FWI and the “Noise” Quandary

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Full waveform inversion (FWI)

FWI performs an inversion that honors every sample of the input data as real signal. How does the “noise” affect these results?

Model-based Inversion

(Lines and Treitel, 1984, among others)

- Define model parameters, x.
- Compute model response, f.
- Compare f to data values, y.
- Minimize e=y-f, “error of fit”.
- We can minimize e (often in a least squares sense). $e^T e$
- Solve for parameter change vector,

$$A\Delta x = b = y - f^0$$

Jacobian Matrix

- The Jacobian matrix, A, is a rectangular matrix of size n by p and its cost of computation can control the cost of the inversion.
- n =no. of data points
- p = no. of model parameters:

$$A_{ij} = \frac{\partial f_i}{\partial x_j}$$

What is “Seismic Noise”?

Definition: (Sheriff’s Dictionary)

1. An unwanted signal.

2. Seismic energy other than primary reflections; includes microseisms, source – generated noise, multiples, …

3. Sometimes divided into coherent noise and random noise.

Revised definition of “Seismic Noise”

With multiples and microseisms being used in inversion and considered as “signal”, a revised definition might be:

Noise: Any recorded signal that is unrelated to properties of the solid earth. Examples could include effects of wind, ocean waves, traffic, power lines, and recording instrument noise.

FWI noise tests for random and correlated noise

This talk will deal with simple tests of FWI on random noise and on multiples (an“undesired signal”).

Two examples will be examined:

1. The effect of random noise on a simple inversion example.
2. The inversion of primaries and multiples for reflection coefficients.

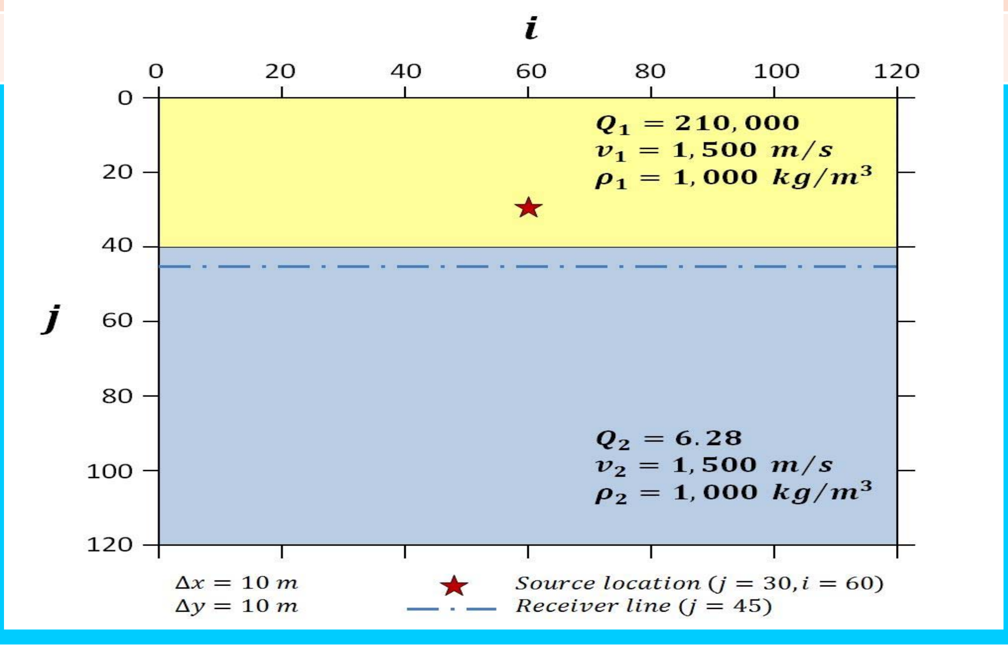
A Simple Synthetic Example – Transmission-Dominated Models

source in row 30, column 60;

receivers in every column, row=45

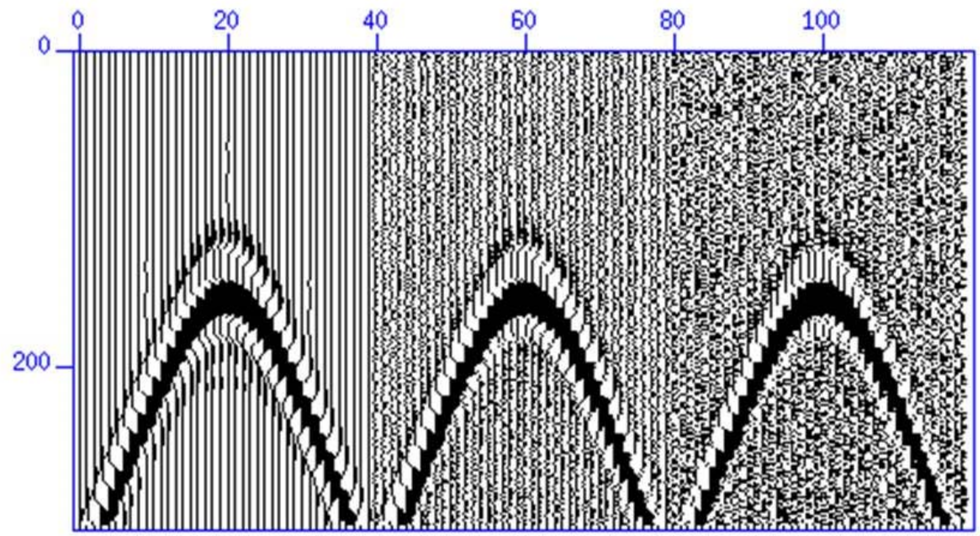
Solve for Q in layer 2

Layers	Cells	Q	
			Model 2
1 (yellow)	1-40		Q=210000 V=1500m/s Rho=1000.
2 (blue)	41-120		Q=6.28 V=1500 m/s Rho=1000.



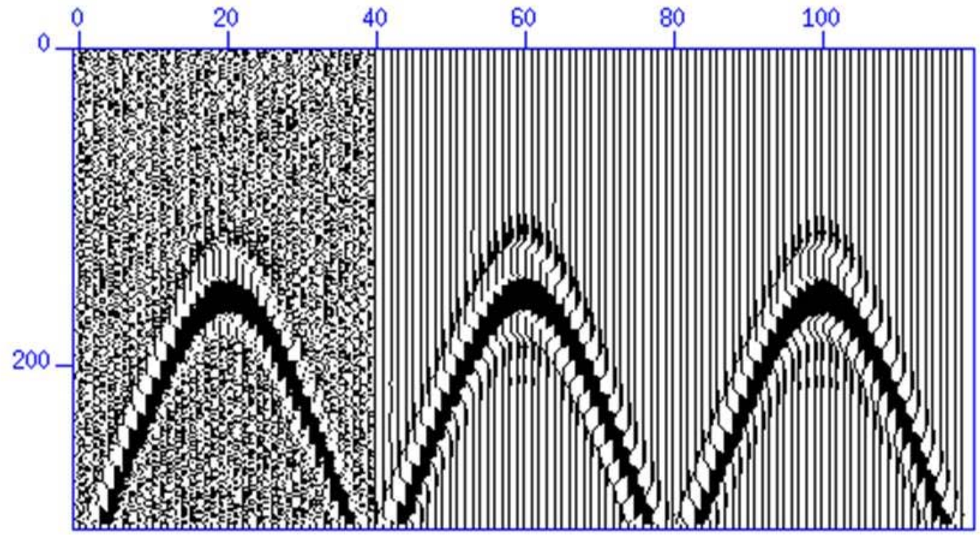
Inversion tests

- Synthetic data used in inversion tests. Left seismic section that is noise-free, middle seismic section has S/N =5.0 and right section with S/N = 2.5.



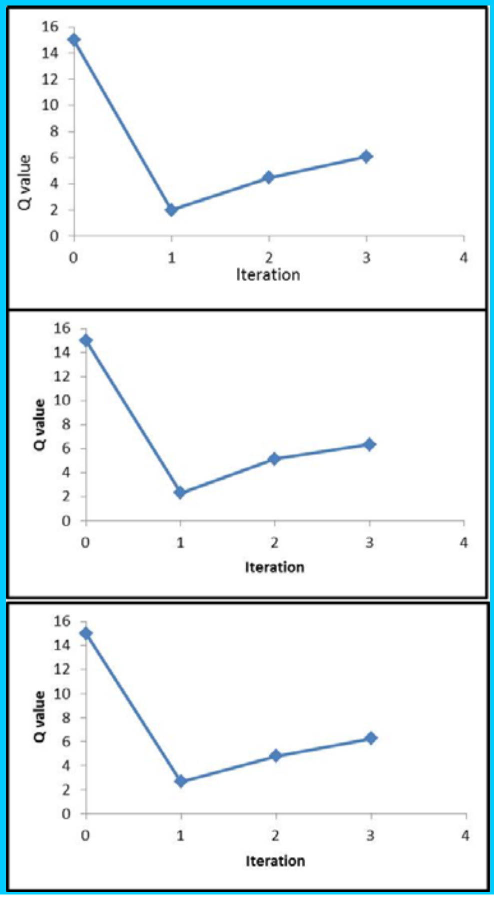
Inversion of Noisier Data

- Inversion of model data with S/N=2.5: Left Input data with Q=6.28 with additive noise, middle seismic section for initial guess Q=15 and right section with converged result after 3 iterations (Q=6.26).



Full waveform inversion (FWI)

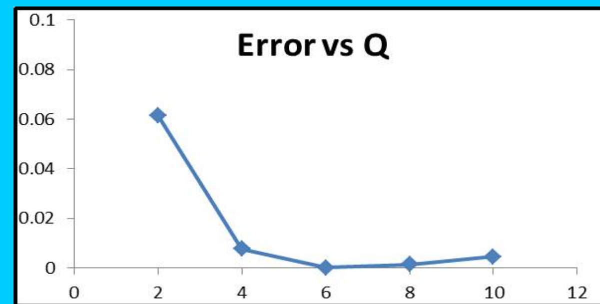
- Q estimates accurate to within 3% after 3 iterations.
- Inversions of data with varying amounts of noiseconverged to the answer within 3 iterations.
- FWI appears to be robust.

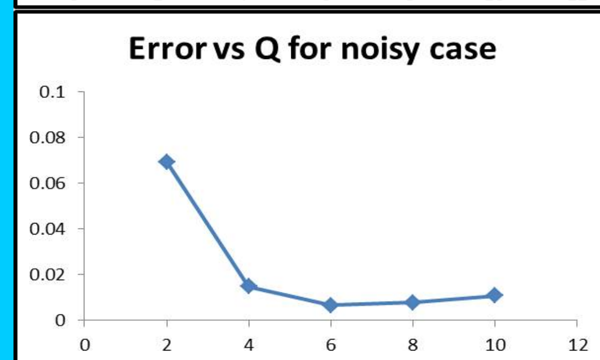


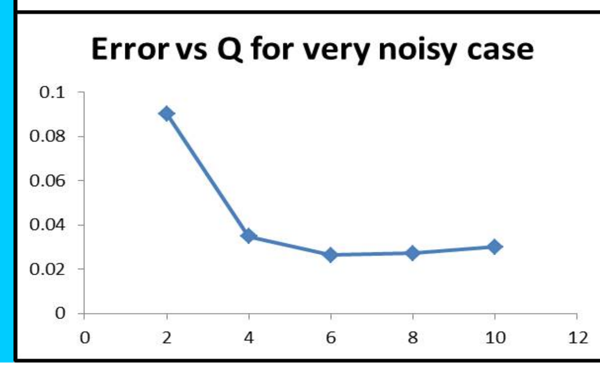
Actual Q value = 6.28
Initial guess Q=15
Converged answers after 3 iterations

Error vs Q for different noise levels

- Q estimates accurate to within 3% after 3 iterations.
- **Although error of fit worsens with decreased S/N, the error minimum is reached near the correct value of Q in each case.**







Top: noise-free case
Middle: noisy
Bottom: very noisy

Robustness of FWI

- The convergence of FWI to the correct answer for modest amounts of additive random noise can be understood if we examine the mathematical formulation for the inversion.

Model-based Inversion

- Recall the least squares solution for noiseless case

$$A\Delta x = b = y - f^0$$

$$A^T A\Delta x = A^T b$$

- Compare to the noisy case where b is replaced by b+n.

$e^T e$

$$A^T A\Delta x = A^T b + A^T n$$

Model-based Inversion

- If noise is random and zero mean, then the crosscorrelation with A is 0.

$$A^T n \approx 0$$

- Hence, for this noisy case, the result is largely unaffected by noise.

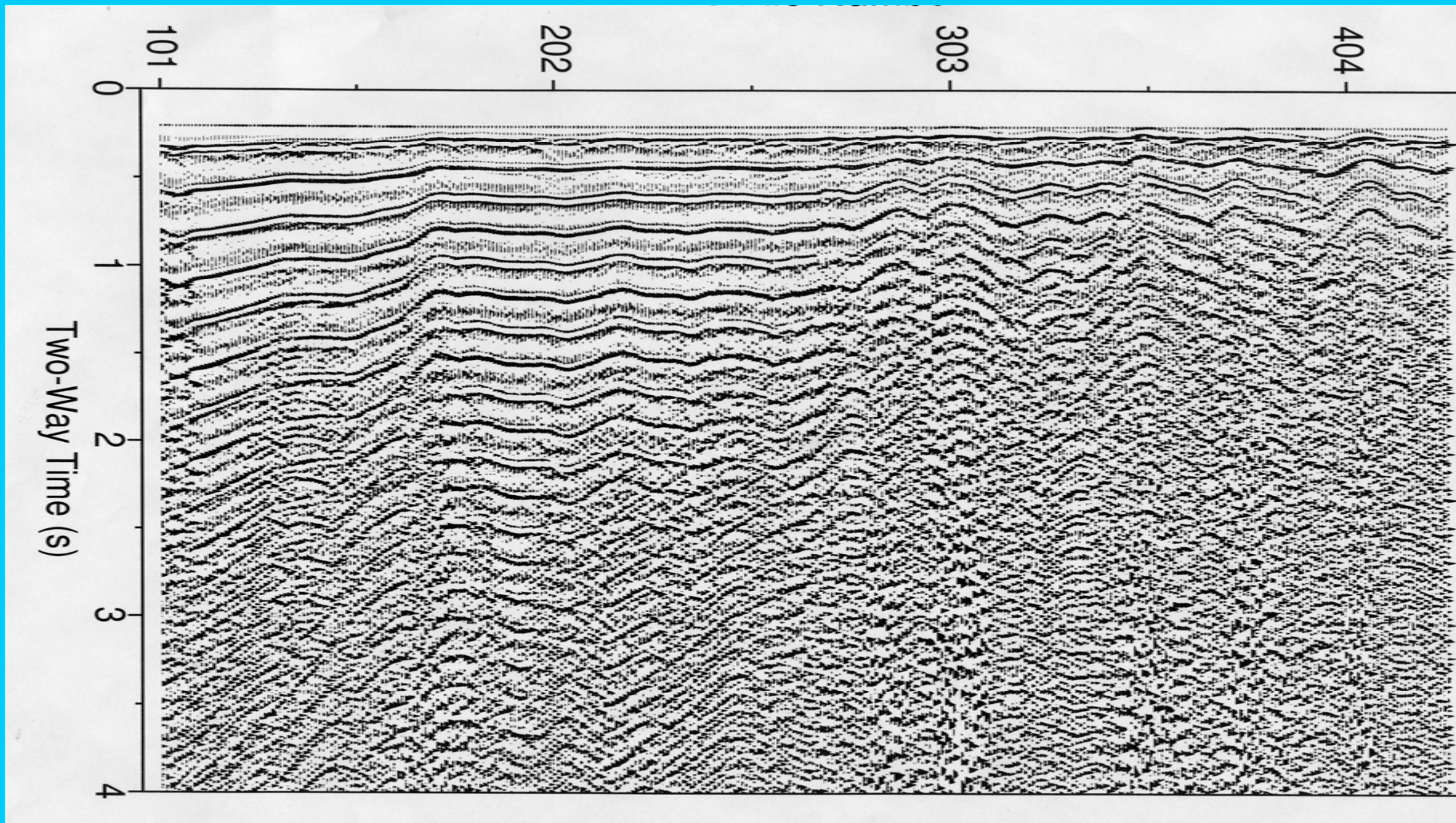
$e^T e$

$$A^T A \Delta x \approx A^T b$$

Preliminary Results

- Full waveform inversion for these simple Q-estimation problems is not overly sensitive to noise
- Error of data fit deteriorates slightly with noise but accuracy of Q estimate is consistent.

How do we use seismic data such as this?

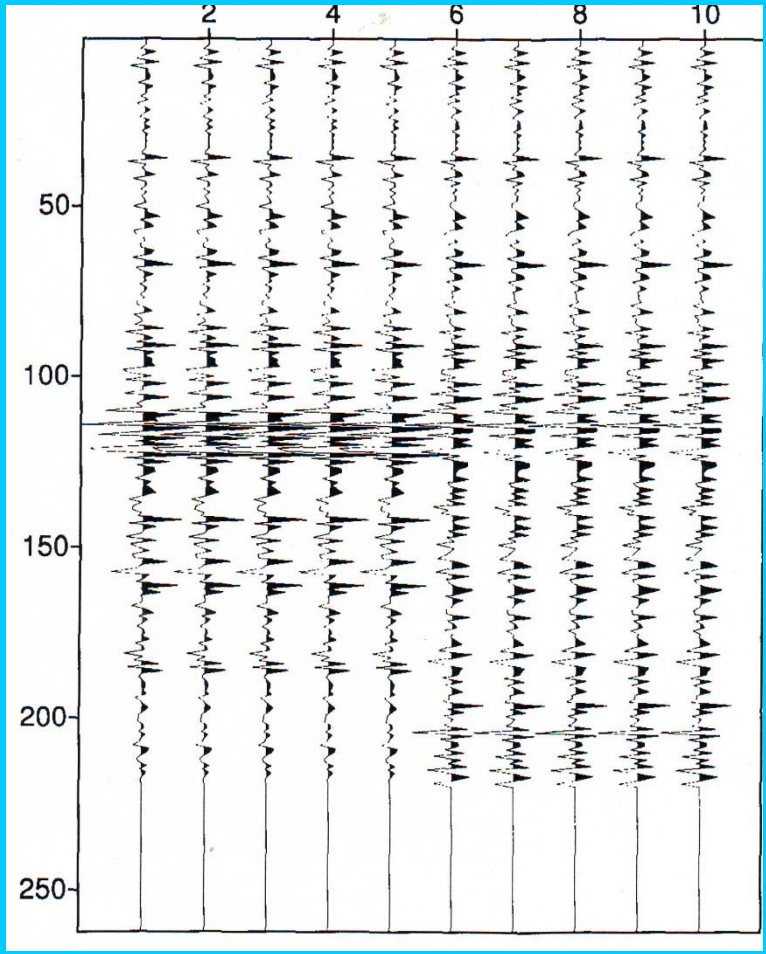


- Sometimes multiple energy can dominate the entire seismic section as in this offshore Newfoundland example from the MUN Ph.D. thesis of Simon O’Brien (1997)

Use multiples as signal to be inverted?

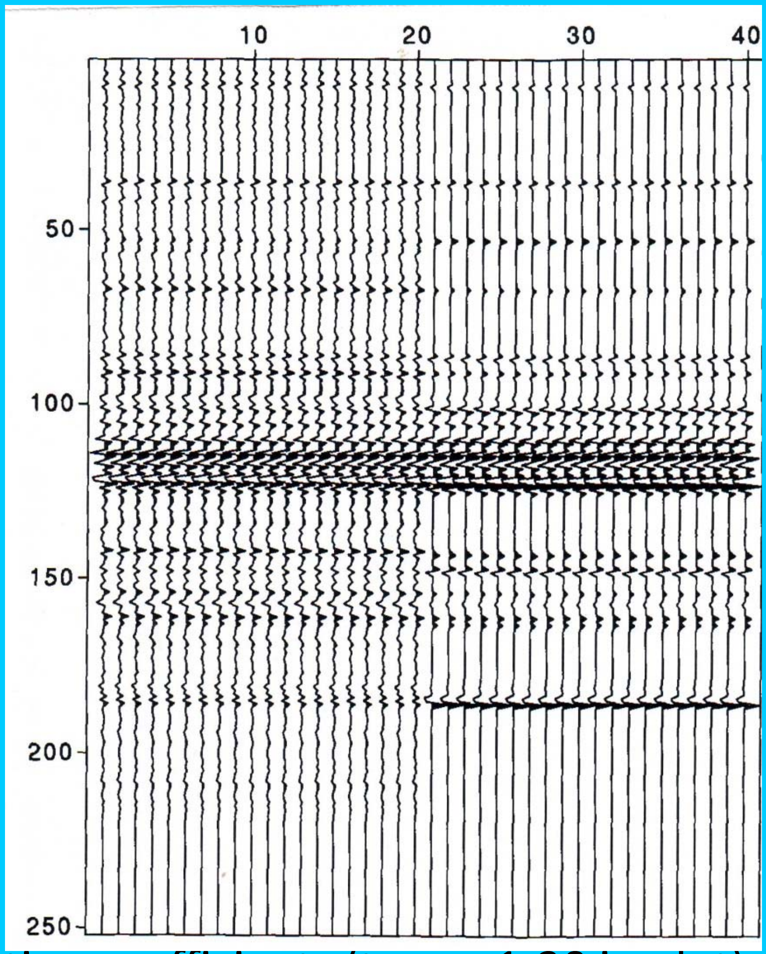
- After trying nearly every multiple suppression known to the industry, O'Brien was able to invert for primary reflections by treating multiple arrivals as "signal" rather than "noise".
- Some model tests were done at about the same time by Lines (1996 CJEG) to compare inversion of multiple energy to deconvolution results.

Comparison of Reflectivity with Impulse Response



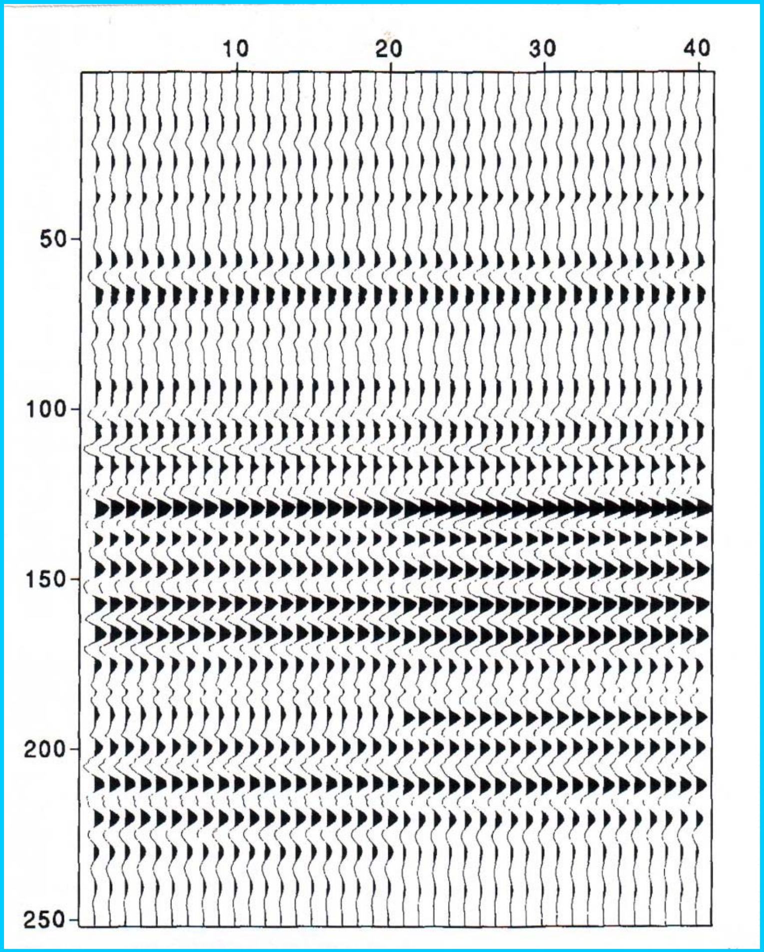
- Comparison of reflectivity (repeated traces 1-5) with impulse response (repeated traces 6-10) from Lines (1996)

Inversion by model fitting of primaries and multiples



- The actual reflection coefficients (traces 1-20 in plot) are compared to the estimated reflection coefficients (traces 21-40) that are obtained from the inversion after 5 iterations (from Lines, 1996).

Comparison of data to model response following inversion



- Data in traces 1-20 is matched well by the model response in traces 21-40 from Lines (1996).

Imaging and inversion of multiples

- Weglein (2014 SEG presentation) stated that the imaging condition of Claerbout (1971) does not handle multiples.
- There was subsequent debate about the validity of imaging multiples. The Schuster "rear view mirror example" served to support the imaging of multiples.
- We can use model-based inversion to estimate primary reflection coefficients.

Conclusions

- Random noise effects on FWI may not be as bad as originally anticipated.
- FWI of multiples may prove useful.
- Imaging of multiples through migration methods – the jury is still out.
- FWI is robust if we start close to the answer.

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