatrix} \begin{bmatrix} g_\kappa(\mathbf{r}) \\ g_\rho(\mathbf{r}) \end{bmatrix}$$

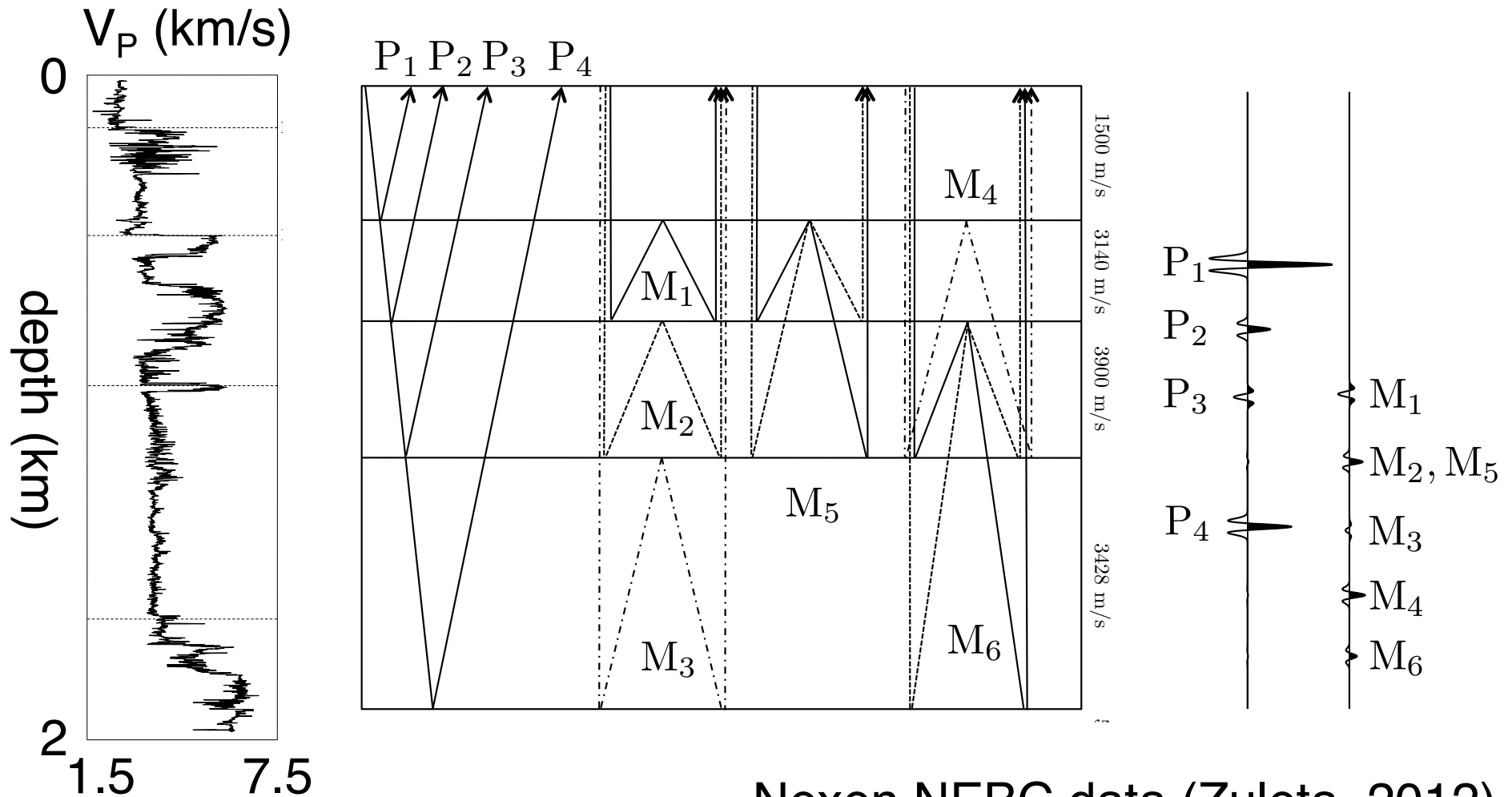
amplitude correction
cross-talk suppression

angle-dependence suppression
cross-talk suppression

# Internal multiple prediction

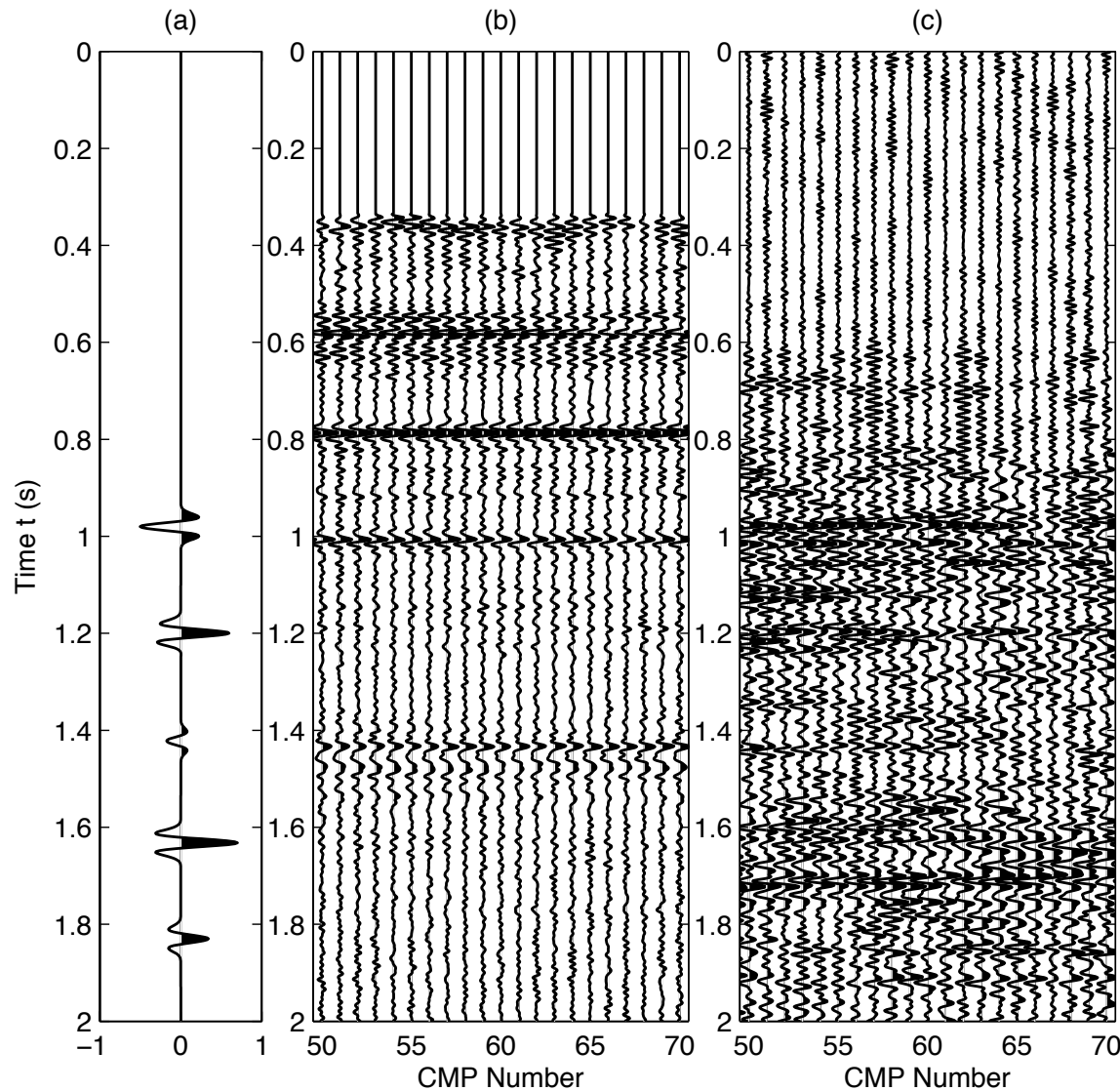
## On noisy land traces

(Hernandez, 2012; following Weglein et al., 1997)



Nexen NEBC data (Zuleta, 2012)

# Internal multiple prediction



(Hernandez & Innanen, 2013)

## Conclusions

Predictions correlate very well with modelled traces

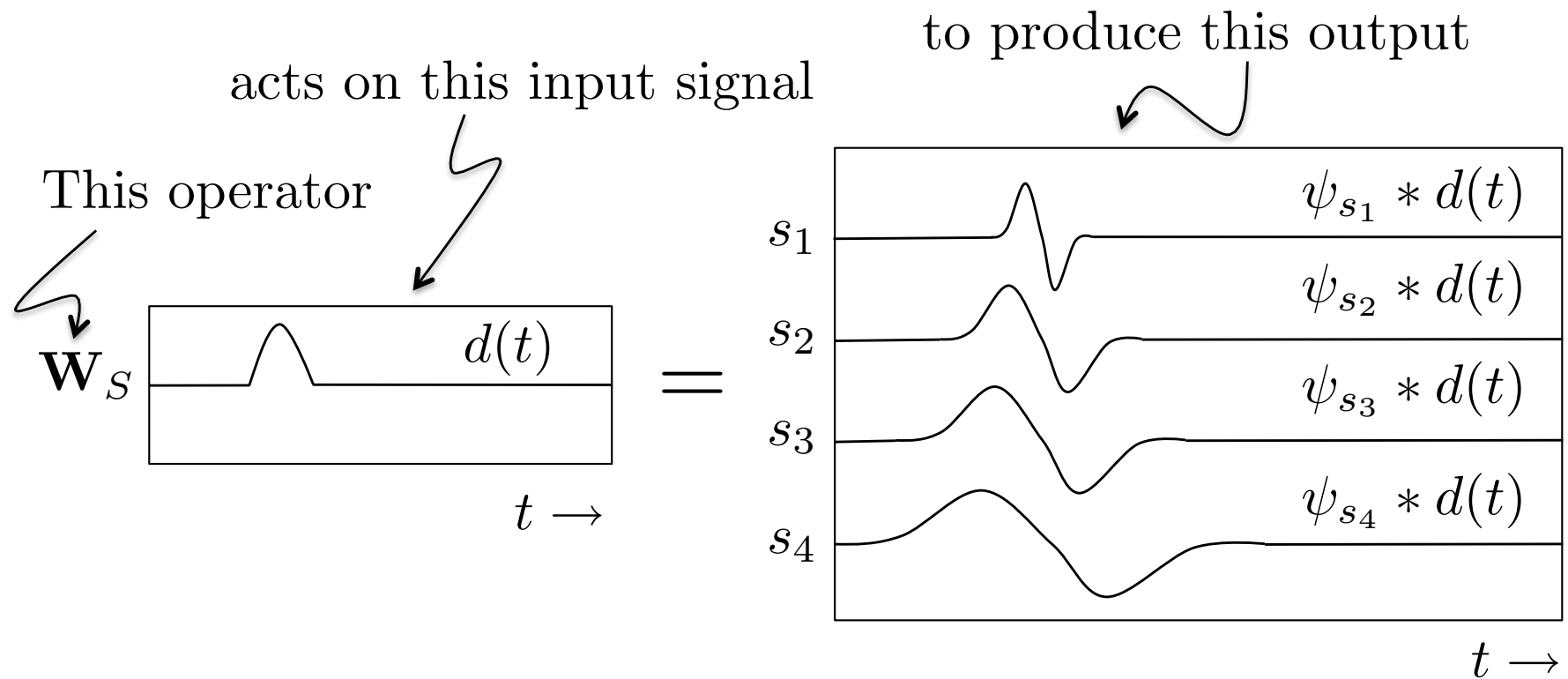
Interpretation tool?  
Probability map of contamination with IMs

Subtraction? Need a way of combating noise...



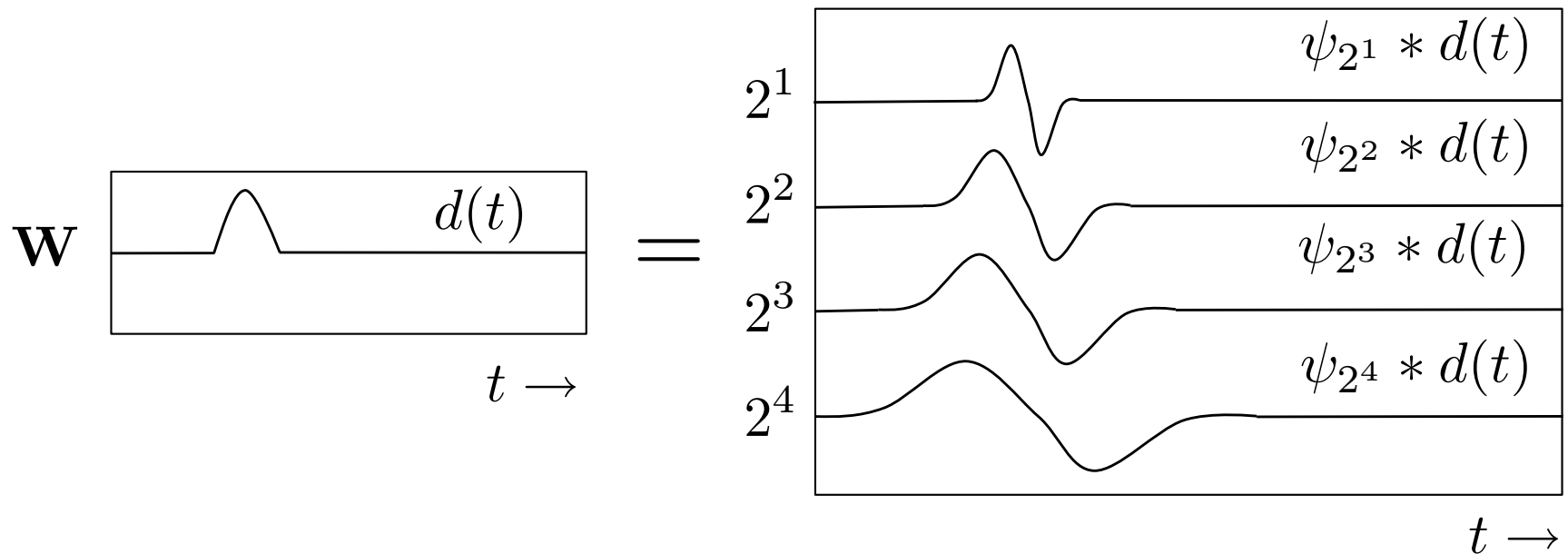
# Continuous wavelet transform

Define the operator  $W_S$



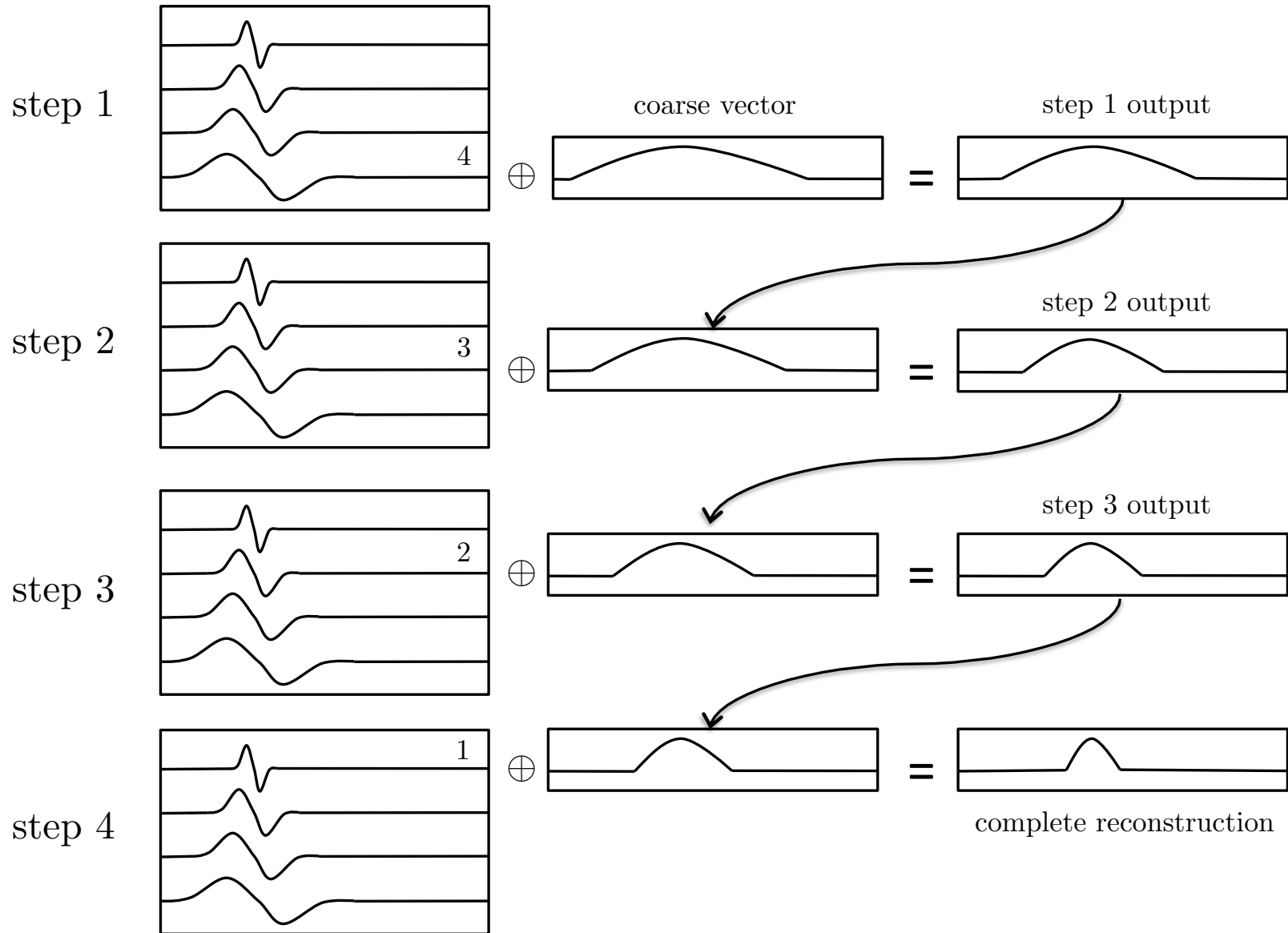
# Continuous wavelet transform

Define the operator  $W$



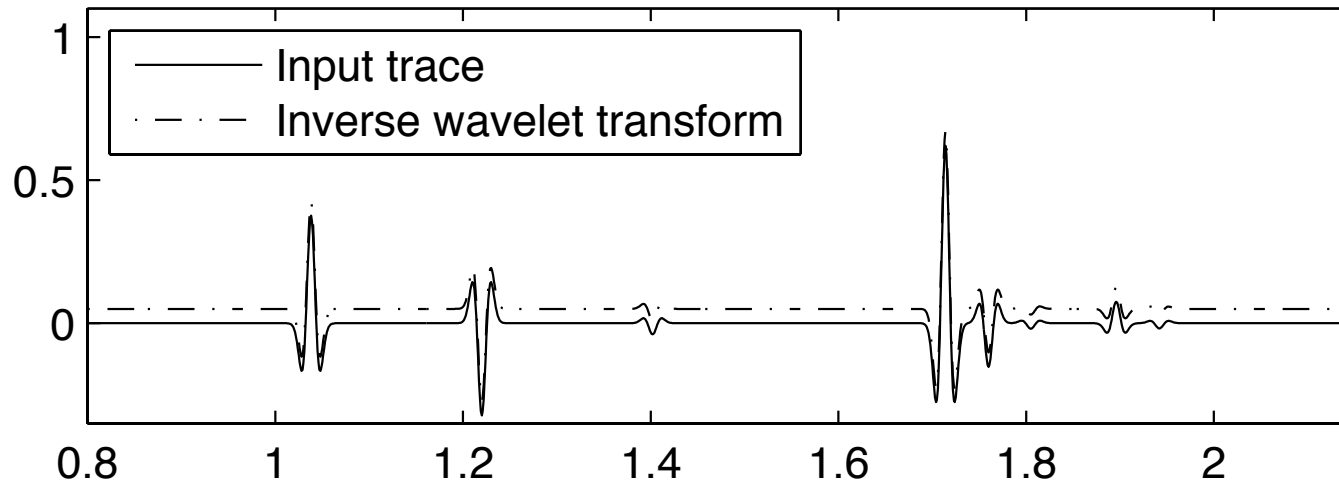
+ a “coarse” signal

# Inversion $W^{-1}$

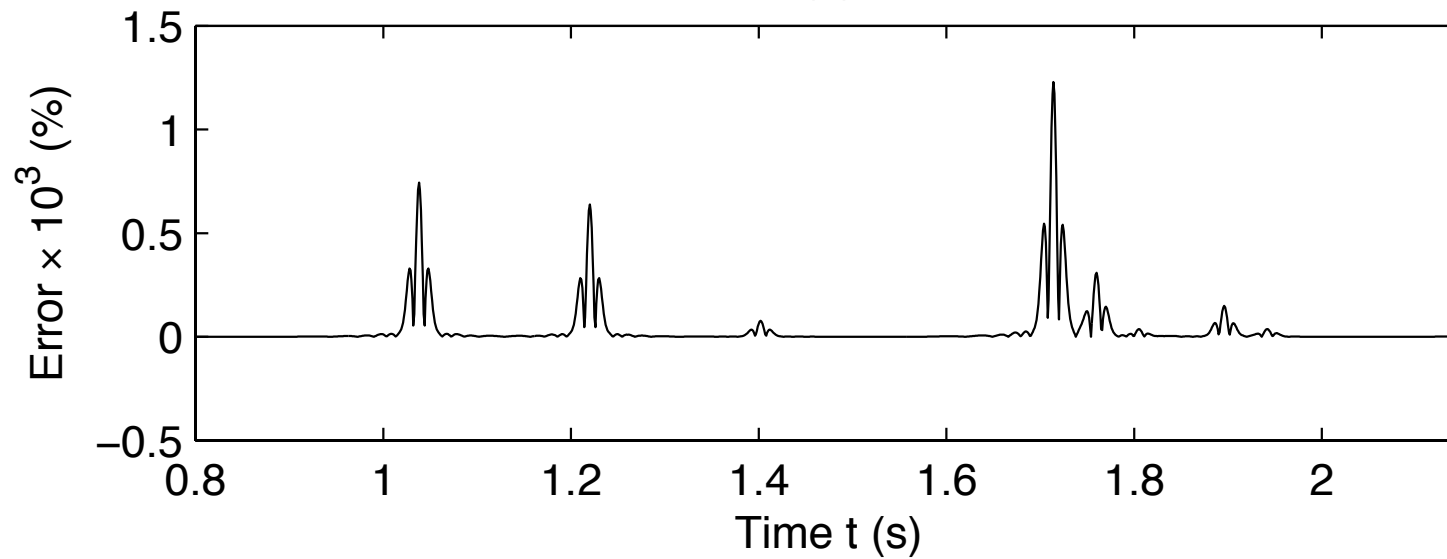


# Inversion $W^{-1}$ accuracy

(a)

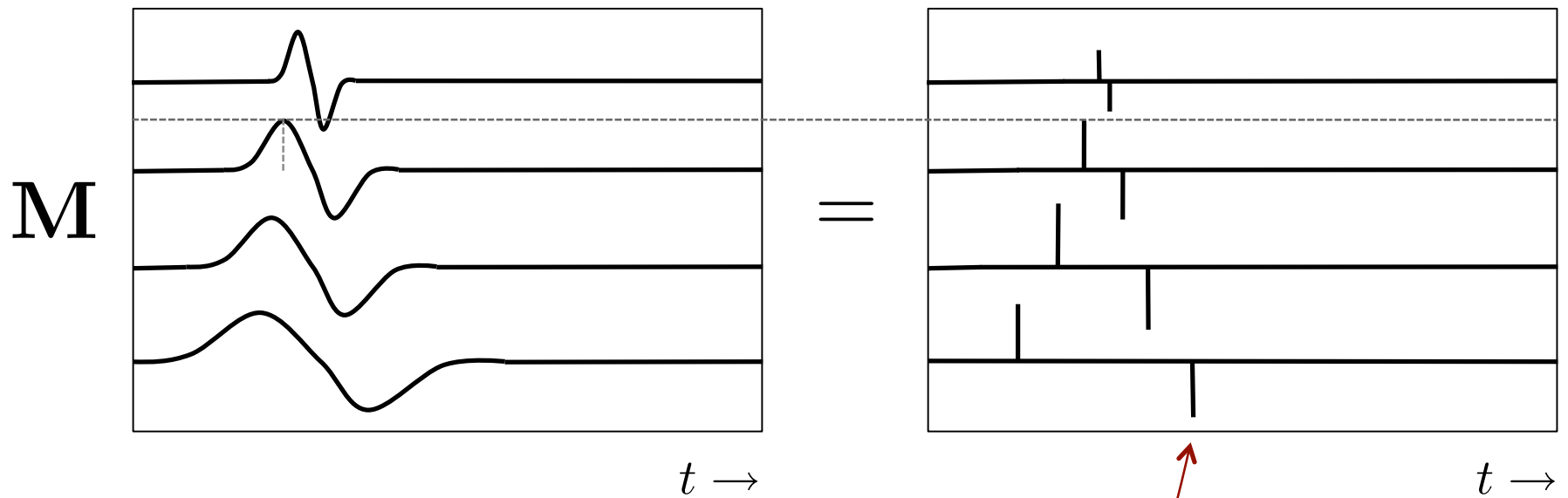


(b)



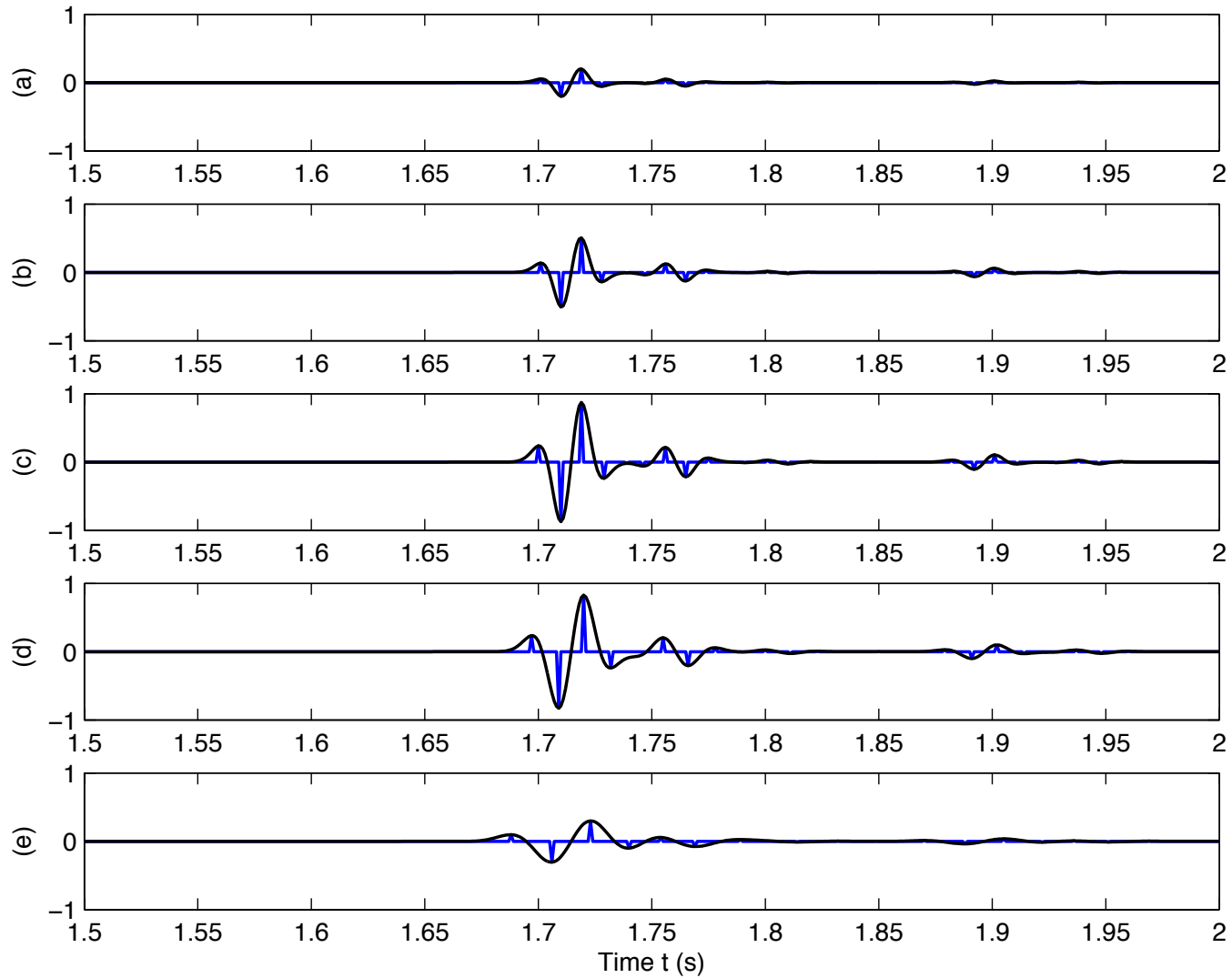
# CWT modulus maxima

Define the operator  $M$



Analysis: e.g., Q estimation  
(Izadi et al., 2011, 2012)

# CWT modulus maxima



# Reconstruction from CWT maxima

## Question

Is there a stable and accurate  $(MW)^{-1}$ ?

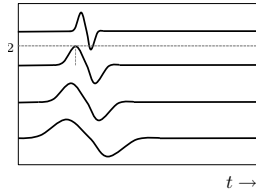
## Answer

Yes! ...approximately...

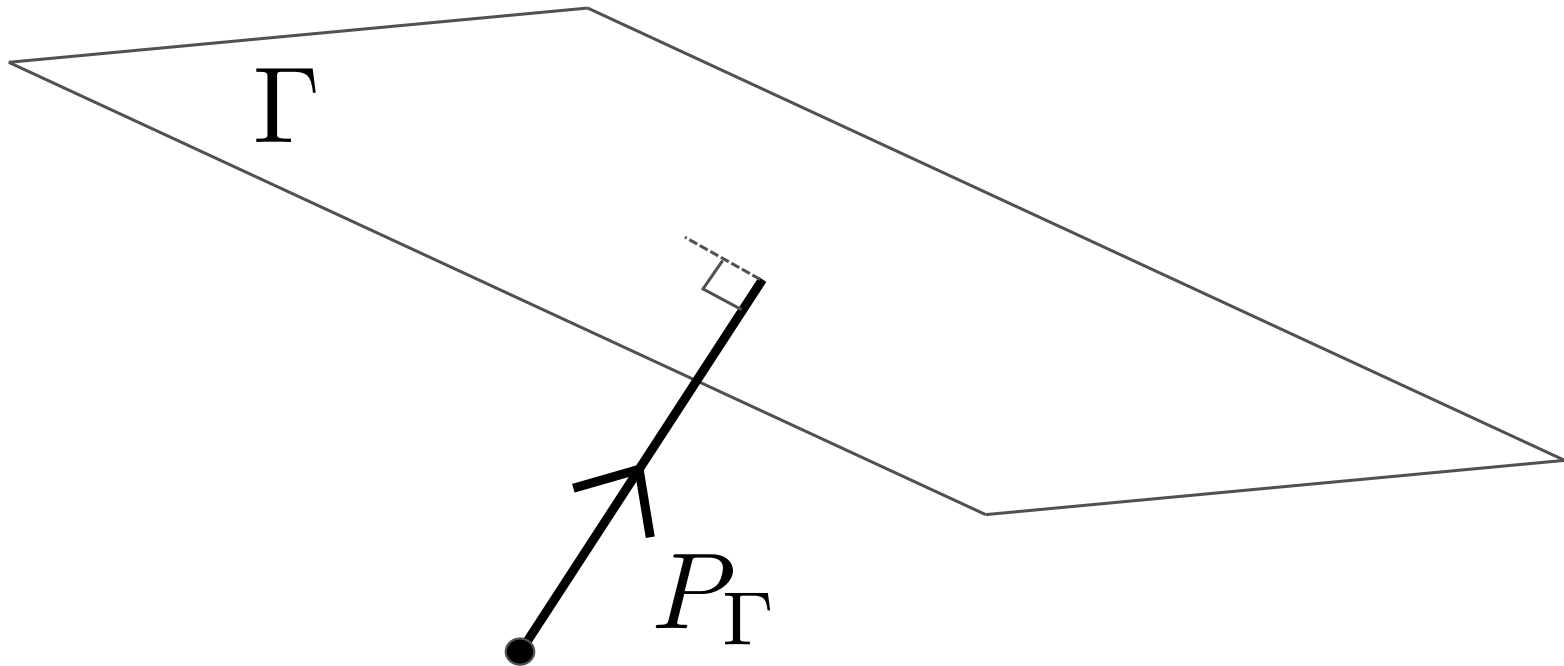
To find it, we need to define two spaces,  $\Gamma$  and  $V$ ,  
and two operators,  $P_\Gamma$  and  $P_V$ .

# Reconstruction from CWT maxima

$\Gamma$ : all matrices



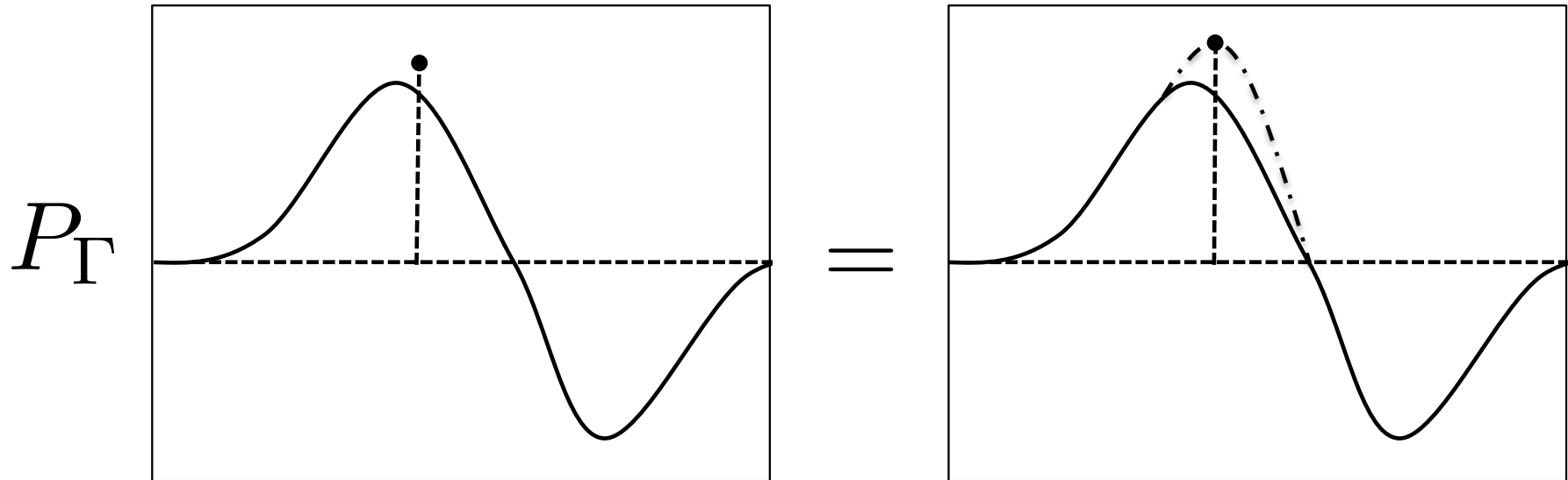
with the same maxima





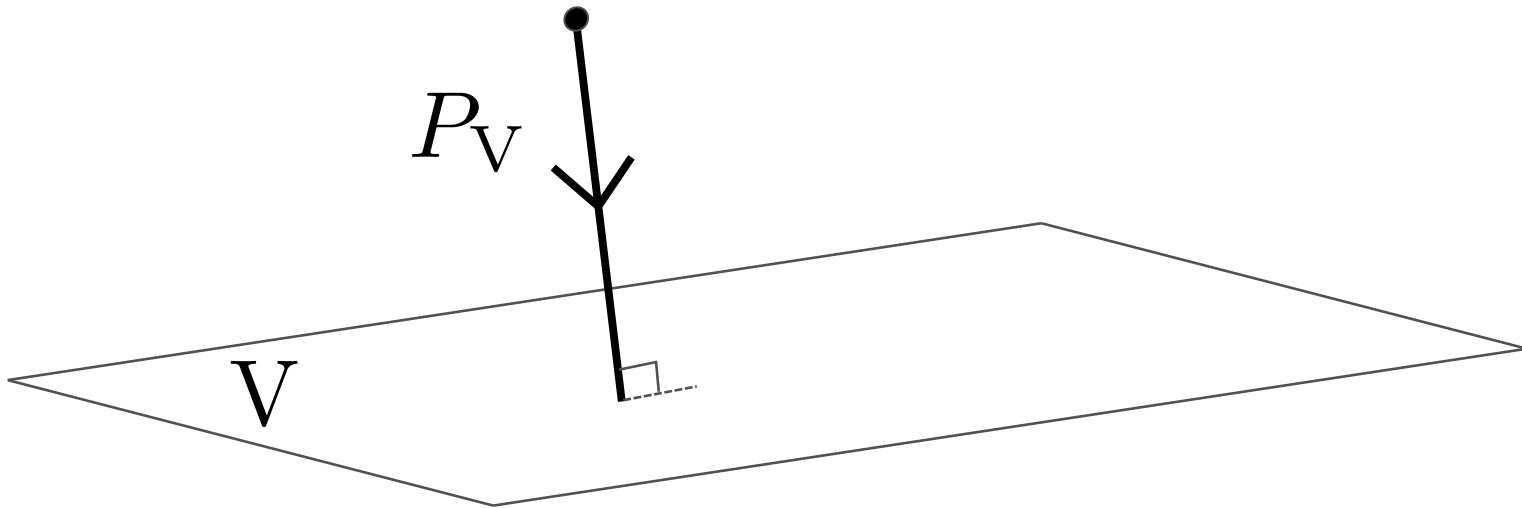
# Reconstruction from CWT maxima

$P_\Gamma$ : projection onto “nearest” element of  $\Gamma$



# Reconstruction from CWT maxima

$V$ : all legitimate CWTs



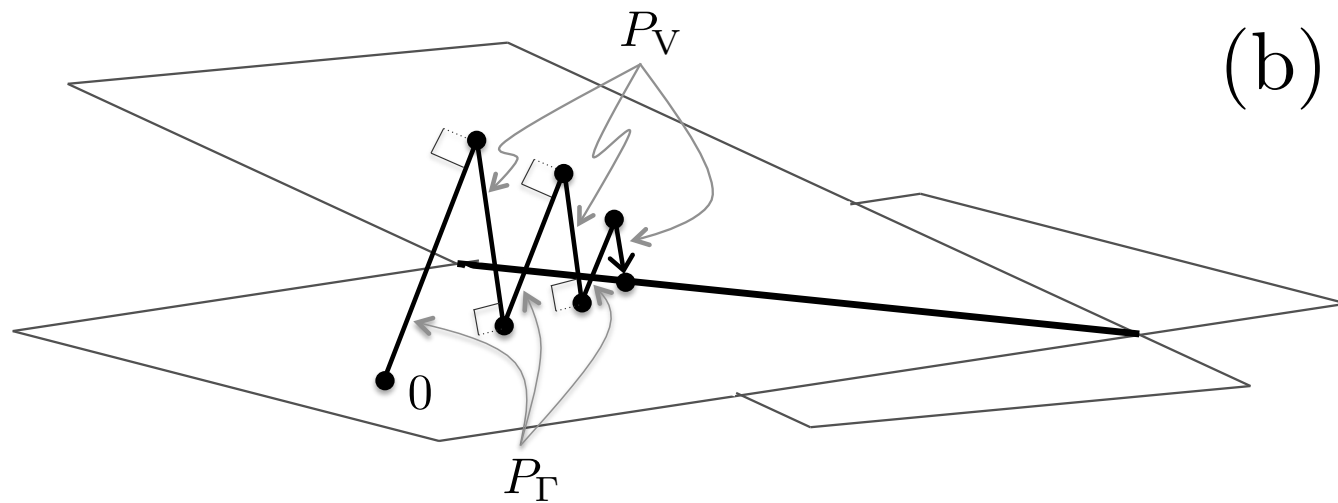
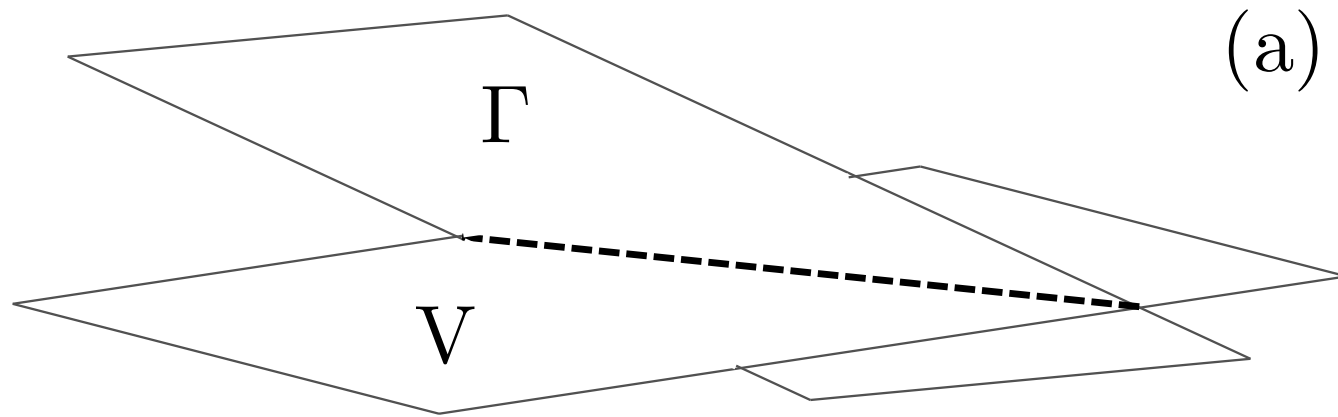
# Reconstruction from CWT maxima

$P_V$ : projection onto “nearest” element of  $V$

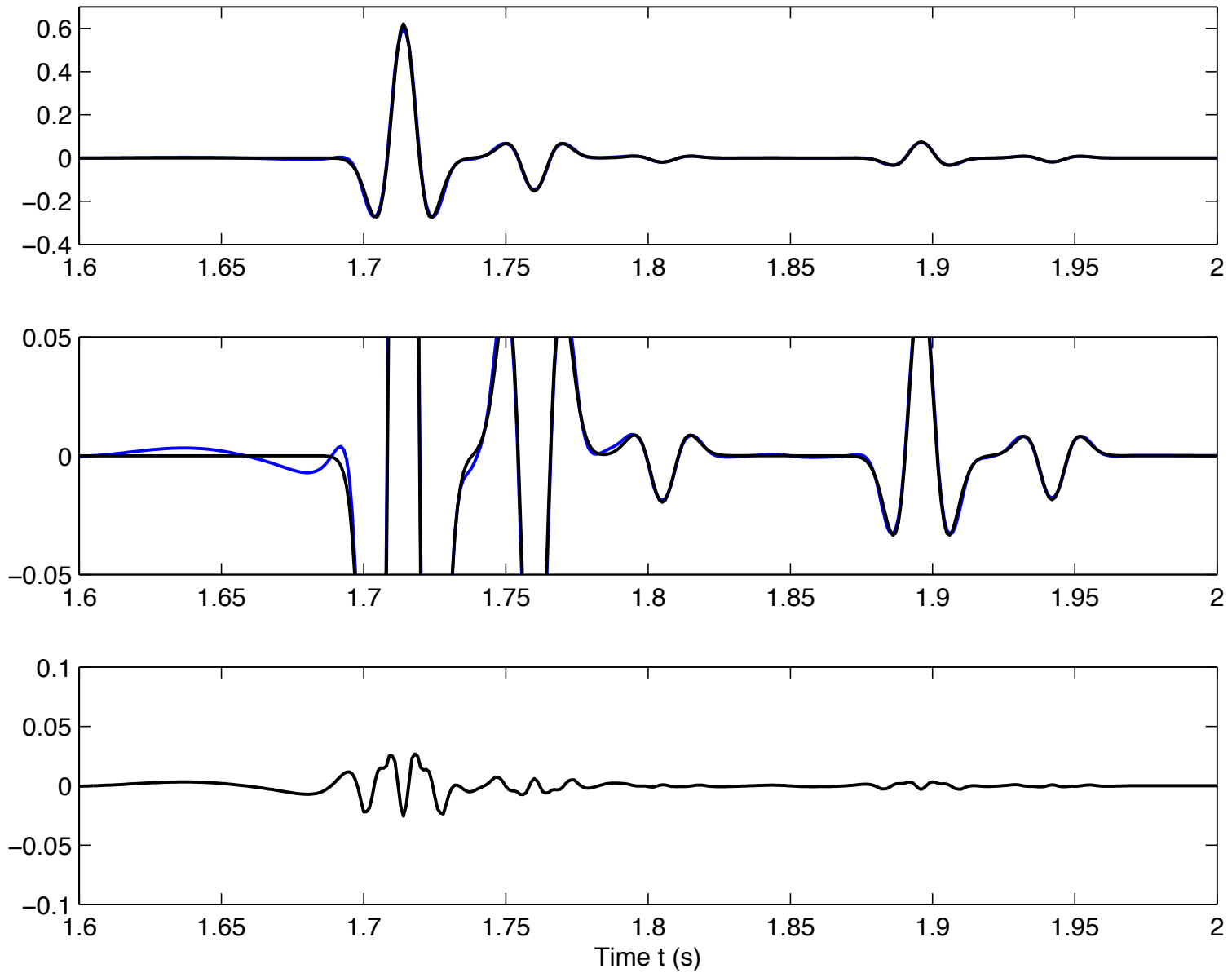
$$P_V = WW^{-1}$$

# Reconstruction from CWT maxima

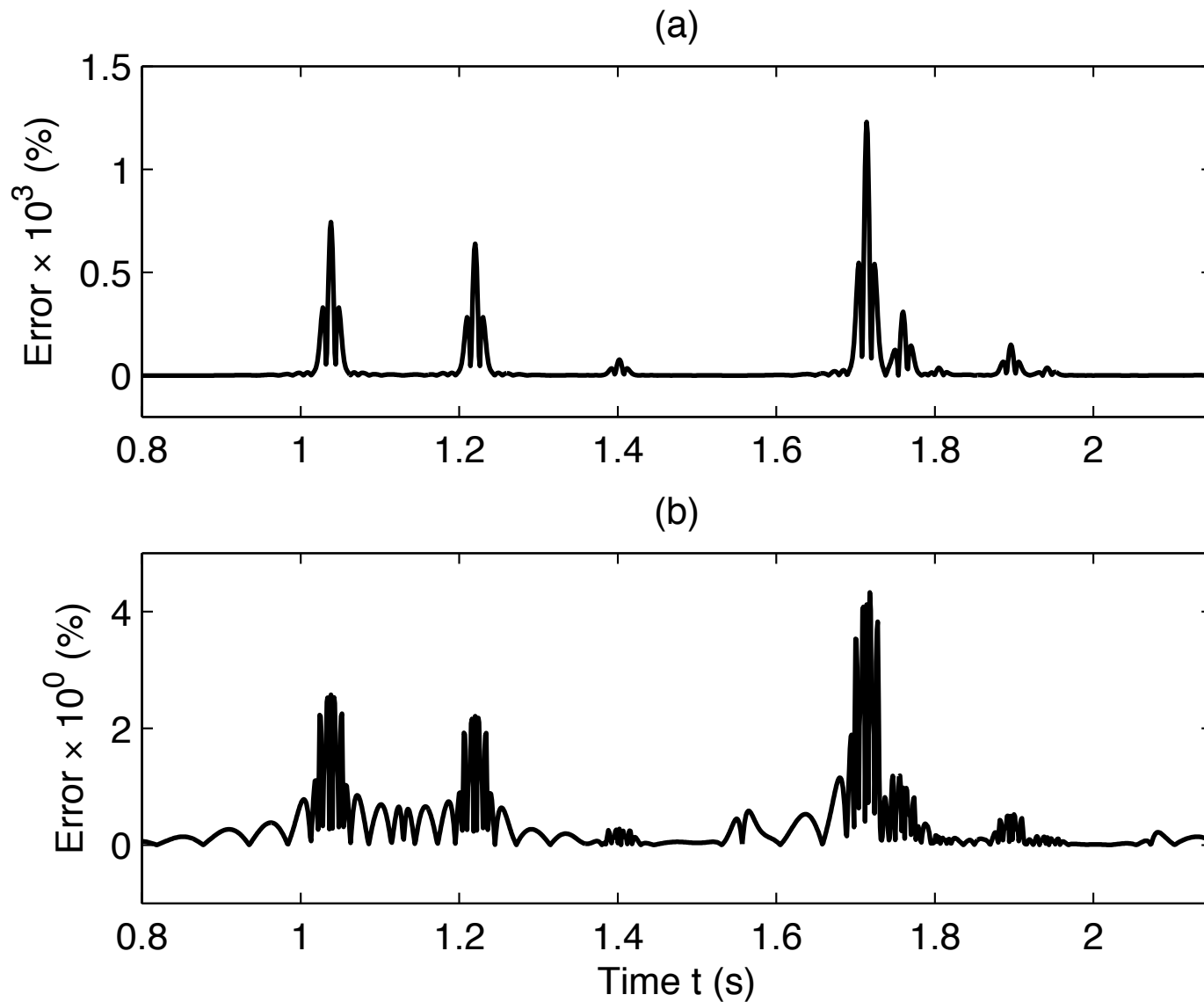
## Iterative reconstruction



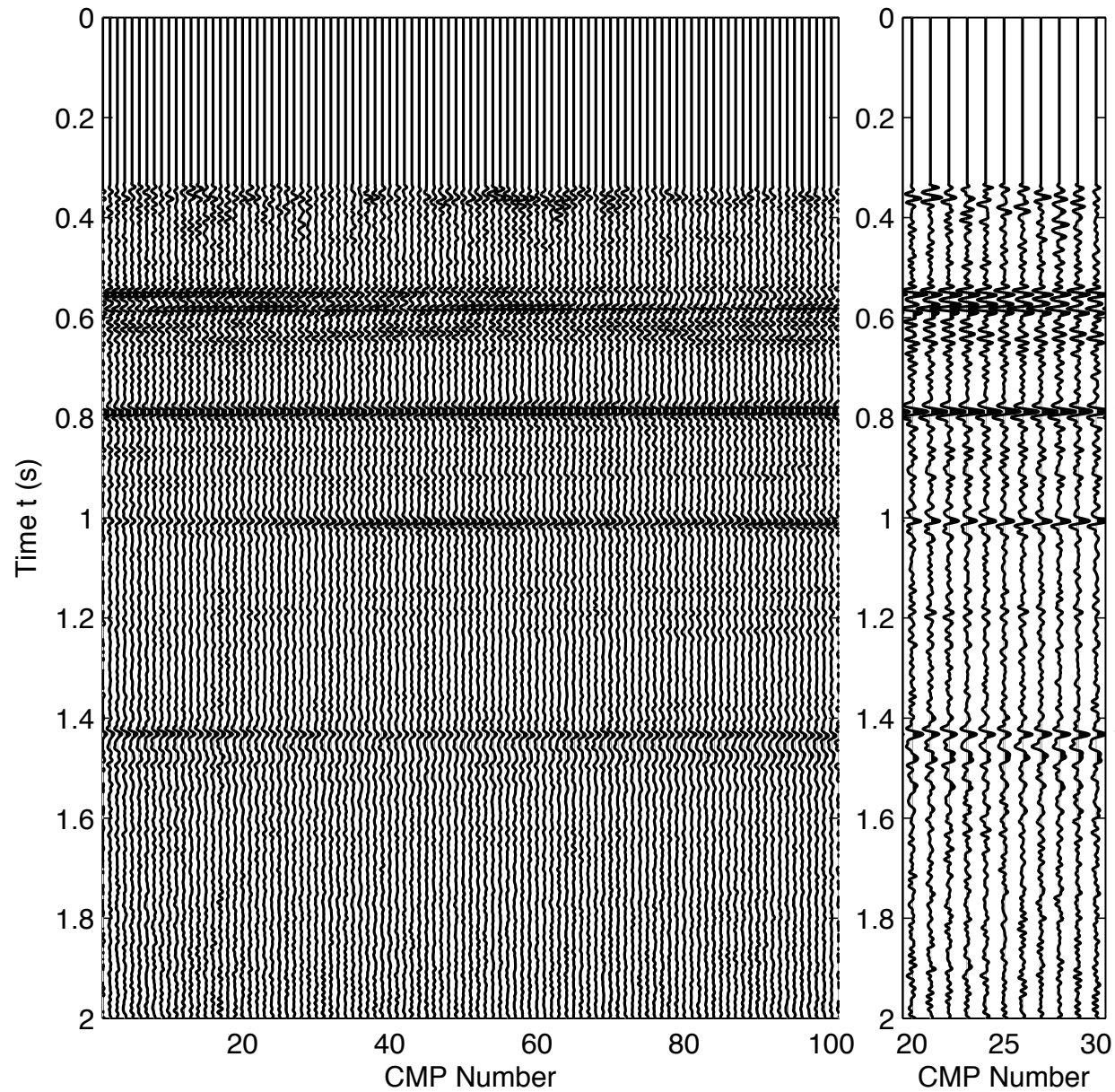
# Reconstruction from CWT maxima



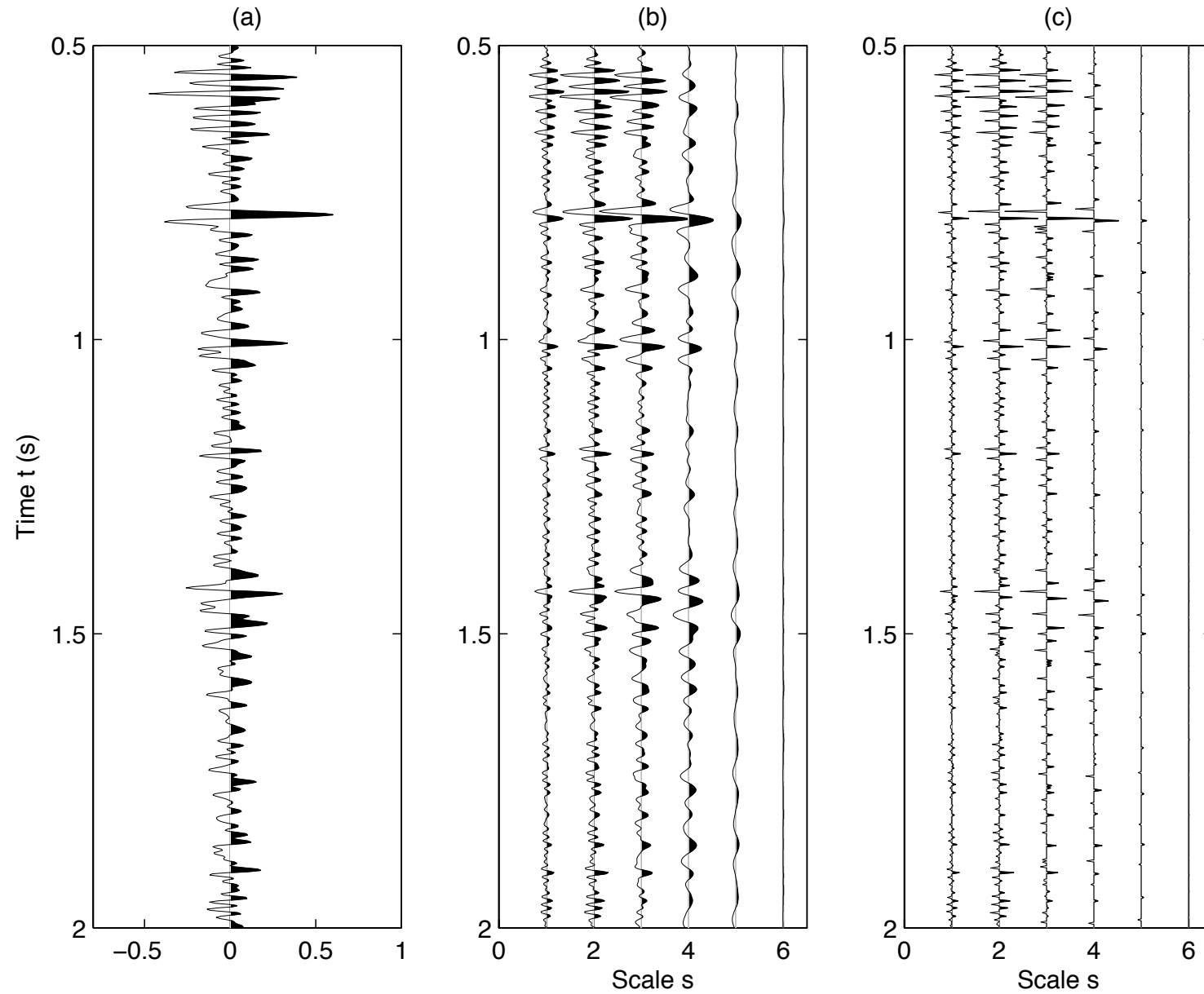
# Reconstruction from CWT maxima



# Aggressive denoising with thresholds

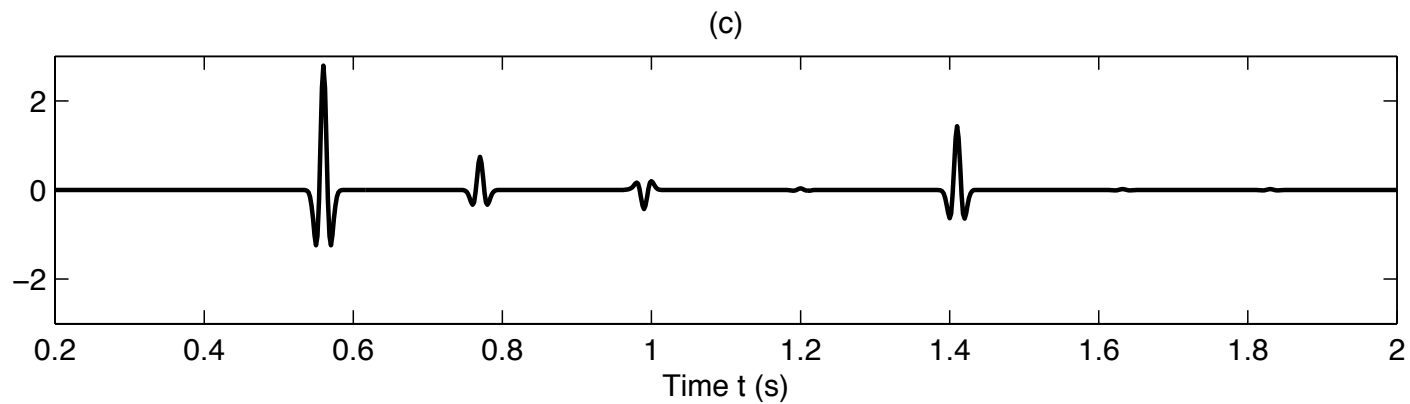
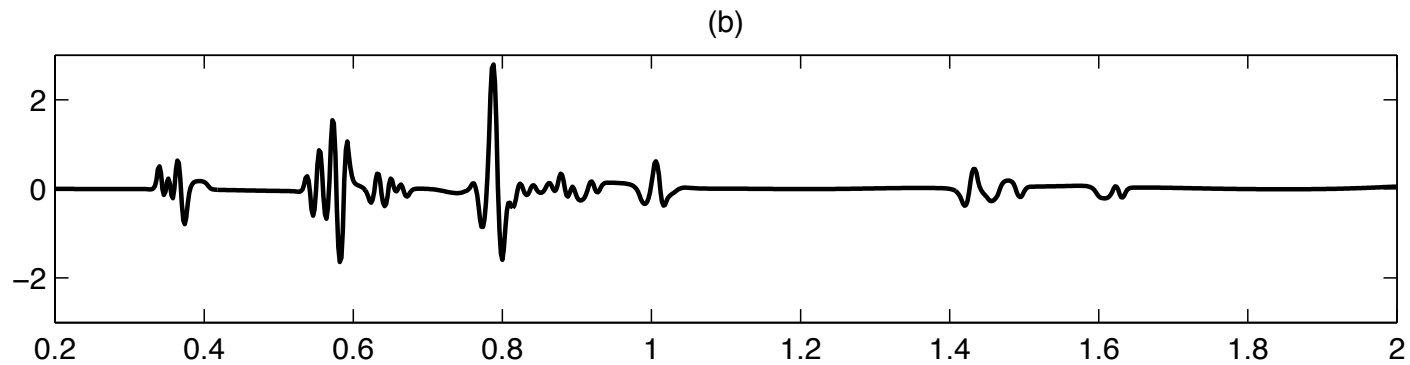
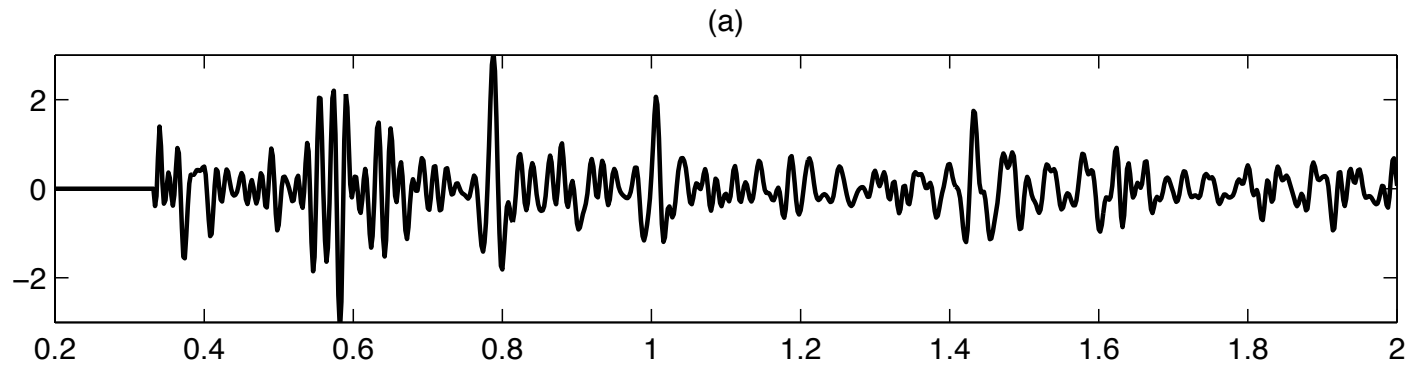


# Aggressive denoising with thresholds





# Aggressive denoising with thresholds



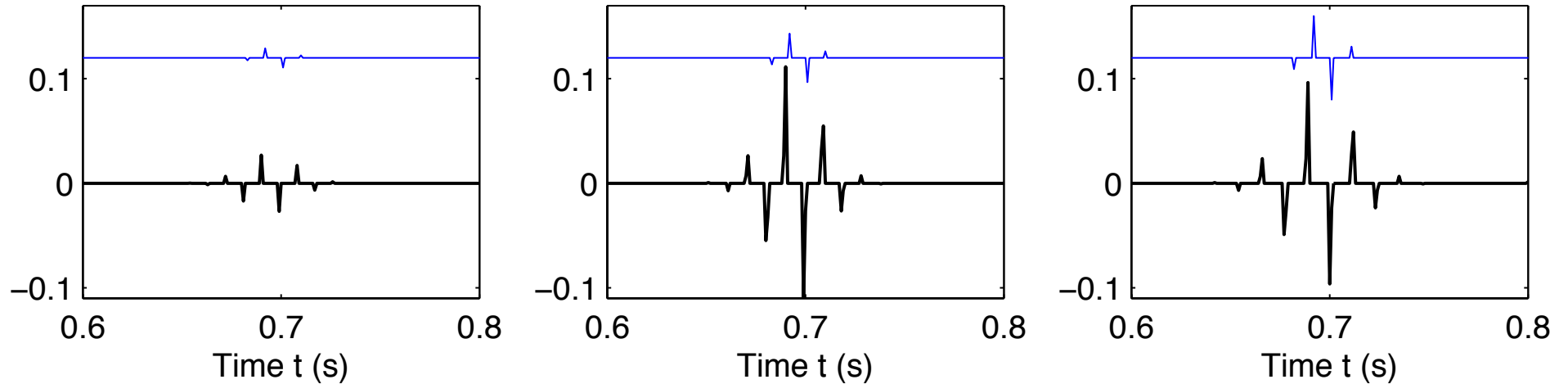
# Internal multiple prediction

PREDICTION = DATA × DATA × DATA

...nonlinear. Question:

prediction of wavelet maxima  
=  
wavelet maxima of prediction?

# Internal multiple prediction

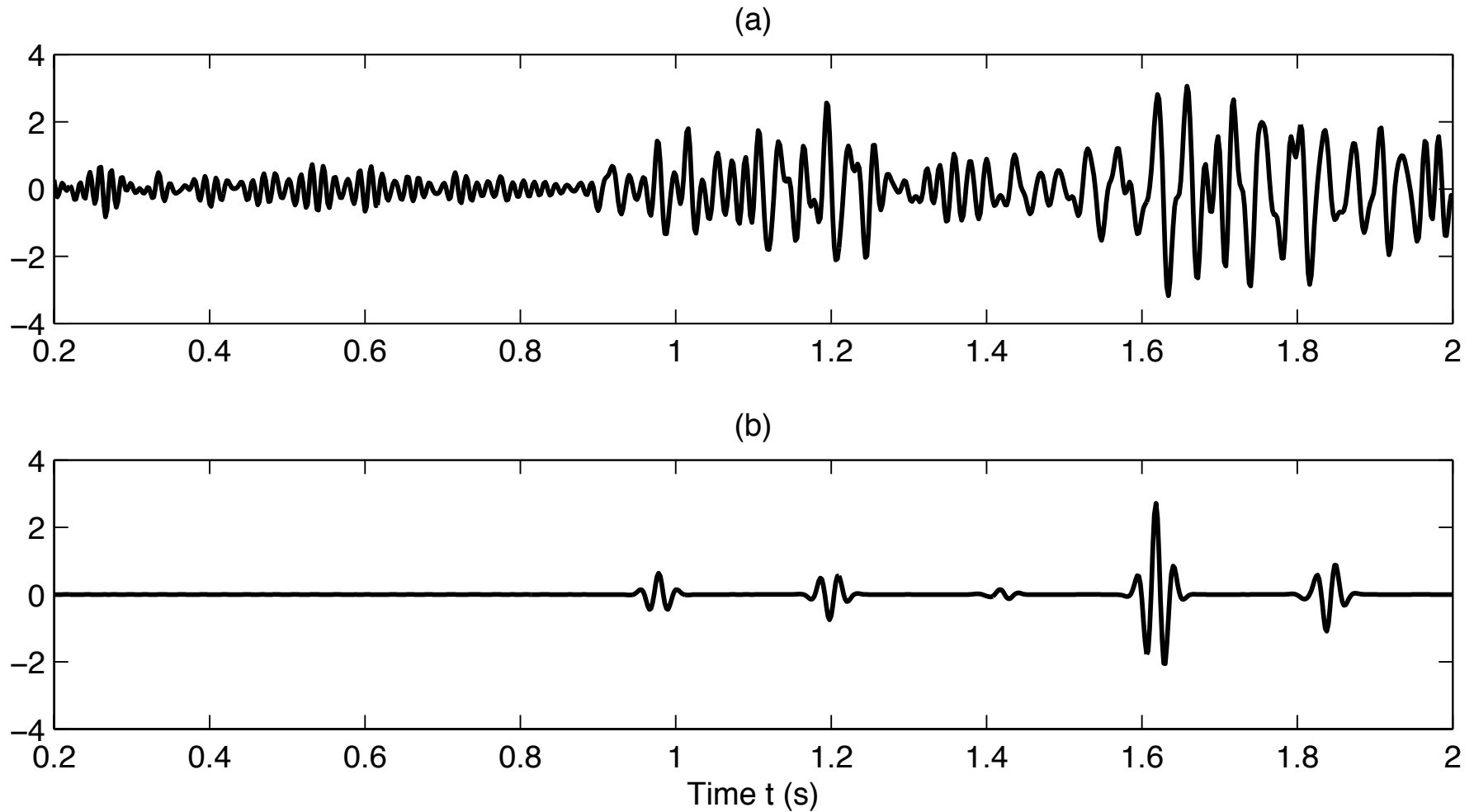


...yes, very close.

Opportunity for making IMP parameters vary with scale – surgical prediction

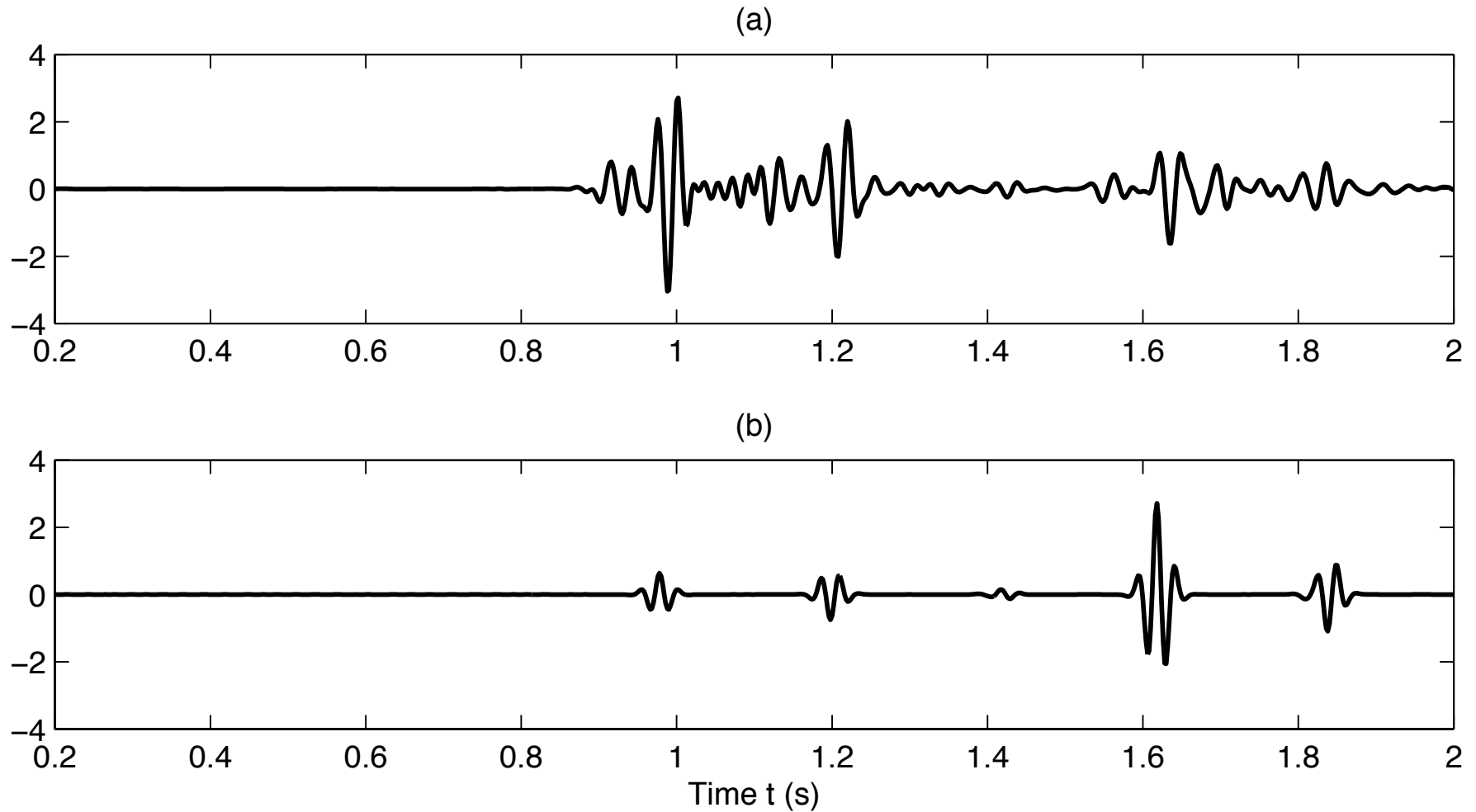
# Internal multiple prediction

Aggressively denoised data in prediction operator



# Internal multiple prediction

Aggressively denoised data in prediction operator



# Codes

MATLAB 1D internal multiple prediction

2014 in CREWES toolbox

(standalone available upon request)

MATLAB 1.5D internal multiple prediction

2014

MATLAB continuous wavelet transform ( $W$ ,  $W^{-1}$ )

MATLAB reconstruction from maxima  $(MW)^{-1}$

2014

# Acknowledgments



Nexen

M. J. Hernandez, H. Izadi