Drift time estimation by dynamic time warping

Tianci Cui and Gary F. Margrave
Outline

- Drift time
- Well-based 1D seismogram models
- Matching stationary and nonstationary seismograms
- Dynamic time warping
- Inclusion of internal multiples
- Conclusions and future work
Drift time estimation by DTW

- Drift time
- Well-based 1D seismogram models
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Drift time: well logs

Hussar well 12-27

- density (kg/m$^3$)
- p-velocity at 12.5 kHz (m/s)
Drift time: fake Q log

\[ v(f_s) = v(f_w)[1 + \frac{1}{\pi Q} \left( \frac{f_s}{f_w} \right)] \]

- \( f_s \): Seismic frequency
- \( f_w \): Well logging frequency

Kjartansson (1979)
Drift time: frequency-dependent velocity

\[ v(f_s) = v(f_w) \left[ 1 + \frac{1}{\pi Q} \left( \frac{f_s}{f_w} \right) \right] \]

- \( f_w \): Well logging frequency
- \( f_s \): Seismic frequency

Kjartanssen (1979)
Drift time

time-depth curve

- $f_w = 12.5\text{kHz}$
- $f_s = 30\text{Hz}$

theoretical drift time
Drift time estimation by DTW

- Drift time
- Well-based 1D seismogram models
- Matching stationary and nonstationary seismograms
- Dynamic time warping
- Inclusion of internal multiples
- Conclusions and future work
Stationary seismogram: $s(t)$

- Reflectivity
- Source wavelet
- $s(t)$: stationary seismogram

Minimum-phase dominant frequency: 30 Hz
Nonstationary seismogram: $sq(t)$

Synthetic zero-offset VSP model with Q effects

(Margrave and Daley, 2014)

upgoing field with Q effects
Nonstationary seismogram: $sq(t)$

Synthetic zero-offset VSP model with Q effects

(Margrave and Daley, 2014)

Surface receiver

$sq(t)$: nonstationary seismogram

(time(s))

(amplitude)

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

0 200 1200

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
1 D seismogram models

Q effects:
1 Diminishing amplitude
2 Widening wavelets
3 Delaying events ↔ Drift time
1 D seismogram models

max corr=0.48685, at lag=8.3

Theoretical drift time

time(s)

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

0 0.01 0.02

Drift time estimation by DTW

- Drift time
- Well-based 1D seismogram models
- **Matching stationary and nonstationary seismograms**
- Dynamic time warping
- Inclusion of internal multiples
- Conclusions and future work
Matching without drift time correction

Time-variant balancing and time-variant constant-phase rotation

max corr=0.48058, at lag=7.6

max corr=0.69087, at lag=2.3
Drift time correction

\[ s_{corr}(t) = s(t + \text{drift}(t)) \]

\text{drift}(t): \text{drift time}

\text{S}_{corr}(t): \text{stationary seismogram after drift time correction}
Matching with theoretical drift time correction

Matching perfection: time-variant balancing and time-variant constant-phase rotation

max corr=0.93752, at lag=1

max corr=0.93747, at lag=0.2
Matching with theoretical drift time correction

Matching perfection: time-variant balancing and time-variant constant-phase rotation

Drift time correction is necessary to match the stationary to nonstationary seismograms.

Calculation of drift time in industrial practice needs one of these:

- Knowledge of Q or
- A check-shot survey or
- Manually stretching and squeezing the synthetic seismogram
Drift time estimation by DTW

- Drift time
- Well-based 1D seismogram models
- Matching stationary and nonstationary seismograms
- Dynamic time warping
- Inclusion of internal multiples
- Conclusions and future work
Dynamic time warping (Hale, 2012):
• Estimates the time shift between two seismograms
• Based on constrained optimization algorithm
• Realized by dynamic programming
• Similar to time-variant crosscorrelation but more sensitive to the rapid-varying time shift

We use dynamic time warping (DTW) to estimate the drift time between the stationary and nonstationary seismograms caused by anelastic attenuation
DTW: drift time estimation

[Graph showing seismograms and time(s) zoomed in]
DTW: drift time estimation

seismograms

s(t)
sq(t)

s(n)
sq(n)

time(s)

zoomed in

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UNIVERSITY OF CALGARY
DTW: drift time estimation

\[ e(m, n) = [s(n) - sq(n + m)]^2 \]

\(-50 \leq m \leq 0\)

\[ s(n) \quad sq(n + m) \quad 0 \leq m \leq 50 \]
DTW: drift time estimation

seismograms

\[ e(m, n) = [s(n) - sq(n + m)]^2 \]

\(-50 \leq m \leq 50\)
DTW: drift time estimation

\[ e(m, n) = [s(n) - sq(n + m)]^2 \]

\[-50 \leq m \leq 50, 1 \leq n \leq 829 \]
DTW: drift time estimation

\[ e(m, n) = [s(n) - sq(n + m)]^2 \]
\[-50 \leq m \leq 50, 1 \leq n \leq 829 \]

\[ m(n) = \text{round} \left( \frac{\text{drift}(n)}{dt} \right) \]

\( n \): sample number, \( dt \): time sample rate
\( \text{drift}(n) \): drift time, \( m(n) \): drift lag
The alignment error is nearly zero along the theoretical drift lag.

Choose a path traveling from \( n = 1 \) to 829, sum alignment errors along this path. The estimated drift lag sequence is the path of the minimal cumulative alignment error.
DTW: drift time estimation

Theoretical drift lag \( m(n) \)

101^{829} \Leftrightarrow \text{infinite}

Constraint: \(|m(n) - m(n - 1)| \leq 1\)
Dynamic Programming

Alignment error array

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27 possible paths
Dynamic Programming: accumulation

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Cumulative alignment error array

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27 possible paths
**Dynamic Programming: accumulation**

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27 possible paths

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**Constraint:** \(|m(n) - m(n - 1)| \leq 1\)
Dynamic Programming: accumulation

Alignment error array

\[
\begin{array}{ccc}
1 & 4 & 7 \\
0 & 3 & 5 \\
-1 & 6 & 2 \\
\end{array}
\]

Cumulative alignment error array

\[
\begin{array}{ccc}
1 & 4 & ? \\
0 & 3 & \\
-1 & 6 & \\
\end{array}
\]

27 possible paths

Constraint: \(|m(n) - m(n - 1)| \leq 1|
Dynamic Programming: accumulation

### Alignment error array

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There are 27 possible paths.

**Constraint:** \(|m(n) - m(n - 1)| \leq 1\)
Dynamic Programming: accumulation

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27 possible paths

Constraint: $|m(n) - m(n - 1)| \leq 1$
Dynamic Programming: accumulation

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27 possible paths

Constraint: $|m(n) - m(n - 1)| \leq 1$
Dynamic Programming: accumulation

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Constraint: \(|m(n) - m(n - 1)| \leq 1\)
Dynamic Programming: accumulation

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Dynamic Programming: accumulation

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27 possible paths

Constraint: \(|m(n) - m(n - 1)| \leq 1\)
## Dynamic Programming: accumulation

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*27 possible paths*

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*3 possible paths*

**Constraint:** $|m(n) - m(n - 1)| \leq 1$
Dynamic Programming: backtracking

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27 possible paths

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3 possible paths

Constraint: $|m(n) - m(n - 1)| \leq 1$
Dynamic Programming: backtracking

### Alignment error array

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27 possible paths

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3 possible paths

**Constraint:** \( |m(n) - m(n - 1)| \leq 1 \)

**Estimated drift lag:** \( m(n) = [0\ 0\ 1] \)
Dynamic Programming

Cumulative alignment error array

\[
m(n) = [0, -1]
\]

Dynamic: optimal solution varies at different stage
Warping path: drift lag
DTW: drift time estimation

The drift lag sequence is constrained to change in blocks of $b$ samples.

**Constraint:** $|m(n) - m(n - 1)| \leq 1$

**Further Constraint:**

$$\sum_{k=1}^{b}|m(n - k + 1) - m(n - k)| \leq 1$$

$101^{829} \rightarrow \text{infinite}$
DTW: drift time estimation

The drift lag sequence is constrained to change in blocks of $b$ samples.

- Constraint: $|m(n) - m(n - 1)| \leq 1$
- Further Constraint:
  $$\sum_{k=1}^{b}|m(n - k + 1) - m(n - k)| \leq 1$$

$101^{829} \Rightarrow \text{infinite}$
DTW: drift time estimation

\[ \text{drift}(t) = m(t) \times dt \]

\( m(t) \): estimated drift lag

\( \text{drift}(t) \): estimated drift time
DTW: matching seismograms

max corr=0.91072, at lag=0.6

max corr=0.94505, at lag=0.2
Drift time estimation by DTW

• Drift time
• Well-based 1D seismogram models
• Matching stationary and nonstationary seismograms
• Dynamic time warping
• Inclusion of internal multiples
• Conclusions and future work
Inclusion of internal multiples

\[ sq(t) \]

upgoing field with Q effects

\[ sqi(t) \]

upgoing field with Q and internal multiple effects
Inclusion of internal multiples: sqi(t)

\[ sq(t) \text{: stationary seismogram} \]
\[ sq(t) \text{: nonstat}(Q \text{ effects}), \text{max corr}=0.48685, \text{at lag}=8.3 \]
\[ sqi(t) \text{: nonstat}(Q+IM \text{ effects}), \text{max corr}=0.38385, \text{at lag}=6.6 \]

\[ \frac{1}{Q_{\text{intrinsic}}} + \frac{1}{Q_{\text{stratigraphic}}} = \frac{1}{Q_{\text{apparent}}} \]

(Richards and Menke, 1983)
Inclusion of internal multiples

max corr = 0.87473, at lag = 0.9

max corr = 0.9253, at lag = 0.2

s(t) + drift correct

s(qi(t)) + TV balans + TV phsrot
Conclusions

• DTW succeeds in estimating drift time automatically without knowledge of Q or a check-shot survey.

• Application of drift time correction results in a much simpler residual phase.

• DTW estimates drift time associated with apparent Q including both intrinsic and stratigraphic effects.
Future work

- Conduct stationary and nonstationary deconvolution on the seismic trace and tie the deconvolved seismic trace to well log reflectivity by DTW

- Estimate Q value from the drift time estimated by DTW
Acknowledgements

- CREWES sponsors
- NSERC: grant CRDPJ 379744-08
- CREWES staff and students

THANK YOU!
Choosing $b$ values

(a) $max \text{ corr}$

(b) $max \text{ corr (zoomed in)}$

(c) lag where corr is max

(d) lag where corr is max (zoomed in)
Choosing $b$ values

- **Drift time**
  - theory
  - $tdr(b=1)$
  - $tdr(b=10)$
  - $tdr(b=30)$

- **Seismograms**
  - $b=1$: max corr=0.84115 at lag=0.5
  - $b=10$: max corr=0.87748 at lag=0.1
  - $b=30$: max corr=0.84547 at lag=0.1
Applications of DTW

- Tying synthetic to recorded seismograms
- Registration of P– and S–wave images
- Residual normal moveout correction
- Alignment of images computed for different source-receiver offsets or propagation angles.