2D Internal multiple prediction in the $\tau - p_s - p_g$ domain

Jian Sun and Dr. Kris Innanen
Outline

- Review of Inverse scattering series (ISS) in multiple prediction
- Internal multiples (IMs) prediction: 1.5D to 2D
- Double $\tau - p_s - p_g$ transform
- Synthetic example of 2D IMs prediction
- Conclusion and future work
- Acknowledgements
Inverse scattering series

\[ z_1 + z_2 - z_3 = W \]
$b_{3IM}(p, \omega) = \int_{-\infty}^{+\infty} d\tau e^{i\omega \tau} b_1(p, \tau) \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega \tau'} b_1(p, \tau') \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega \tau''} b_1(p, \tau'')$

where $p$ is the horizontal slowness, or ray parameter, $\tau$ is the intercept times for primaries.

$b_1(p, \tau) = -i2q_sD(p, \tau)$
IMs prediction: 1.5D to 2D

Synthetic in offset domain

Synthetic in $\tau - p$ domain

Prediction in $\tau - p$ domain

Prediction in offset domain

$d(x, t) \Rightarrow D(p, \tau) \Rightarrow b_1(p, \tau) \Rightarrow b_{3IM}(p, \tau) \Rightarrow b_{3IM}(x, t)$

$$b_{3IM}(k_g, k_s, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} dk_1 e^{-iq_1(\epsilon_g-\epsilon_s)} \int_{-\infty}^{+\infty} dk_2 e^{-iq_2(\epsilon_g-\epsilon_s)} \int_{Z^-}^{Z^+} dz e^{i(q_1+q_1)z} b_1(k_g, k_1, z) b_1(k_1, k_2, z') b_1(k_2, k_s, z'')$$

Where,

$$q_X = \frac{\omega}{c_0} \sqrt{1 - \frac{k_X^2 c_0^2}{\omega^2}};$$

$$k_z = q_g + q_s;$$

Coates (1996):

$$b_{3IM}(p_g, p_s, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} dp_1 e^{-i\omega(\tau_1-p_g)} \int_{-\infty}^{+\infty} dp_2 e^{-i\omega(\tau_2-p_s)} \int_{\tau^-}^{\tau^+} d\tau e^{i\omega\tau} b_1(p_g, p_1, \tau) b_1(p_1, p_2, \tau') b_1(p_2, p_s, \tau'')$$

$$b_1(p_g, p_s, \tau) = -i2q_s D(p_g, p_s, \tau)$$
Double $\tau - p_s - p_g$ transform

\[ t = px + 2 \sum_i z_{si}q_i \]
\[ \tau = 2 \sum_i z_{si}q_i \]

\[ \psi(p, \tau) = \int \Psi(x, \tau + px)dx \quad \text{Time domain} \]
\[ \phi(p, \omega) = \int \varphi(x, \omega)e^{i\omega px}dx \quad \text{Frequency domain} \]
Double $\tau - p_s - p_g$ transform

$$X_{SM} + X_{RM} = X$$

$$T = X_{SM}p_s + X_{RM}p_g + \sum_i z_{Mi}(q_{si} + q_{gi})$$

$$p_s = \frac{\sin\theta_s}{v_0} > 0; \quad p_g = \frac{\sin\theta_g}{v_0} > 0.$$
Double $\tau - p_s - p_g$ transform

\[ T = -x_sp_s + x_gp_g + \sum_i z_i(q_{si} + q_{gi}) \]

For source:

\[ \theta_s > 0 \quad \Rightarrow \quad p_s = \frac{\sin\theta_s}{v_0} > 0 \]

\[ \theta_s < 0 \quad \Rightarrow \quad p_s = \frac{\sin\theta_s}{v_0} < 0 \]

For receiver:

\[ \theta_g < 0 \quad \Rightarrow \quad p_g = \frac{\sin\theta_g}{v_0} < 0 \]

\[ \theta_g > 0 \quad \Rightarrow \quad p_g = \frac{\sin\theta_g}{v_0} > 0 \]

For source and receiver:

\[ T = x_sp_s + x_gp_g + \sum_i z_i(q_{si} + q_{gi}) \]

\[ \theta < 0 \quad \Rightarrow \quad p = \frac{\sin\theta}{v_0} < 0 \]

\[ \theta > 0 \quad \Rightarrow \quad p = \frac{\sin\theta}{v_0} > 0 \]
Double $\tau - p_s - p_g$ transform

$\theta_s < 0 \iff p_s = \frac{\sin \theta_s}{v_0} < 0$

$\theta_g > 0 \iff p_g = \frac{\sin \theta_g}{v_0} > 0$

$T = x_s p_s + x_g p_g + \sum_i z_i (q_{si} + q_{gi})$

**Time domain:**

$D(p_s, p_g, \tau) = \int \int d(x_s, x_g, \tau + p_s x_s + p_g x_g) d x_s d x_g$

**Frequency domain:**

$\tilde{D}(p_s, p_g, \omega) = \int \int_{-\infty}^{+\infty} d(x_s, x_g, \omega) e^{+i\omega(p_s x_s + p_g x_g)} d x_s d x_g$
Double $\tau - p_s - p_g$ transform

- $d(x_s, x_g, t)$
- $x_s = 636m$

$\frac{p_g}{k m} \geq 0$

$\delta(x_s, p_g, \tau)$

$\hat{d}(p_g, \tau)$

$x_s = 636m$
Double $\tau - p_s - p_g$ transform

\[ \hat{d}(x_s, p_g, \tau) \]

\[ D(p_s, p_g, \tau_0) \]

\[ \hat{d}(x_s, \tau) \]

\[ p_g = 0 \]

\[ p_s = -\frac{d\tau}{dx_s} < 0 \]

\[ \tau_0(s) \]

\[ D(p_s, \tau_0) \]

\[ p_g = 0 \]
Matrix multiplication

\[
|p_s| - |p_g| \leq \frac{2 \sin \alpha}{v_{\min}}
\]

\[\text{(Liu et al., 2000)}\]
Synthetic example

106m
320m
844m
1500m/s

330m
96m

2200m/s

844m

4500m/s

Recored data 3D display

Receiver location (m)
Source location (m)

1000
500
0
0
500
1000

0
0.2
0.4
0.6
0.8
1

0
0.1
0.2
0.3
0.4
Synthetic example

![Synthetic example](image)
Synthetic example

\[ d(x_s, x_g) \]

15Hz

25Hz

\[ D(p_s, p_g) \]
Synthetic example

15 Hz

\[ b_1(p_s, p_g) \]

\[ p_s = -0.3, 0, 0.3 \]

25 Hz

\[ b_1(p_s, p_g) \]

\[ p_g = -0.3, 0, 0.3 \]
Synthetic example
Synthetic example

Common shot gather \((x_s = 630\text{ m})\) of raw data

Common shot gather \((x_s = 630\text{ m})\) of IMs prediction
Conclusion and future work

- Internal multiples can be reconstructed using inverse scattering series (ISS) algorithm in an automatic and stepwise way.
- Double $\tau - p_s - p_g$ transform with respect to source, receiver coordinates was discussed and applied to prepare the input data for ISS algorithm.
- 2D internal multiple prediction using ISS algorithm in double plane wave domain was performed.
- Preliminary results exemplify that ISS algorithm in double plane wave domain can provide more relevant and practical benefits.

Future work:

- Computation burden and practical tests.
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