

# **FWI with source illumination: A synthetic case study**

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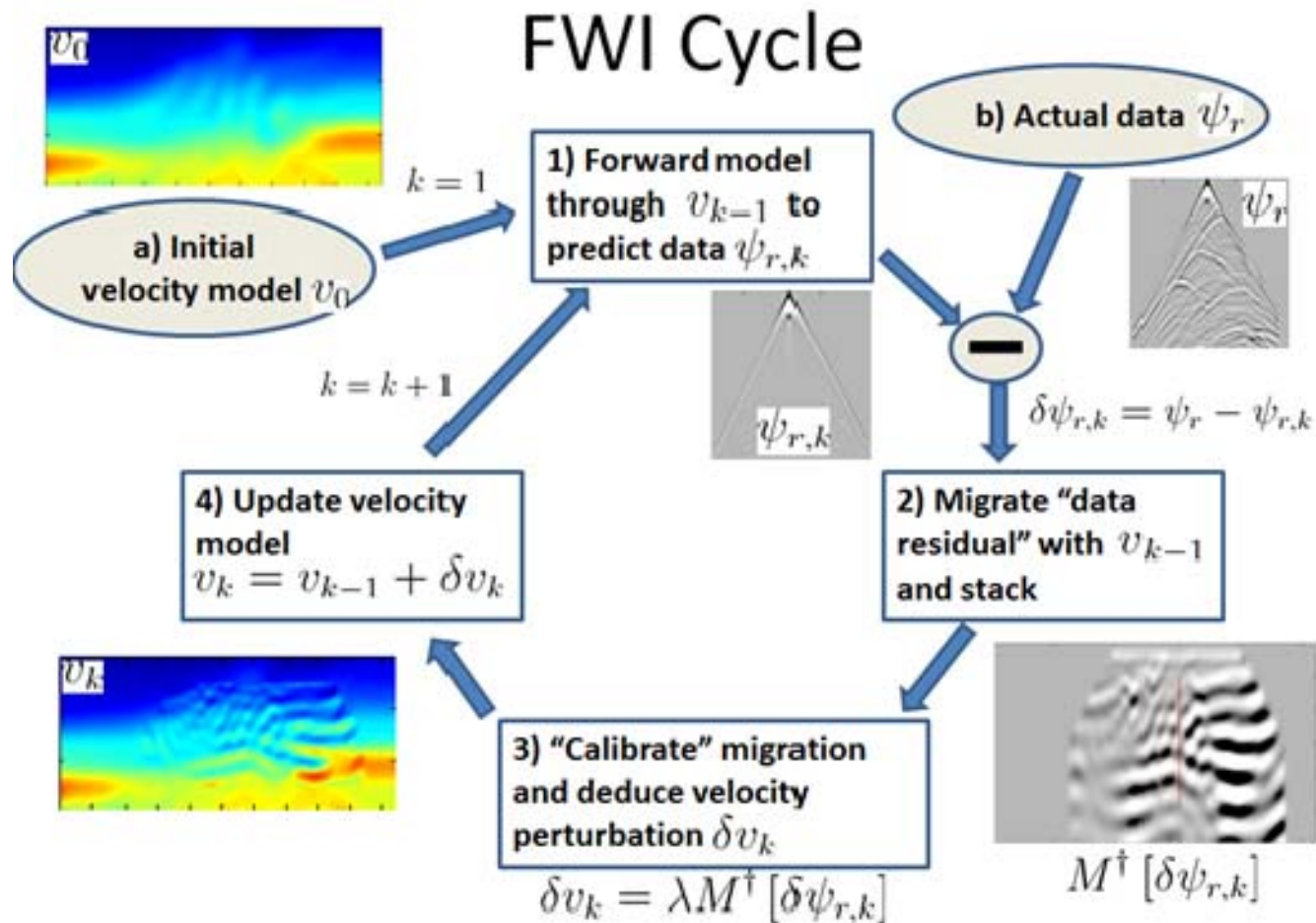
# Outline

- Introduction to FWI
- Synthetic Examples
  - *Model with flat layers*
    - ❖ Comparing a linear velocity  $v(z)$  model with a smooth version of the real model
    - ❖ Comparing a line search method with well update for step length calculation
  - *Model with structures*
    - ❖ Comparing a line search method with well update for step length calculation
- Results
- Conclusions and Discussions
- Future work

# Introduction

**Full waveform inversion is an optimization technique that seeks to find a model of the subsurface that best matches the observed field data at every receiver location. The method begins from a best guess of the true model, which is iteratively improved using linearized inversions methods**

# Introduction



Margrave et al, 2012

# Introduction

## Objective functions (functionals)

$$1) \quad \phi_k = \sum_{s,r} (\psi - \psi_k)^2$$

Conventional FWI, **validates** the predicted data  $\psi_k$  against the observed data  $\psi$

$$2) \quad \beta_k = \left\| \lambda G_k - (V_{well} - V_{BG})_k \right\|^2$$

**Validates** the migration velocity model  $V_{BG}$  against the known velocity model  $V_{well}$  at the well. Using well control (IMMI) as an alternative to line search methods

- We can search for a scalar  $\lambda$  using a line search method - Steepest Gradient Method
- Or we can calculate a scalar  $\lambda$  by minimizing 2) above w.r.t  $\lambda$

$$\lambda = \frac{\sum_j \delta V_j G_j}{\sum_j G_j^2} \quad \delta V_j = (V_{well} - V_{BG})_j \quad j \text{ indicates sample number.}$$

# Synthetic Examples: Model with flat layers

## Forward modelling parameters

Shot spacing	50 meters
Receiver spacing	5 meters
Dominant frequency	50 hertz
Number of Shots	40
Number of Receivers	448
Layer thickness of model	500 meters

# Synthetic Examples: Model with flat layers

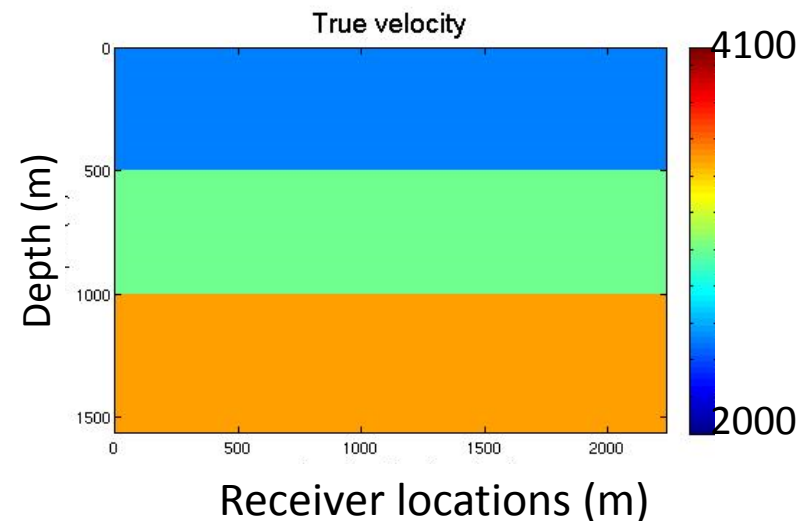
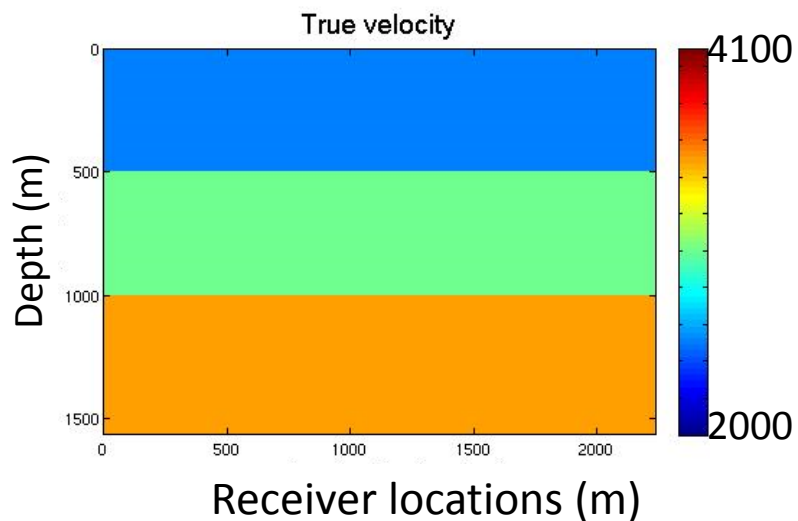
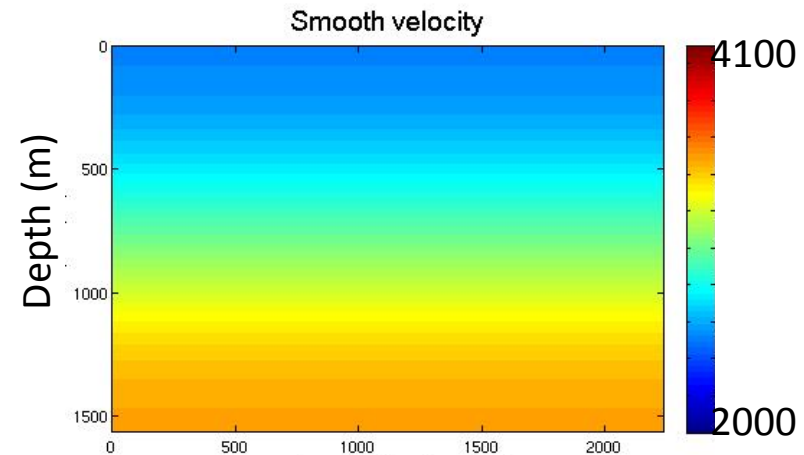
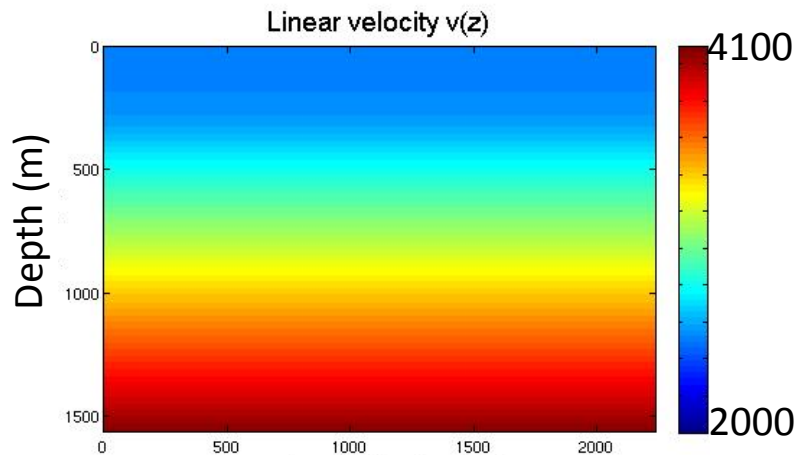
Three Layers of velocities **2.5Km/s**, **3Km/s** and **3.5Km/s**.

Inversion

- **PSPI** migration algorithm with a **Deconvolution IC**
  - ❖ Deconvolution IC has the closest link to estimating reflectivity if wavelet can be estimated correctly
  - ❖ The Cross Correlation IC provides an image amplitude that is the product of source and receiver wavefields and has the unit of amplitude square.
  - ❖ Source-normalized CC IC has the same unit and scaling as the Reflectivity
- Multi-scale approach (Pratt,1999)

# Synthetic Examples: Model with flat layers

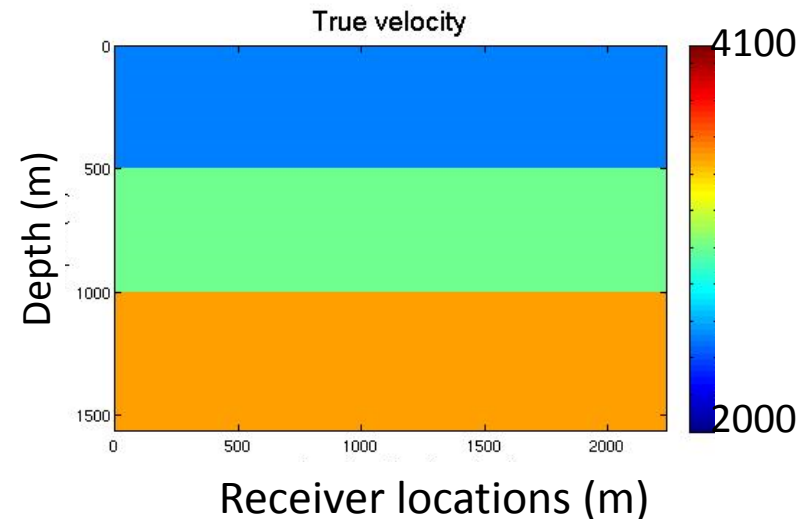
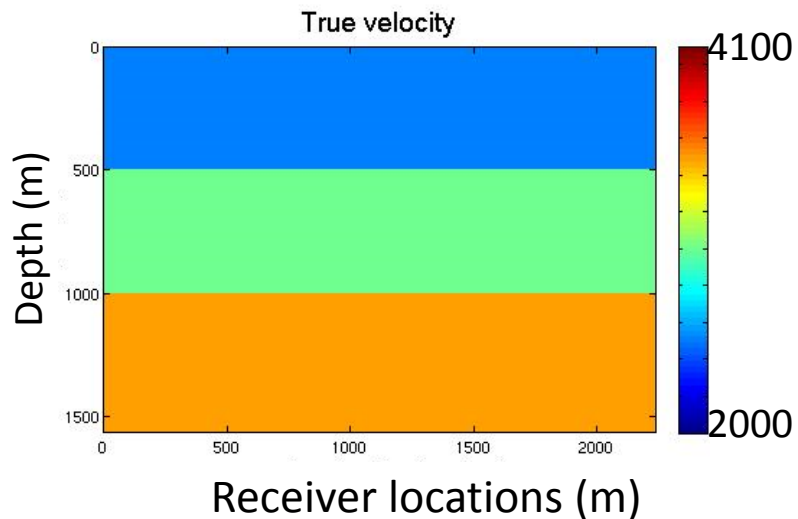
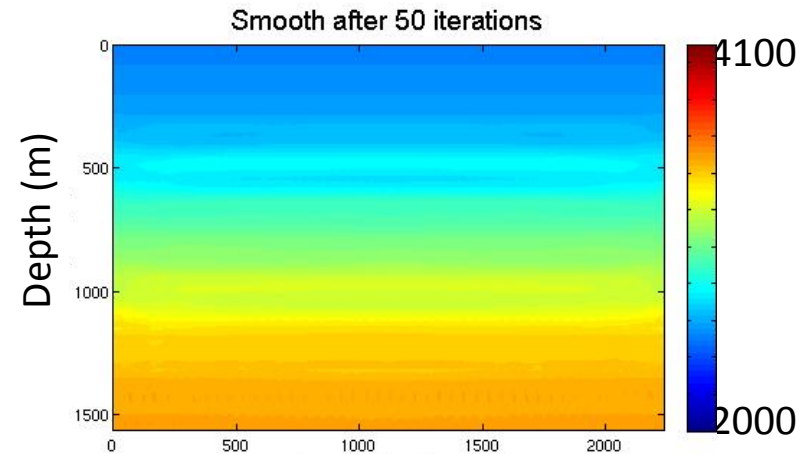
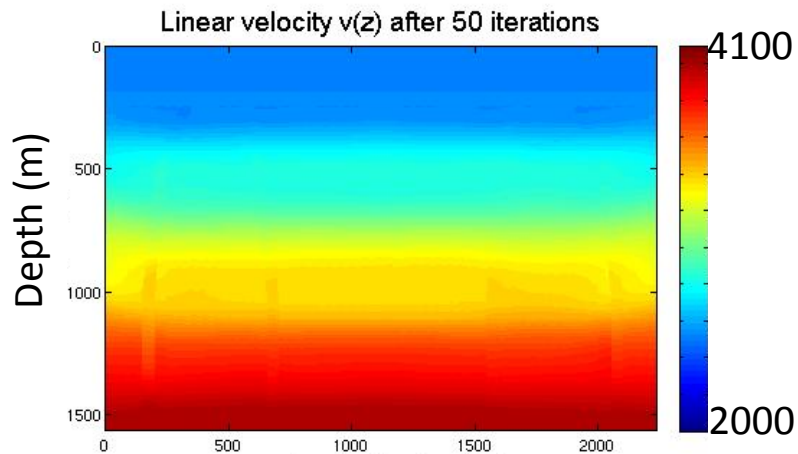
Comparing Linear velocity  $v(z)$  as starting model with a smooth version of the true velocity model





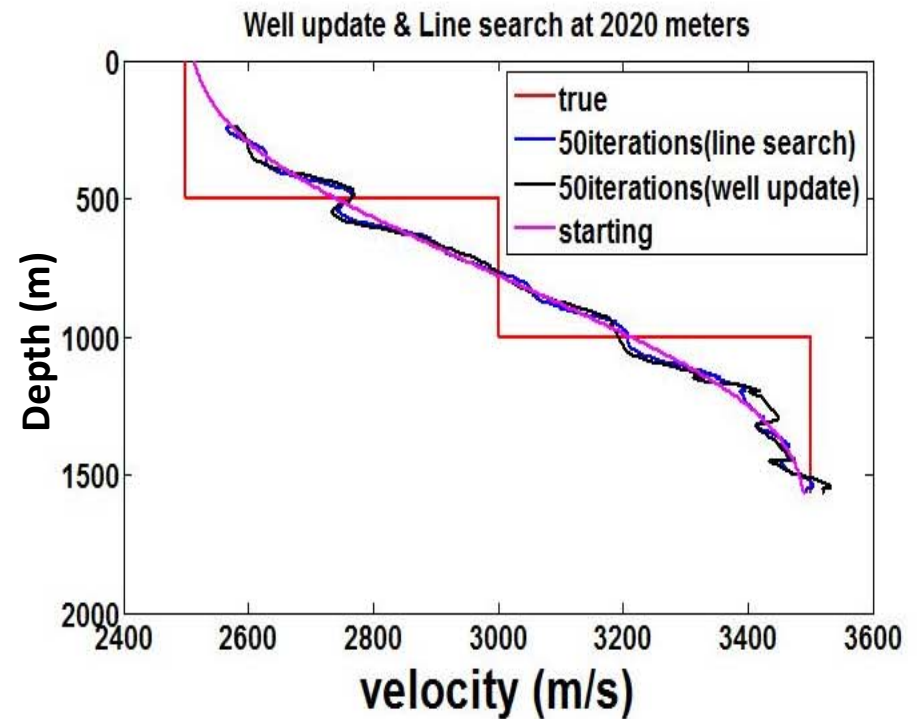
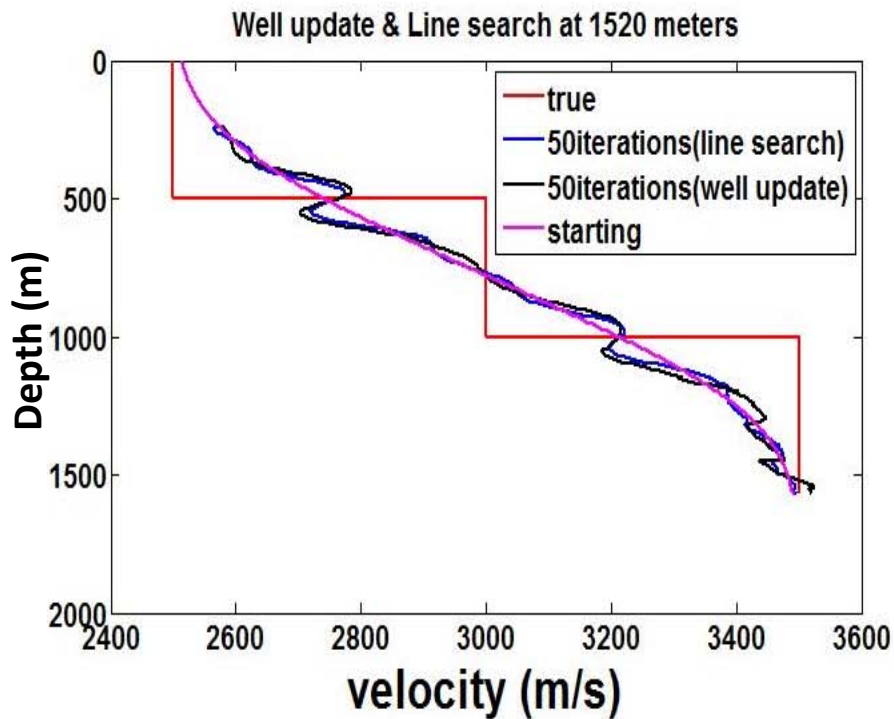
# Synthetic Examples: Model with flat layers

Comparing Linear velocity  $v(z)$  as starting model with a smooth version of the true velocity model



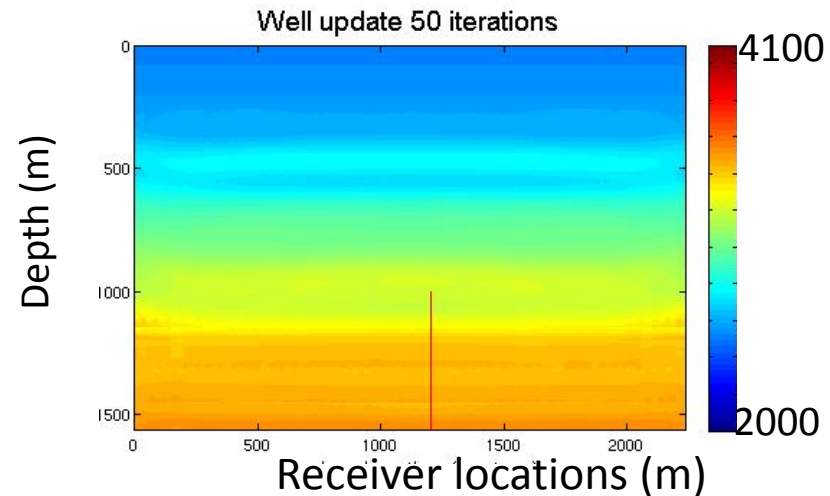
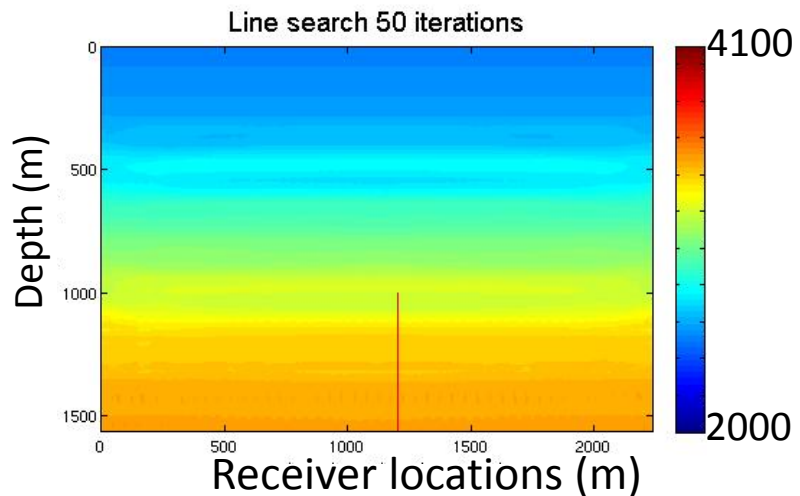
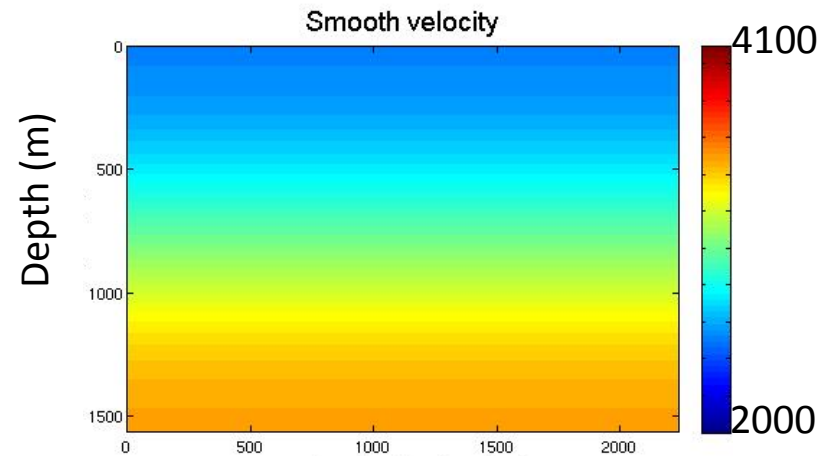
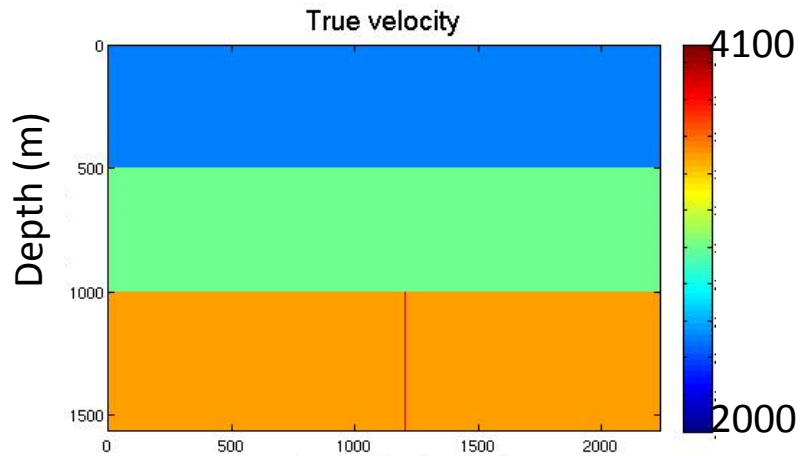
# Synthetic Examples: Model with flat layers

Comparing Line search optimization with a scalar deduced from well information at 1520 meters and 2020 meters.



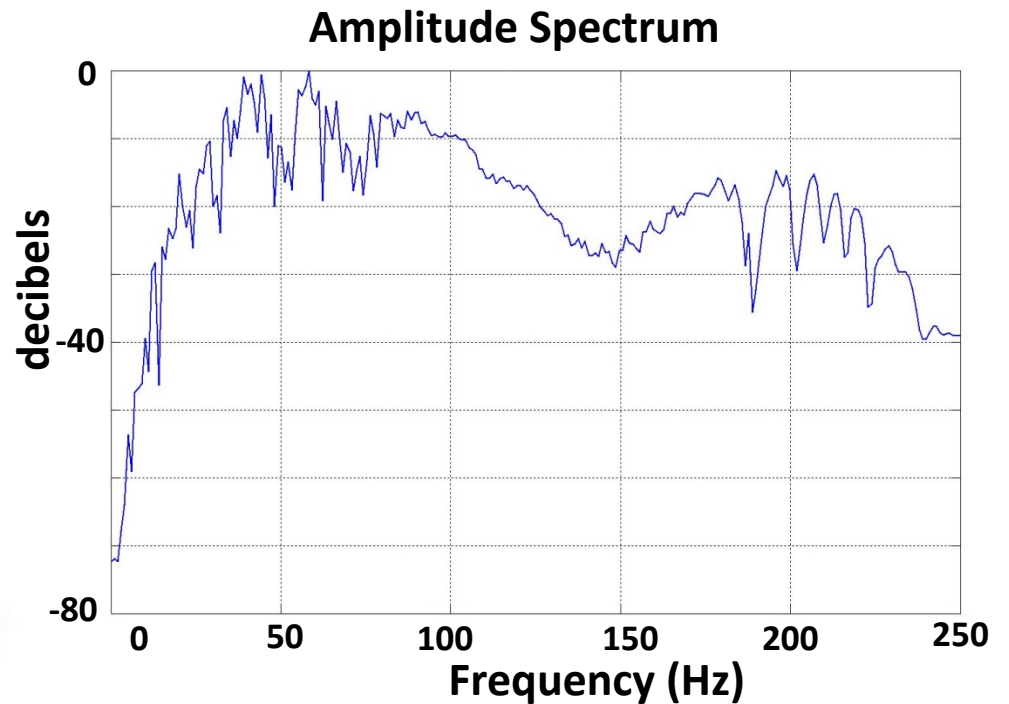
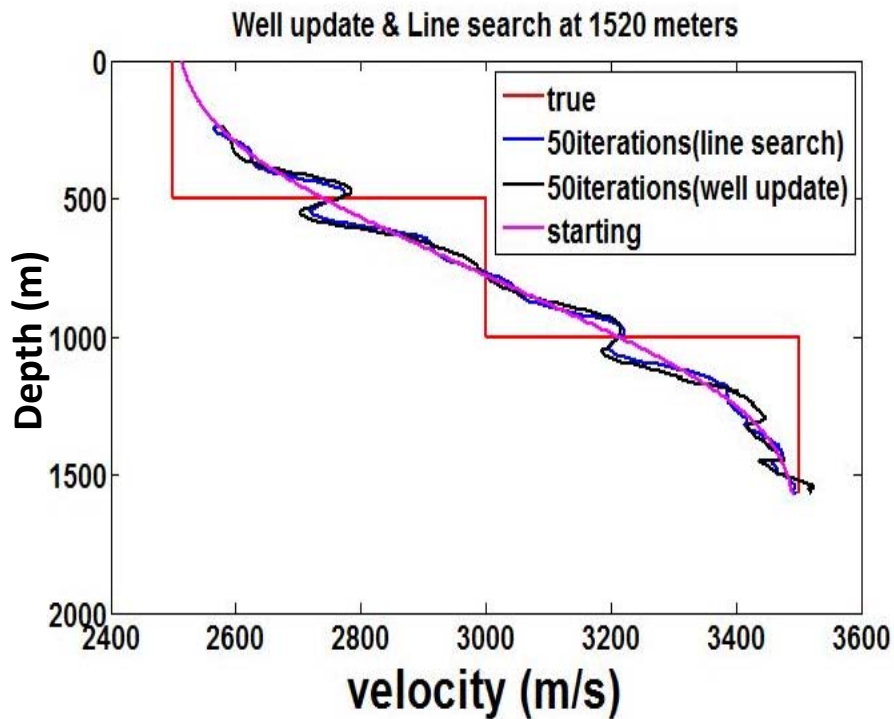
# Synthetic Examples: Model with flat layers

Comparing Line search optimization with a scalar deduced from well information . Starting model is a smooth version of the true velocity model

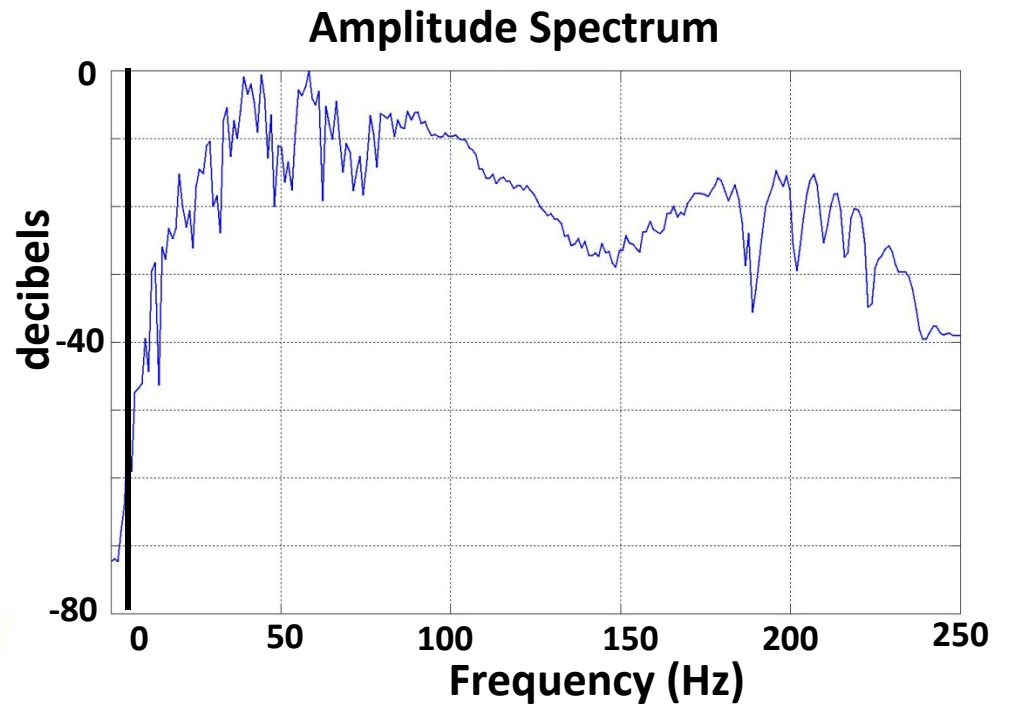
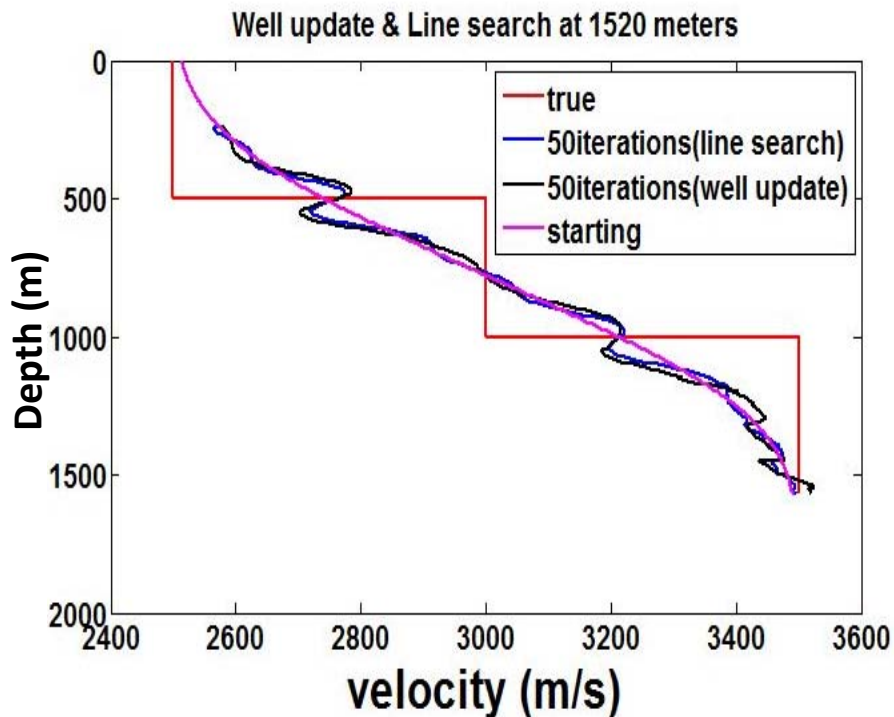


# Synthetic Examples: Model with flat layers

Comparing Line search optimization with a scalar deduced from well information at 1520.



# Conclusions: Model with flat layers



It turns out that our **simple** flat layer model isn't quite **simple** to invert!

## Conclusions: Model with flat layers

- » Using a **Linear  $V(z)$  velocity model** as a starting model did not give 'encouraging' results with **conventional** FWI compared to using a smooth version of the true model.
- » Comparing a line search optimization technique to a scalar derived using well log information (using a smooth version of the true model), suggests that both methods are trying to do the job **but** the inverted models from both methods are still quite far from convergence. **Why is this so and what can we learn?**
- » **We will repeat experiment with a lower dominant frequency, reduced layer thickness and whitening the spectrum before FWI**

# Synthetic Examples: Model with Structures

Structured layers with an **anticlinal structure** at the bottom of the model.

- **Variable** layer thickness.

## Inversion

- **RTM** migration algorithm **Cross Correlation IC**

- ❖ We used the Cross Correlation IC but we normalized the image by the square of the source illumination strength.

- Multi-scale approach (Pratt,1999)
- Used well information and a line search to determine the step-length

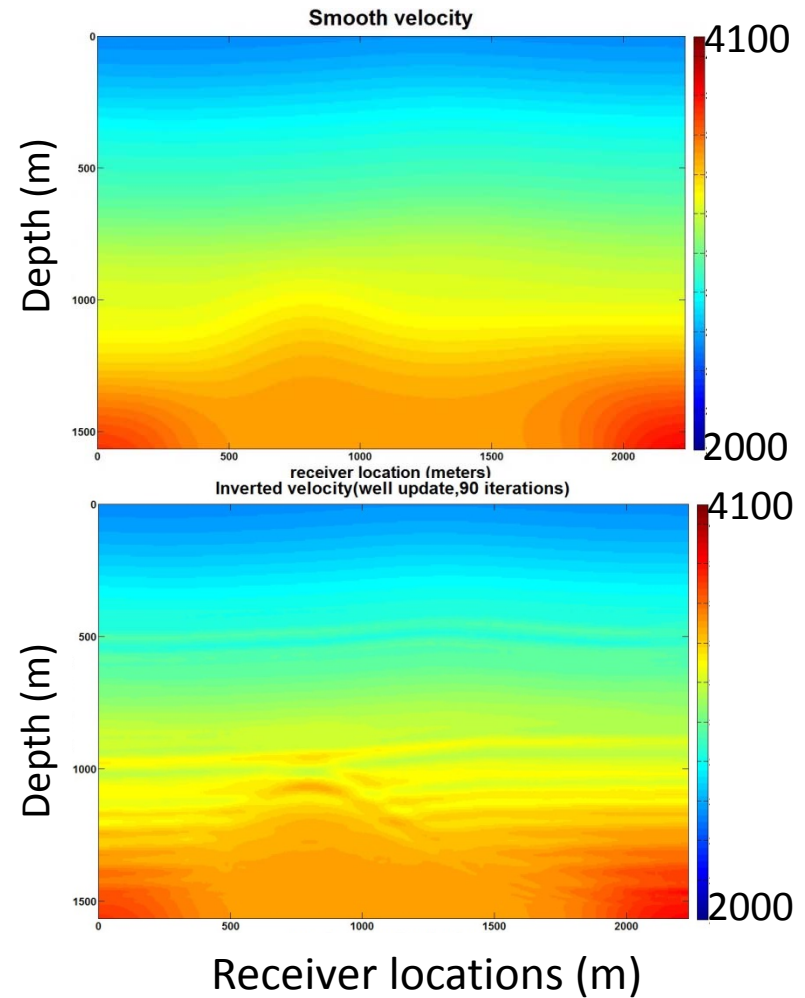
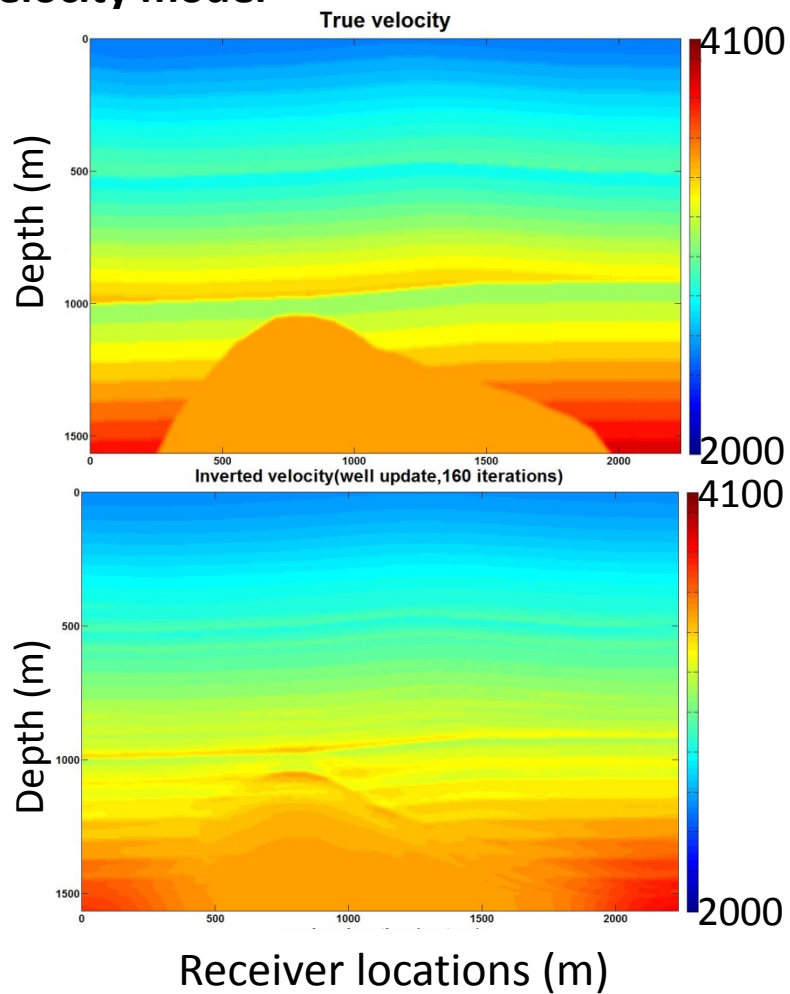
# Synthetic Examples: Model with flat layers

**Scalar deduced from well information.  
Starting model is a smooth version of the true velocity  
model**



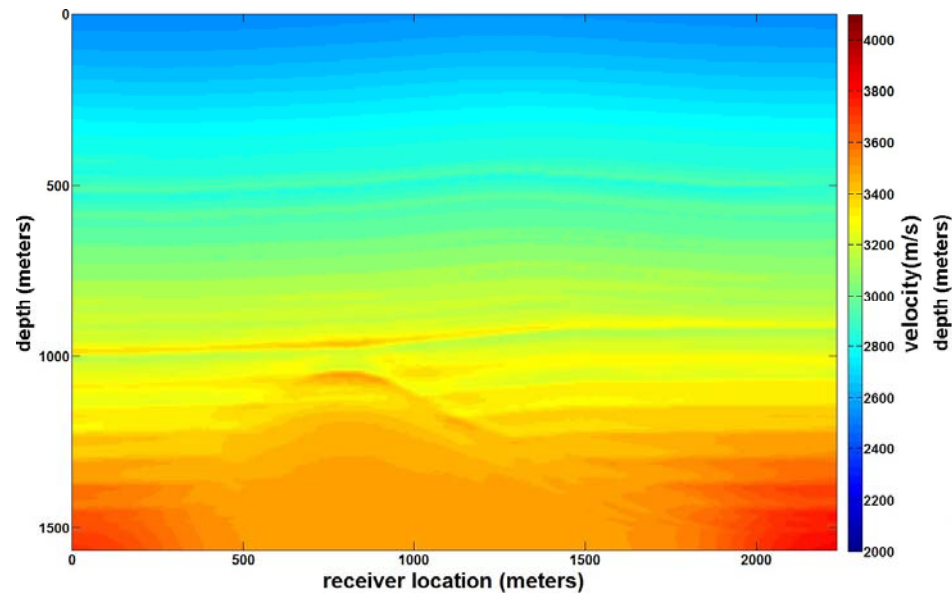
# Synthetic Example: Model with Structures

Step-length deduced from well information . Starting model is a smooth version of the true velocity model

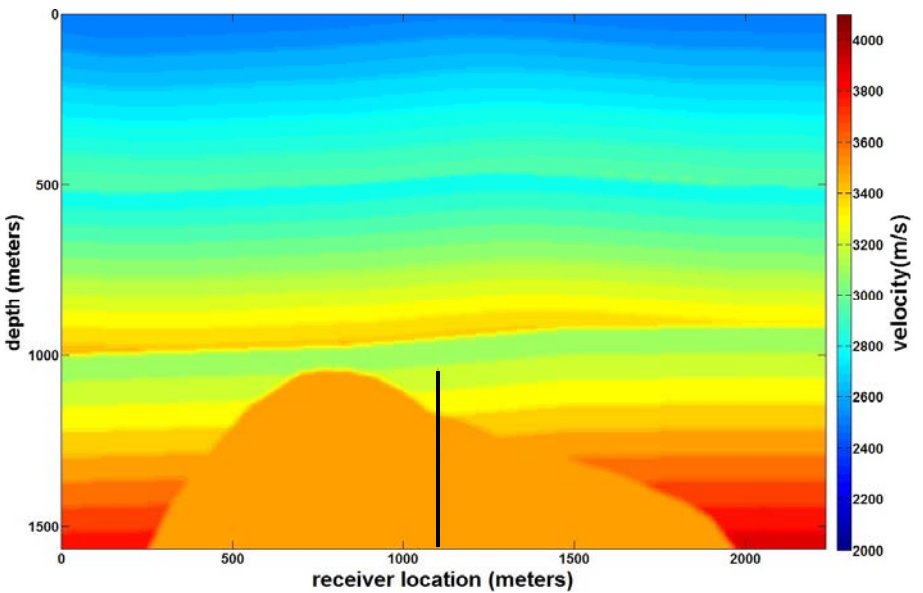


# Synthetic Example: Model with Structures

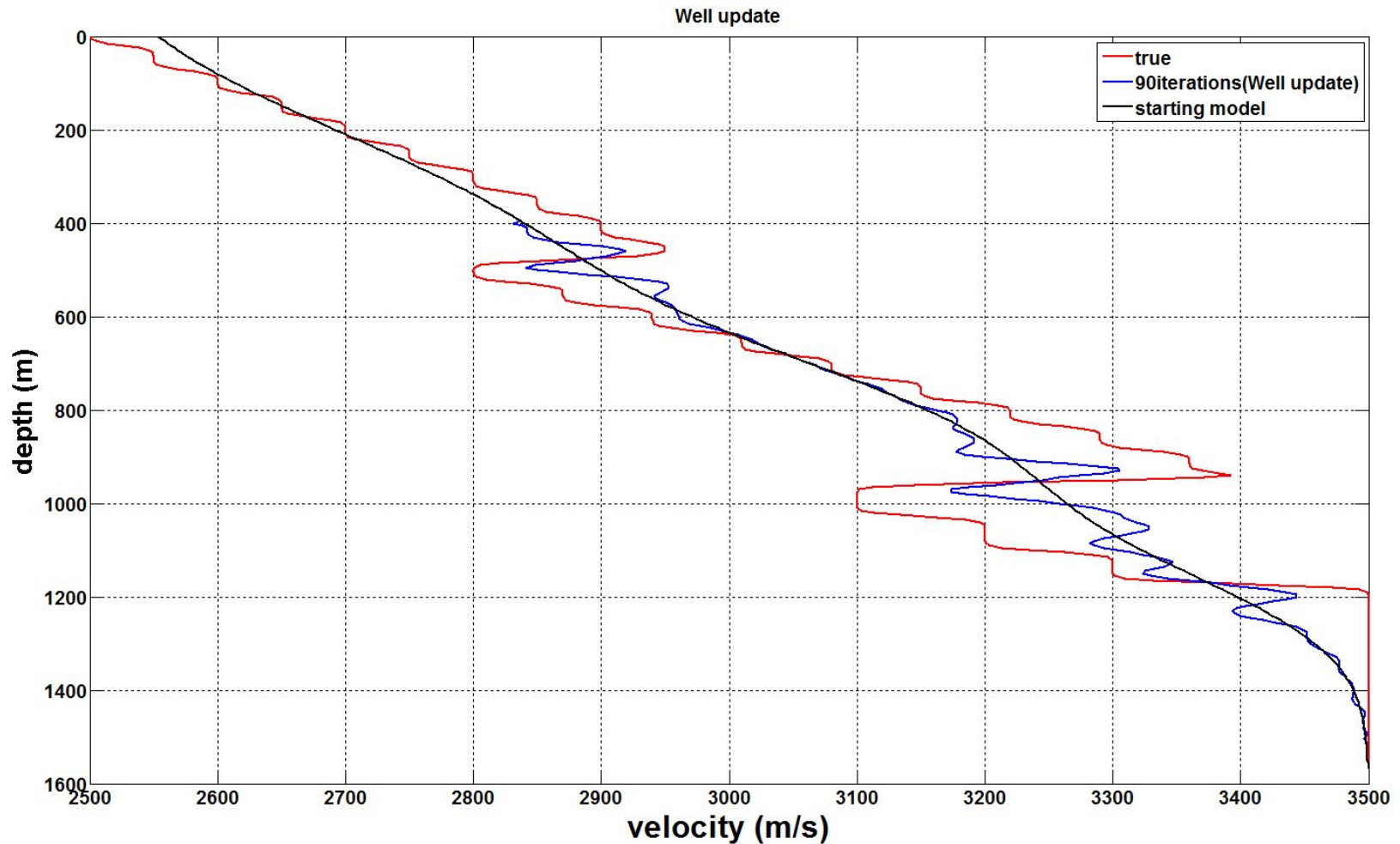
Inverted velocity model(Well update,160 iterations)



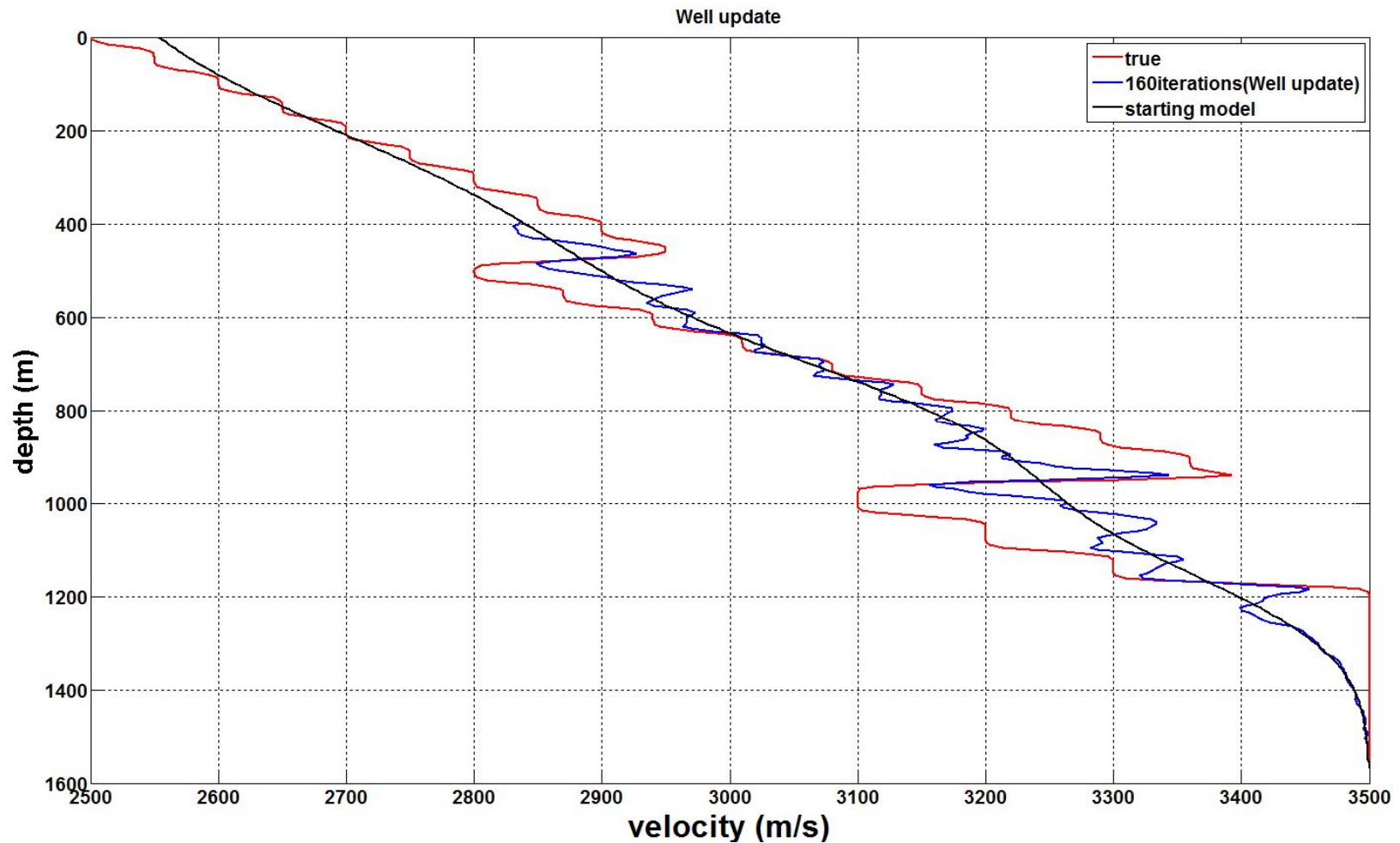
True model



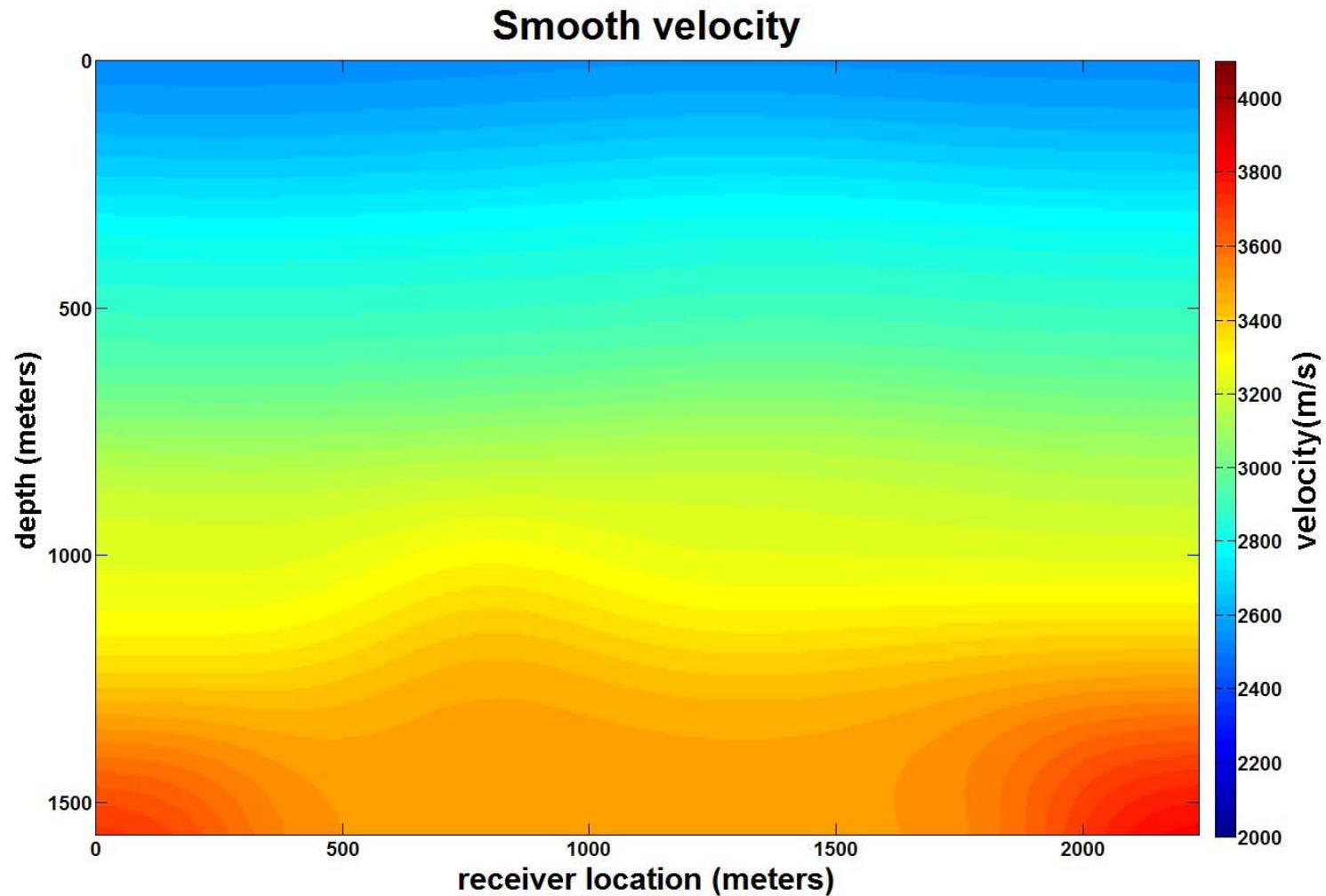
# Synthetic Examples: Model with Structures vertical profile at 520 meters



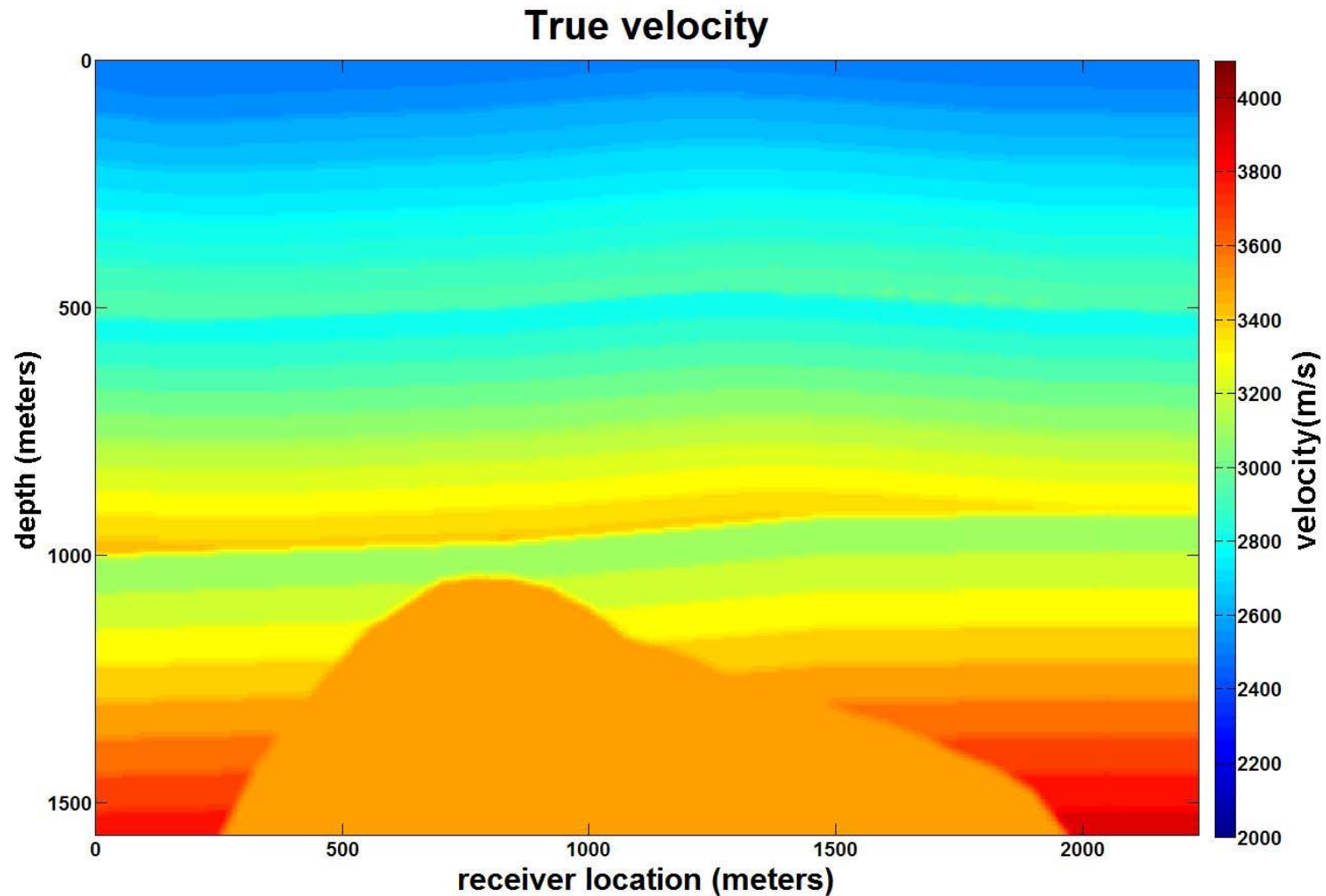
# Synthetic Examples: Model with Structures vertical profile at 520 meters



# Synthetic Examples: Model with Structures

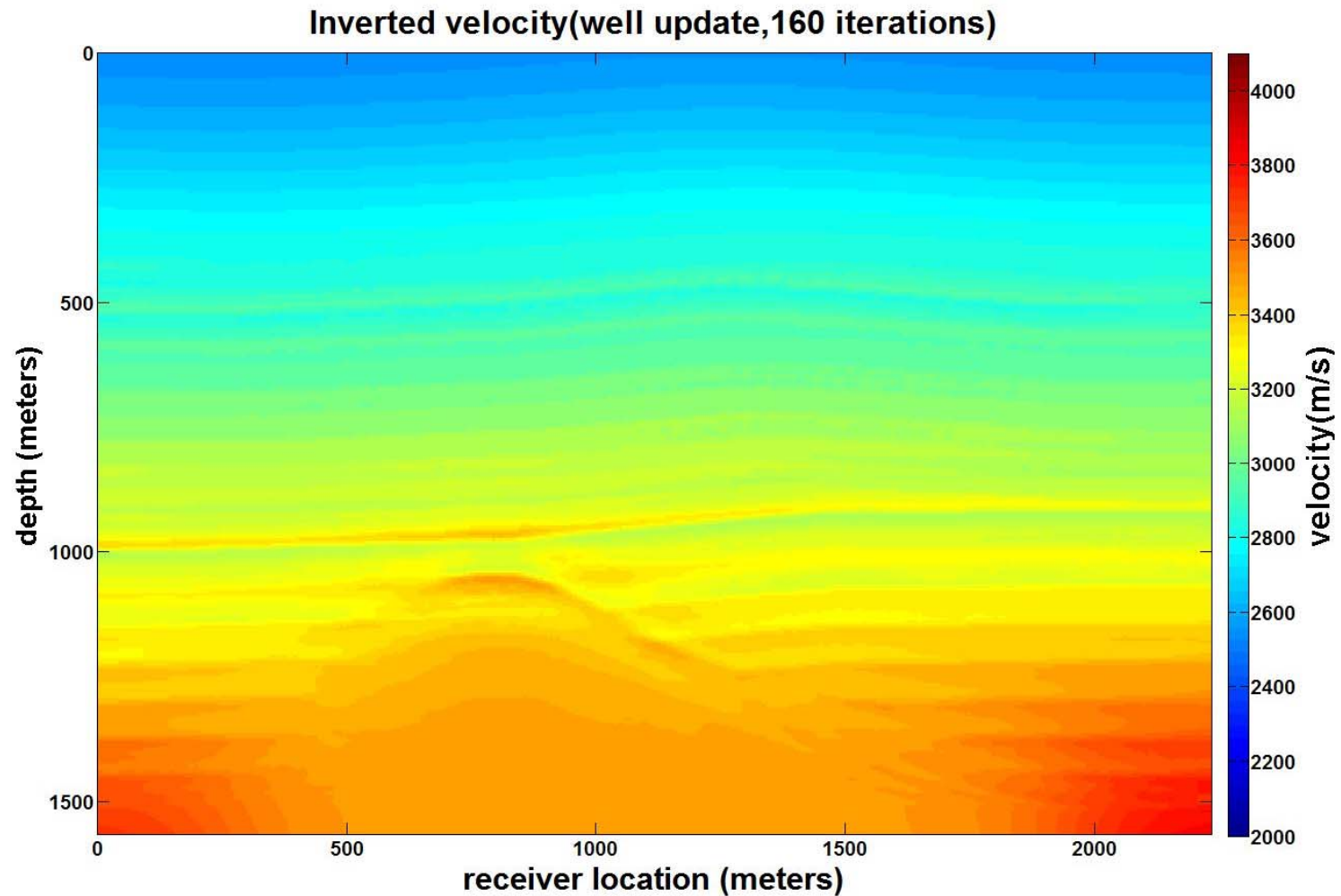


# Synthetic Examples: Model with Structures





# Synthetic Examples: Model with Structures

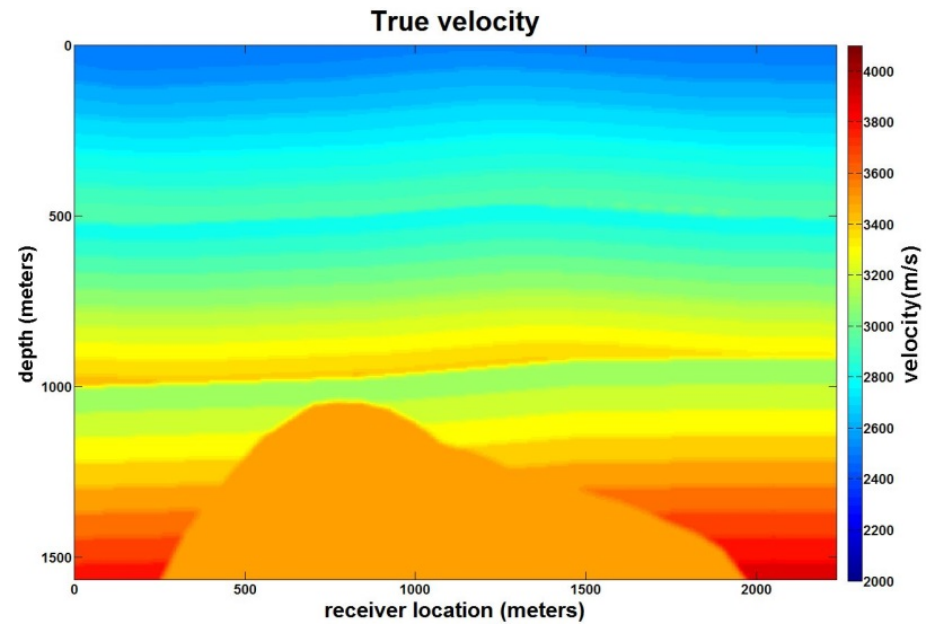
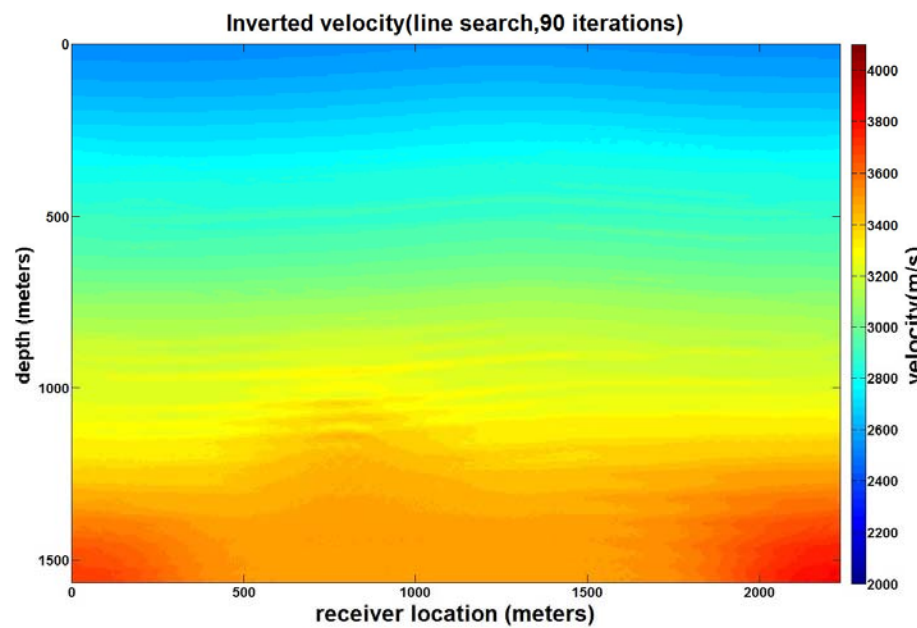


# Synthetic Examples: Model with flat layers

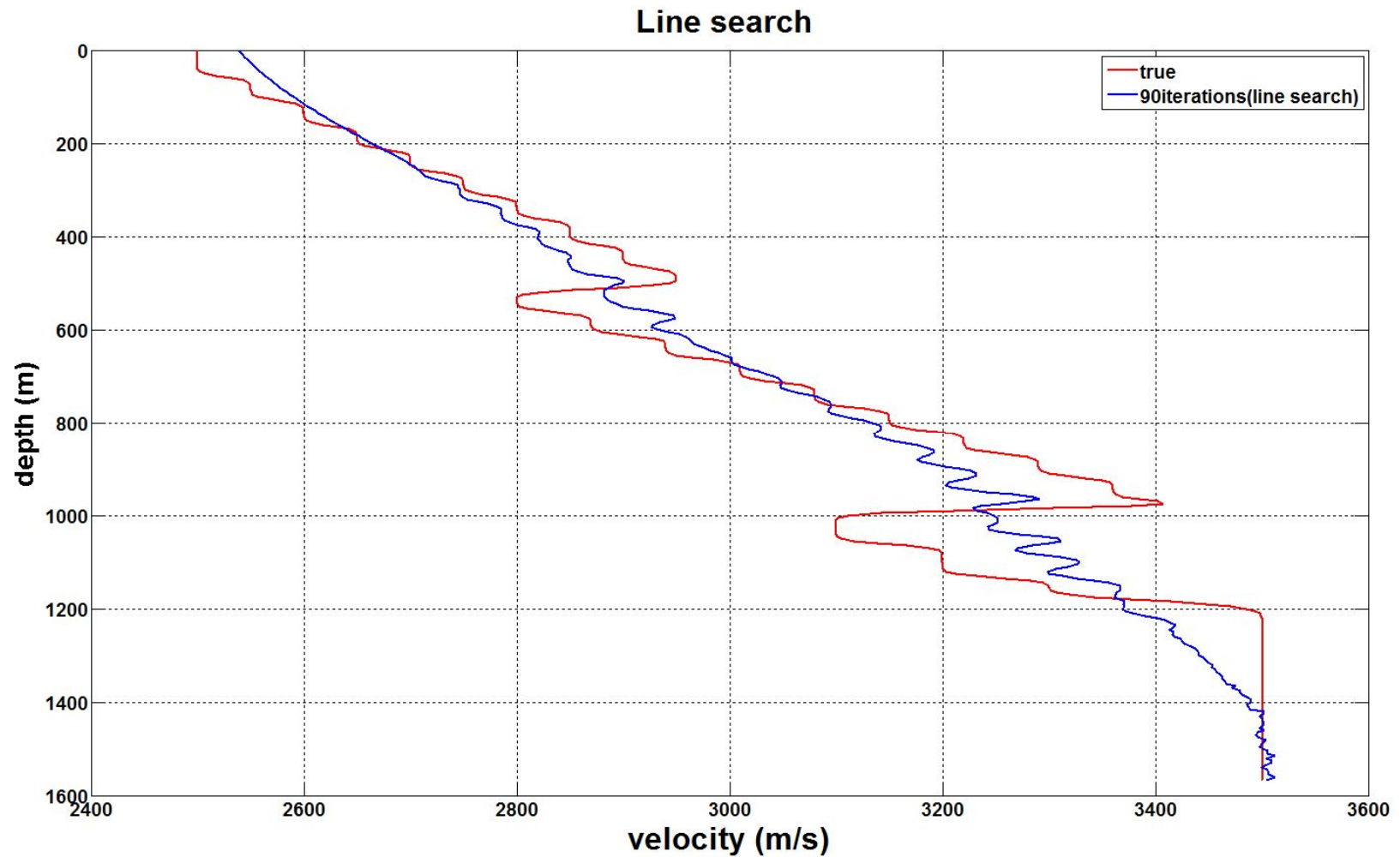
**Scalar deduced using a line search.  
Starting model is a smooth version of the true velocity  
model**



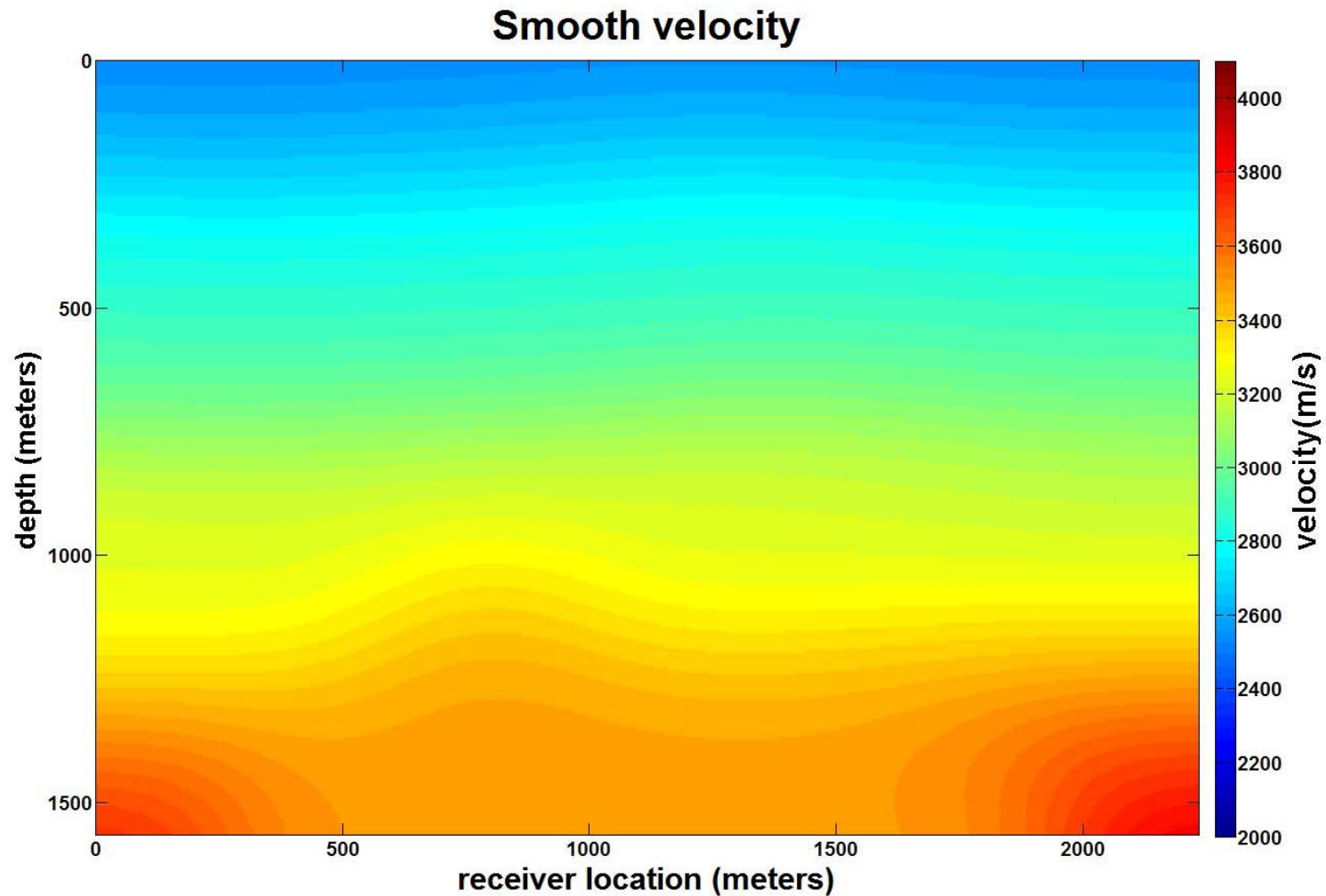
# Synthetic Examples: Model with Structures



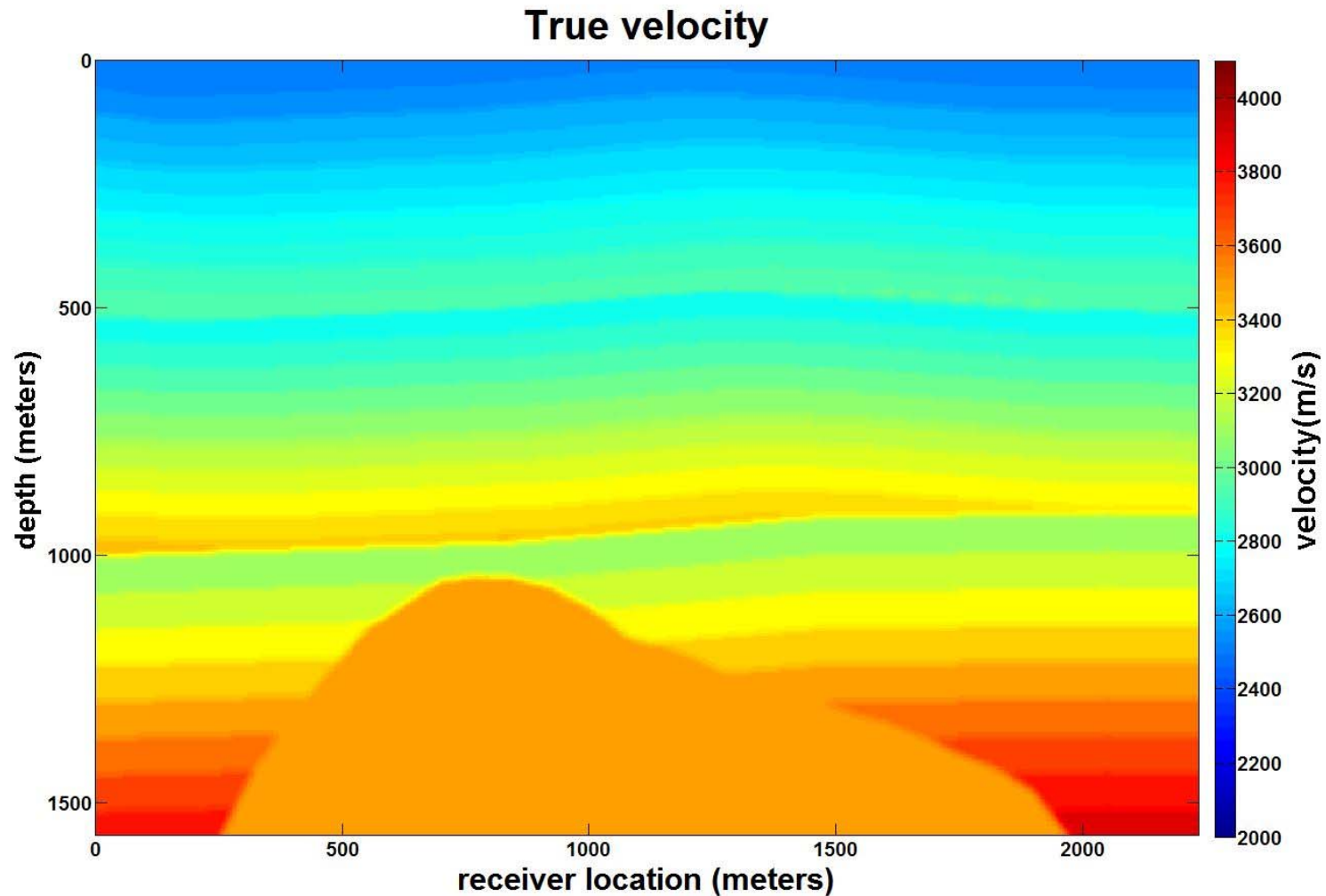
# Synthetic Examples: Model with Structures vertical profile at 520 meters



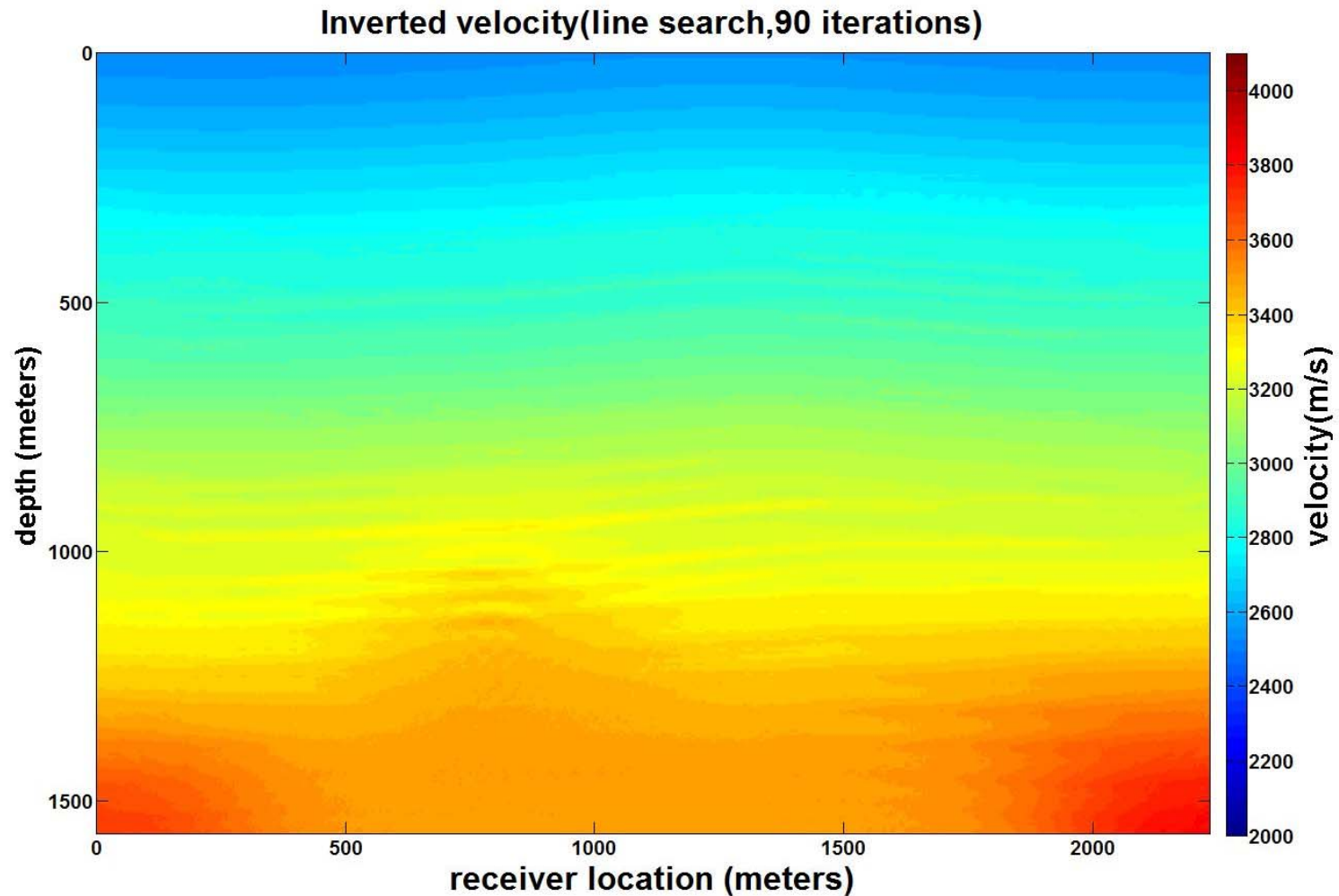
# Synthetic Examples: Model with Structures



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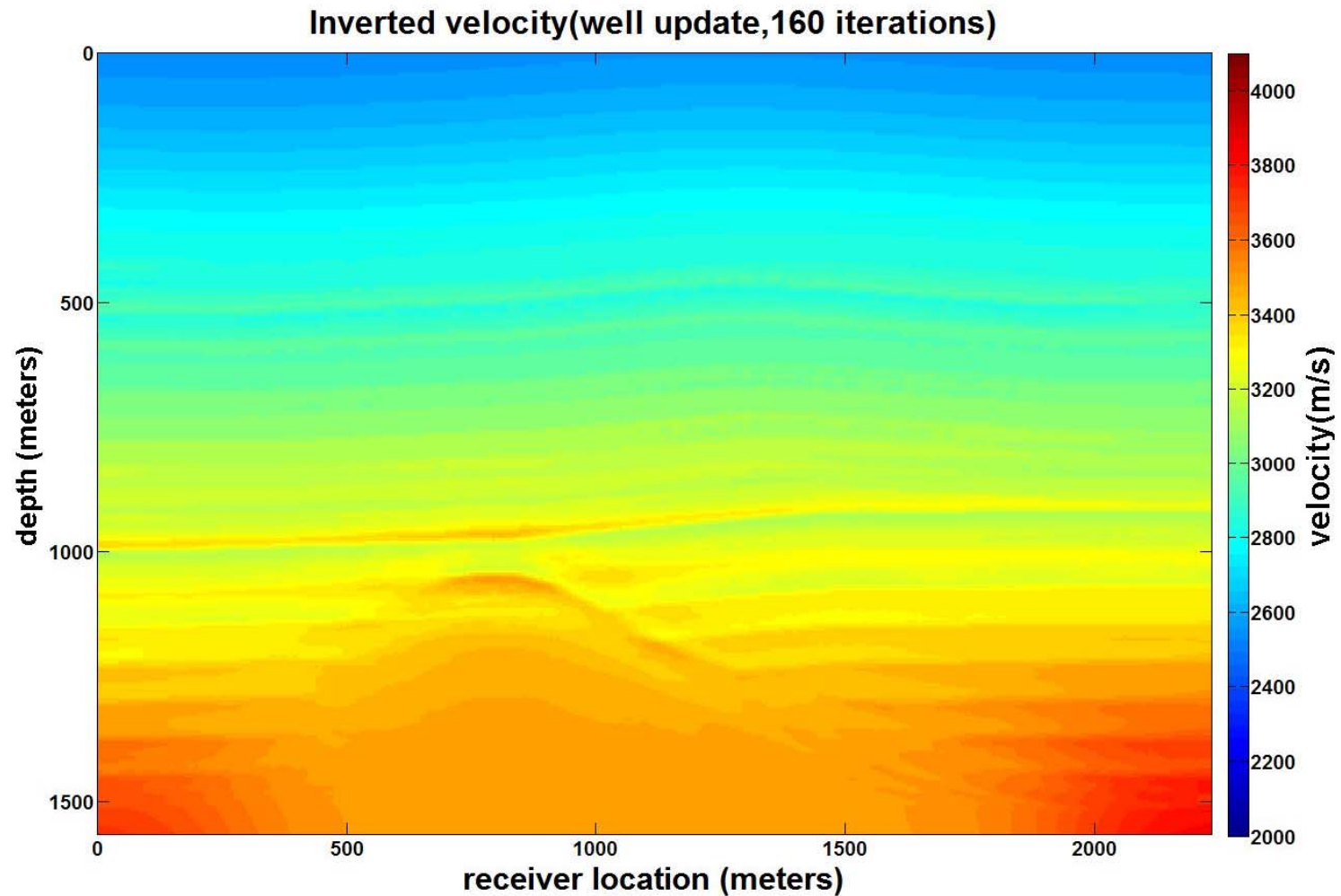
# Synthetic Examples: Model with Structures



# Synthetic Examples: Model with flat layers

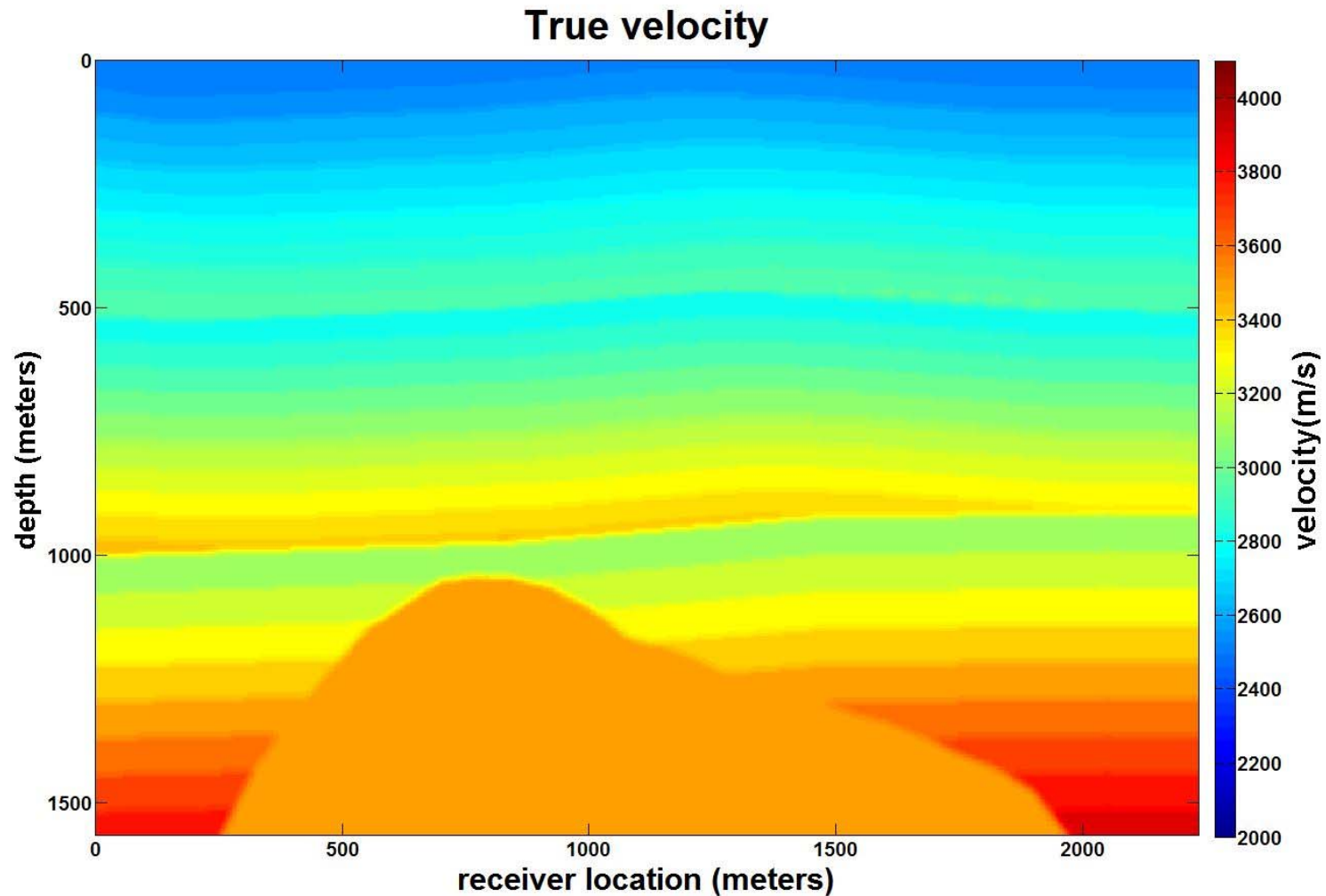
**Comparing Line search optimization with a scalar deduced from well information.**

# Synthetic Examples: Model with Structures



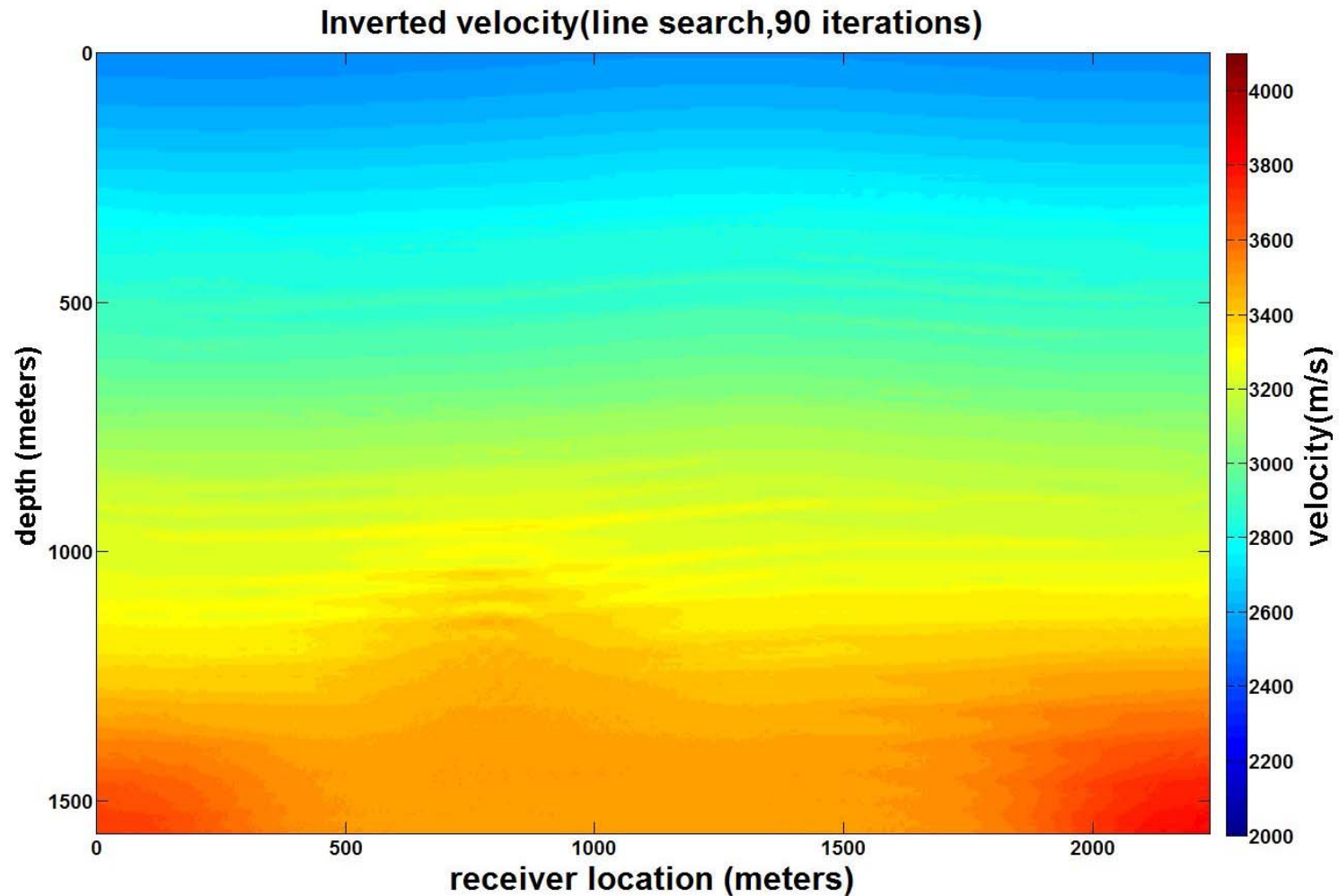


# Synthetic Examples: Model with Structures

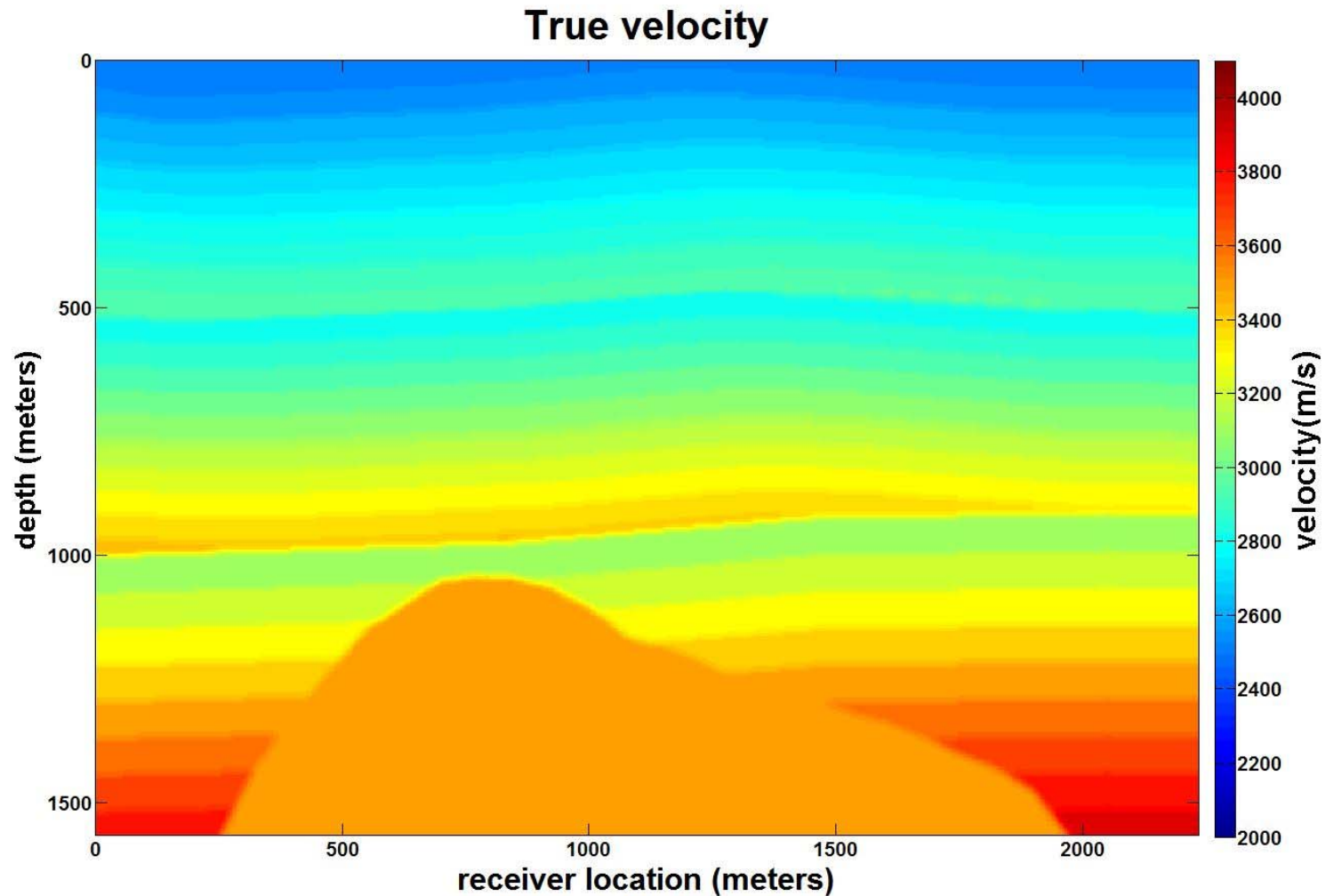




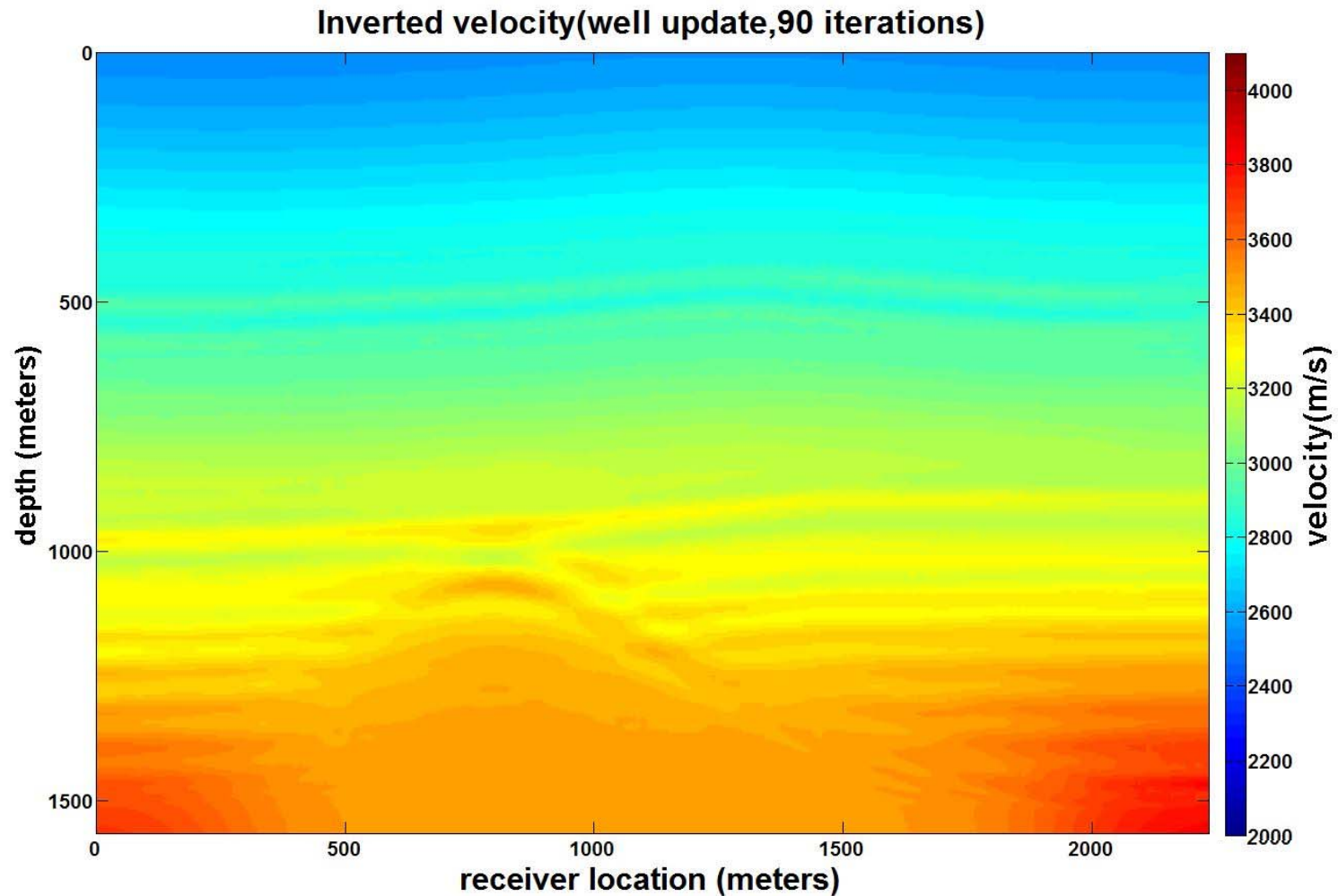
# Synthetic Examples: Model with Structures



# Synthetic Examples: Model with Structures



# Synthetic Examples: Model with Structures



## Conclusions: Model with Structures

- » We have high resolution in the deeper sections than in the shallow from the inverted model using well logs.
- » Using well information was able to resolve the flanks of the anticline. We can get better results if we have good well control and run more iterations.
- » Information from well logs to calculate a scalar for the model update shows encouraging results and could save us some computational time compared with the line search method.
- » Using a line search optimization scheme, we observe that we have better resolution in the shallow sections of the model than in the deeper sections.

# Discussions

- » Results from the flat layer model reiterate the importance of low frequency information in our data.
- » The scalar derived from well information can be used to condition the line search code.
- » We conjecture that a combination of a line search optimization scheme with well information should produce desirable results than if only one method of optimization is used.

# Future Work

- » **Exploit the full possibilities of incorporating well Information into FWI such as having a wavelet update between iterations inside our code.**
- » **Having a combination of a line search optimization and with well information during iterations.**
- » **We will investigate if better results will come from a model using reflectivity from a well log, or using NMO velocities, or a tomographic model.**

# Acknowledgements

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- **Marcelo Guarido**
- **CREWES staff and students**

**THANKS!**



# Theory of FWI: Data validation & Well validation

**DATA Validation (Margrave et al, 2012): Comparing predicted data to recorded data.**

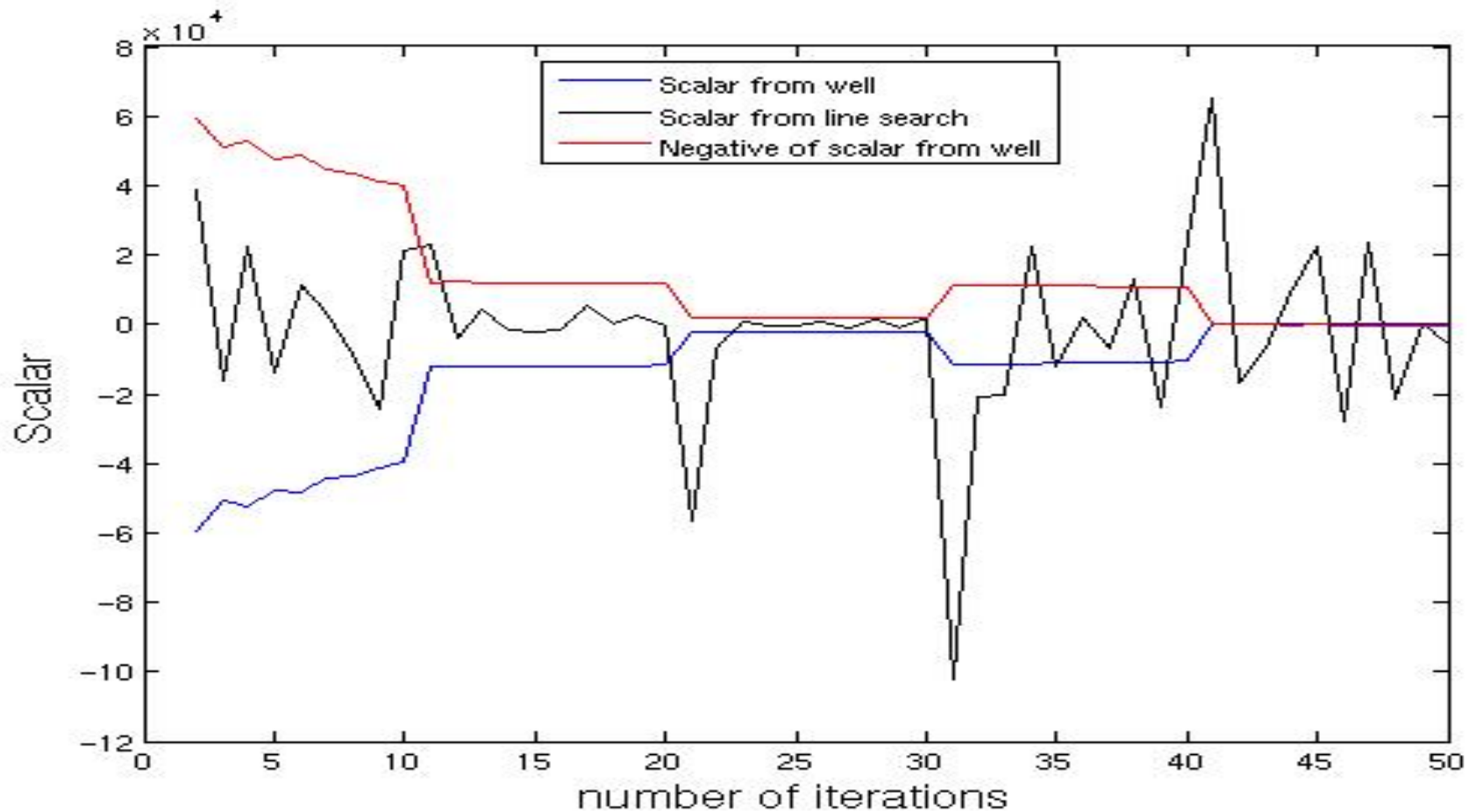
Data validation is far more sensitive to changes in the shallow part of the model than well validation.

**WELL Validation (Margrave et al 2012): Comparing reflectivity image to synthetic seismograms at well locations.**

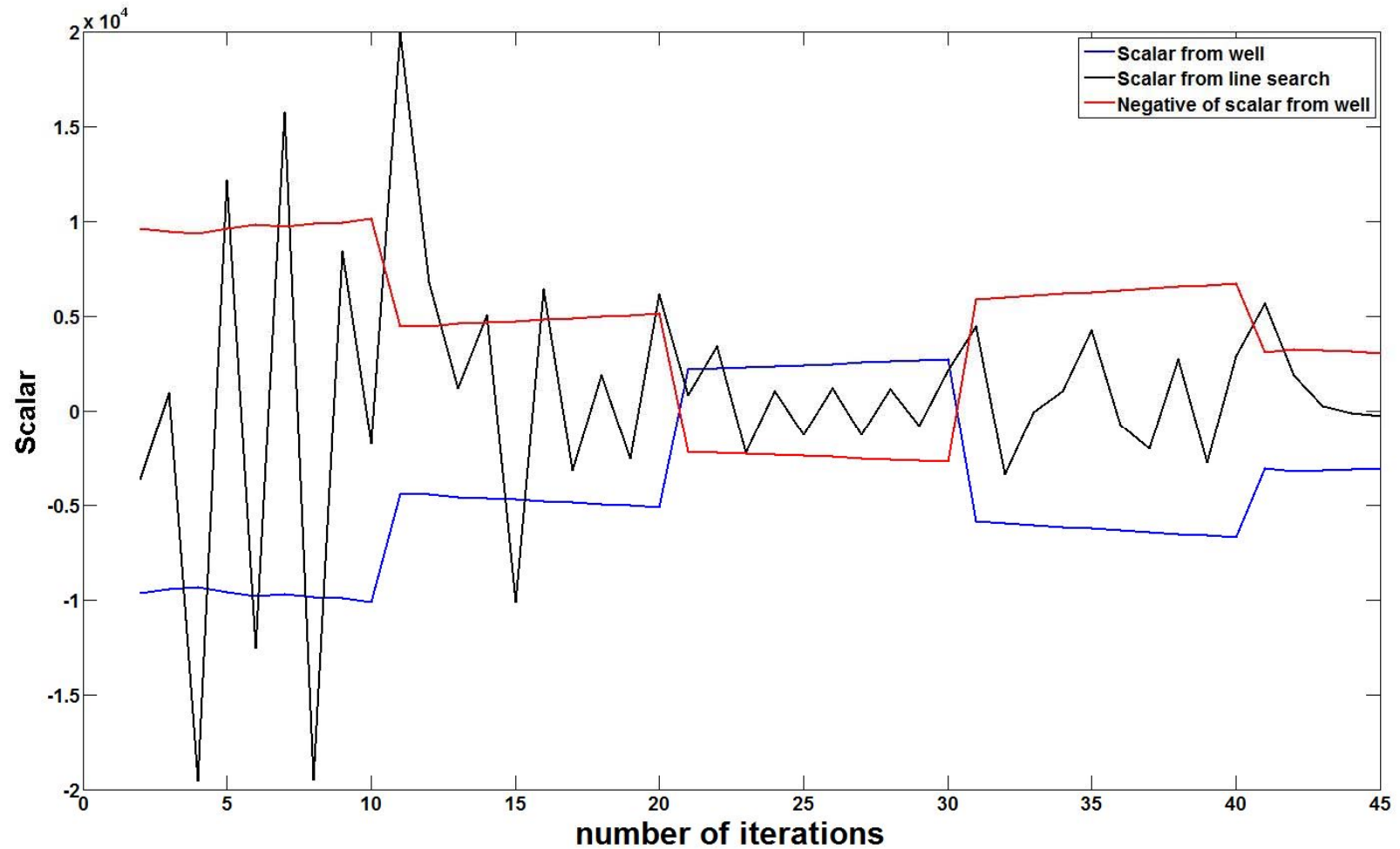
Well validation is limited by the depth range of the logs, hence it is sensitive to changes within the logged part of the model

# Synthetic Examples: Model with flat layers

Comparing the paths of the Line search optimization with the paths of a scalar deduced from well information through 50 iterations

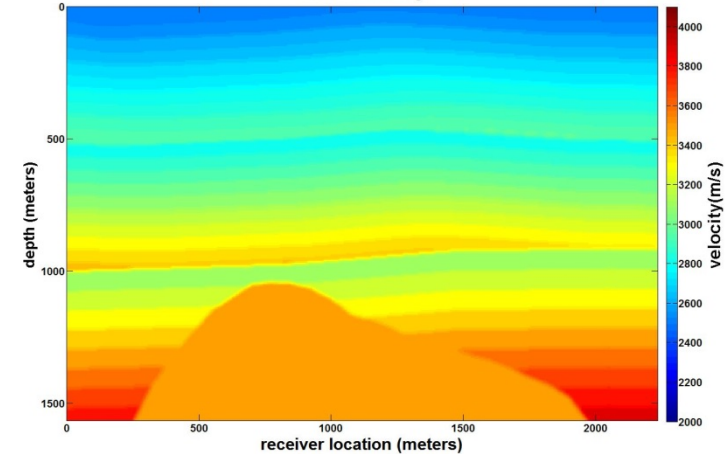
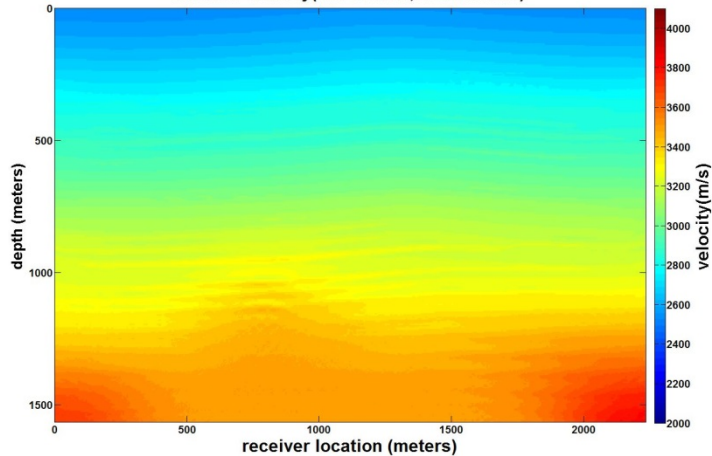
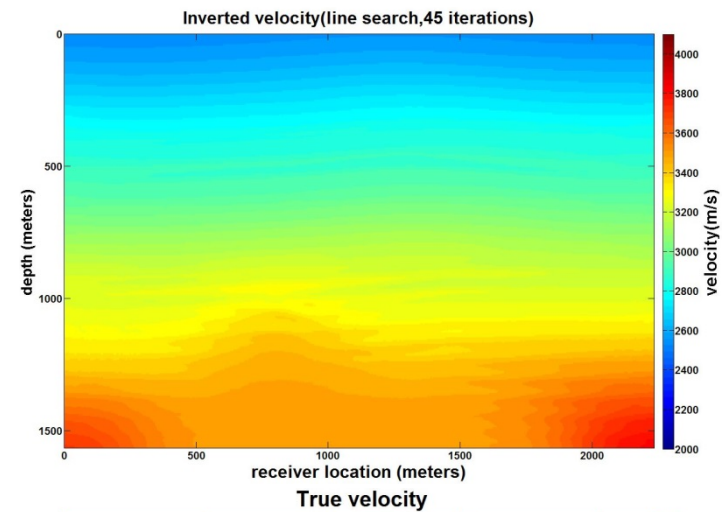
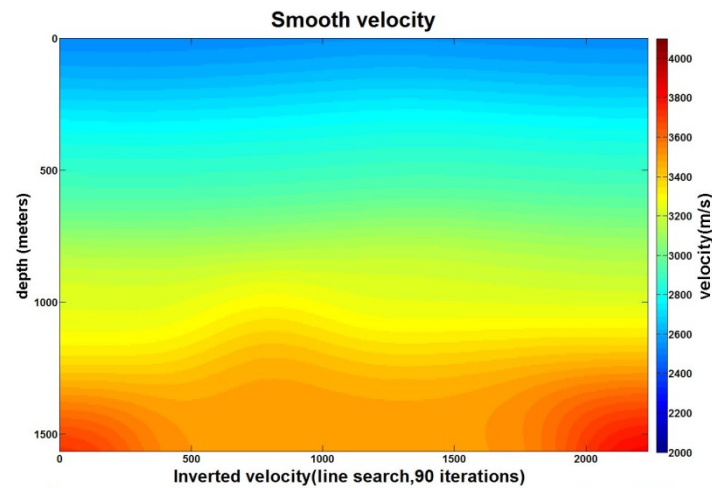


# Synthetic Examples: Model with Structures



# Synthetic Examples: Model with Structures

Step-length deduced from line search. Starting model is a smooth version of the true velocity model



# Theory of FWI

Full waveform inversion is an optimization technique that seeks to find a model of the subsurface that best matches the recorded field data at every receiver location. The method begins from a best guess of the true model, which is iteratively improved using linearized inversions methods although the FWI problem is non-linear (Warner et al, 2013).

$$* \quad \phi_k = \sum_{s,r} (\psi - \psi_k)^2$$

$$* \quad g_k(x, z) = \int \sum_{s,r} \omega^2 \hat{\psi}_s(x, z, \omega) \delta \hat{\psi}_{r(s),k}^*(x, z, \omega) d\omega$$

$$* \quad \delta v_k(x, z) = \lambda \int \sum_{s,r} \omega^2 \hat{\psi}_s(x, z, \omega) \delta \hat{\psi}_{r(s),k}^*(x, z, \omega) d\omega$$

$$* \quad \delta v_k(x, z) = \lambda g_k(x, z)$$

$$* \quad v_{k+1}(x, z) = v_k(x, z) + \lambda g_k(x, z)$$

- How to obtain/search for the scalar  $\lambda$  in the equations above is part of the focus of this presentation

# Synthetic Examples: Model with flat layers

Comparing Line search optimization with a scalar deduced from well information at 520 meters and 1020 meters.

