Viscoelastic AVO modeling and inversion

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Outline

- Introduction and motivation
- Inhomogeneous waves
- Viscoelastic Zoeppritz equations
- Complex Snell’s law and it’s linearization
- AVO equations
- Inversion strategy
- Conclusion

Introduction and motivation

Elastic medium
- density
- P-wave velocity
- S-wave velocity

$$\mathrm{R}^{\mathrm{E}}_{\mathrm{PP}} = A^{\mathrm{E}}_{\mathrm{PP}} + B^{\mathrm{E}}_{\mathrm{PP}} \sin^2\theta_P + C^{\mathrm{E}}_{\mathrm{PP}} (\tan^2\theta_P - \sin^2\theta_P)$$

Zero offset term

$$\frac{1}{2}\left[\frac{\Delta\rho}{\rho} + \frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}\right]$$

AVO gradient

$$-2\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right] + \frac{1}{2}\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}$$

Curvature

$$\frac{1}{2}\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}$$

Quantitative Seismology, Aki and Richards, (2002)

Introduction and motivation

- Sensitivity of the reflection coefficients in a low-loss viscoelastic media to elastic and anelastic properties.
- The effects of change in attenuation angle across the boundary on the AVO equations (has not been considered in the literatures).
- AVO modeling in attenuative media taken into account the inhomogeneity of the wave.
- AVO inversion in viscoelastic media.
- Establish a framework for viscoelastic full wave form inversion.

Inhomogeneous waves

Attenuation in viscoelastic media characterized by quality factors (Q_P, Q_S) and attenuation angles (δ_P, δ_S)

Borcherdt, R. D., 2009. Viscoelastic waves in layered media, Cambridge University Press

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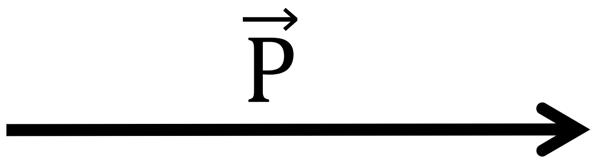
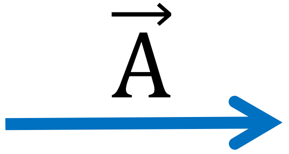


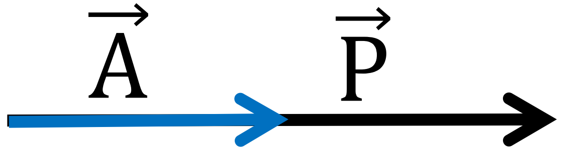
Propagation vector



Attenuation vector

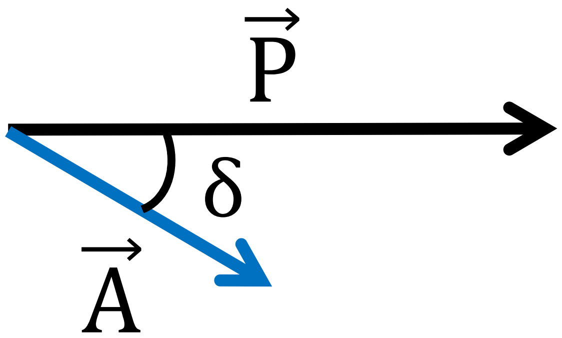
Attenuation angle δ

Homogenous wave



$\delta = 0$

Inhomogeneous wave



$\delta \neq 0$

Inhomogeneous waves

$$e^{-\vec{A}\cdot\vec{r}} e^{-i\vec{P}\cdot\vec{r}} \rightarrow e^{-i(\vec{P}-i\vec{A})\cdot\vec{r}} = e^{-i\vec{K}\cdot\vec{r}} \rightarrow \vec{K} = \vec{P} - i\vec{A}$$

Amplitude damping

Wave number vector is complex

- Ray parameter is complex
- Vertical slowness is complex

Reflectivity is a complex function

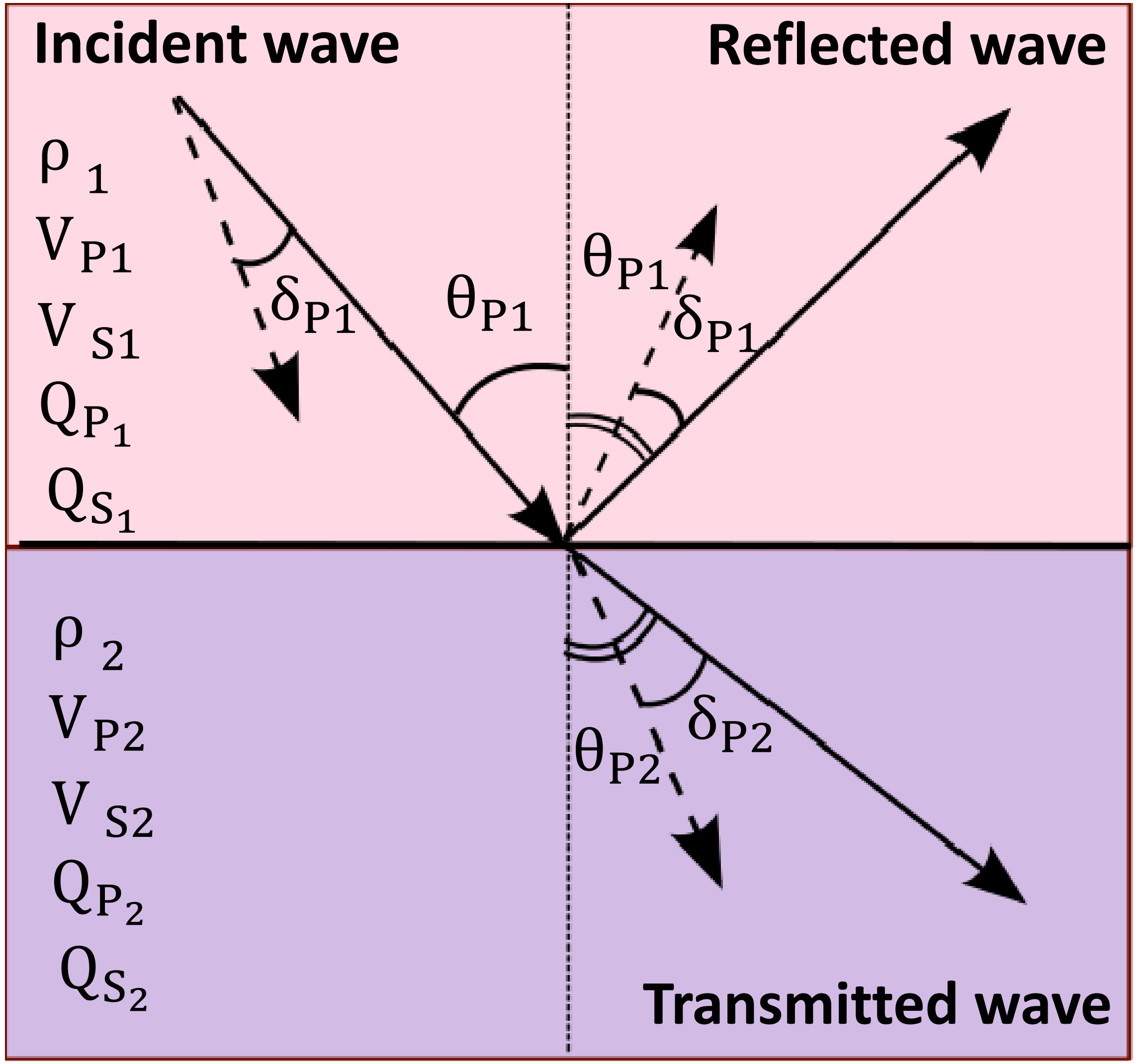
$$R\left(\frac{\Delta\rho}{\rho}, \frac{\Delta V_S}{V_S}, \frac{\Delta V_P}{V_P}, \frac{\Delta Q_P}{Q_P}, \frac{\Delta Q_S}{Q_S}, \theta, \delta\right)$$

Viscoelastic Zoeppritz equations

$$\frac{V_{P2}}{V_{P1}} = \frac{V_{S2}}{V_{S1}} = 2.5$$

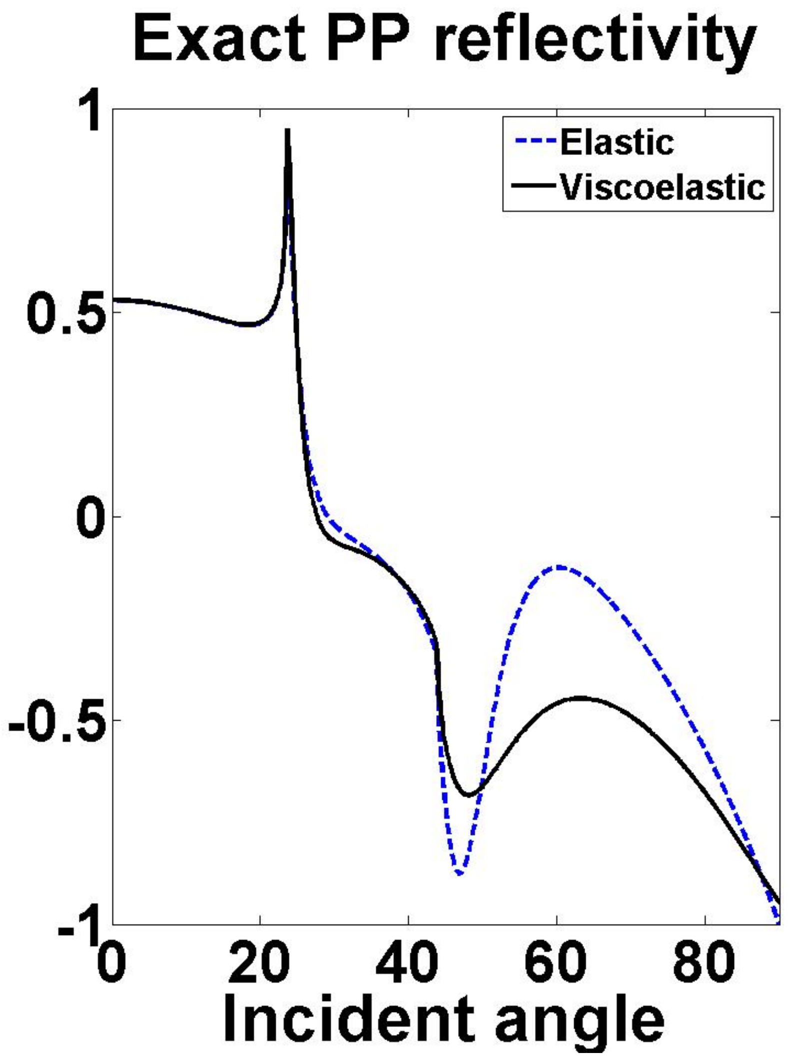
$$\frac{Q_{P2}}{Q_{P1}} = \frac{Q_{S2}}{Q_{S1}} = 1.3$$

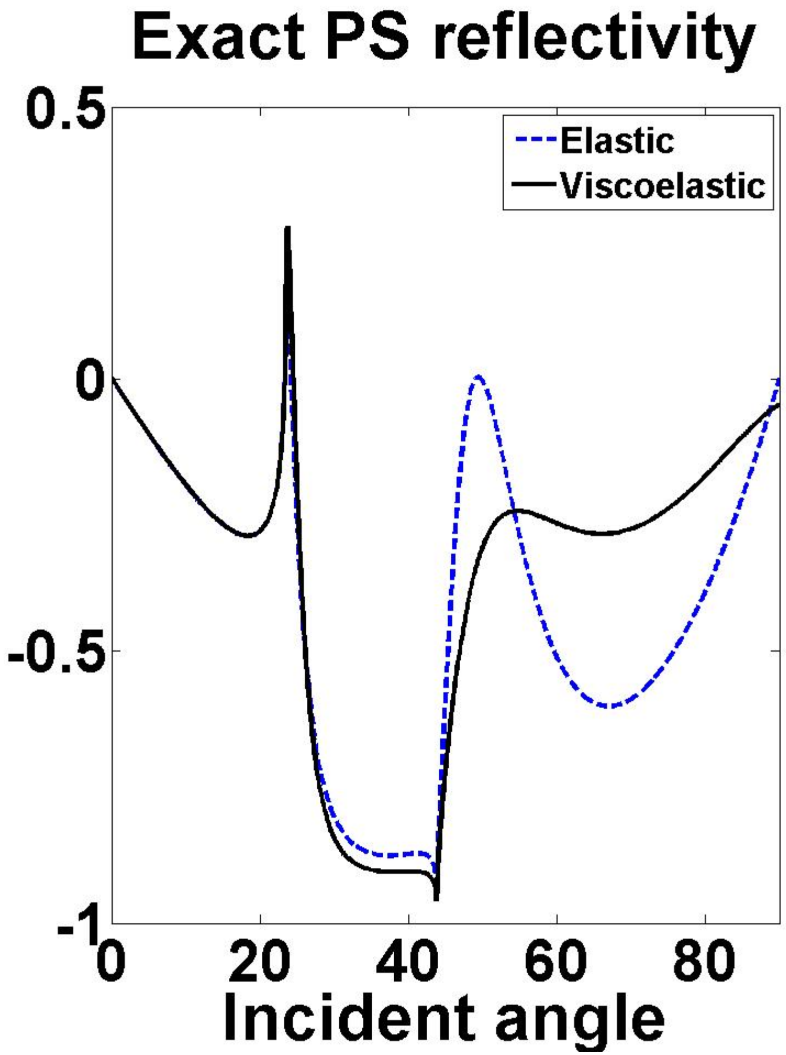
$$\frac{\rho_2}{\rho_1} = 1.3$$



Viscoelastic Zoeppritz equations

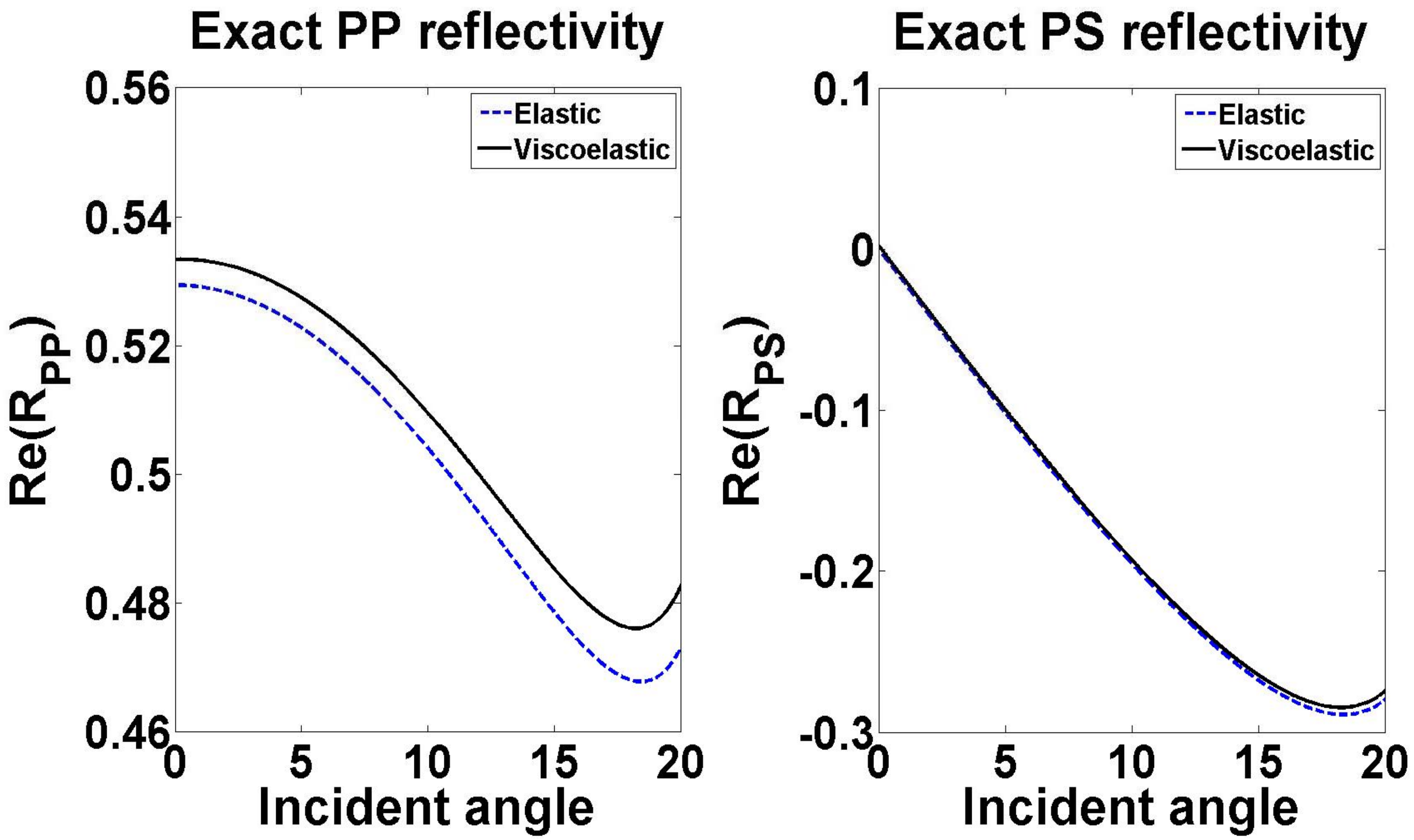
Inhomogeneous wave ($\delta_{\mathrm{P}} = 45^{\circ}$)





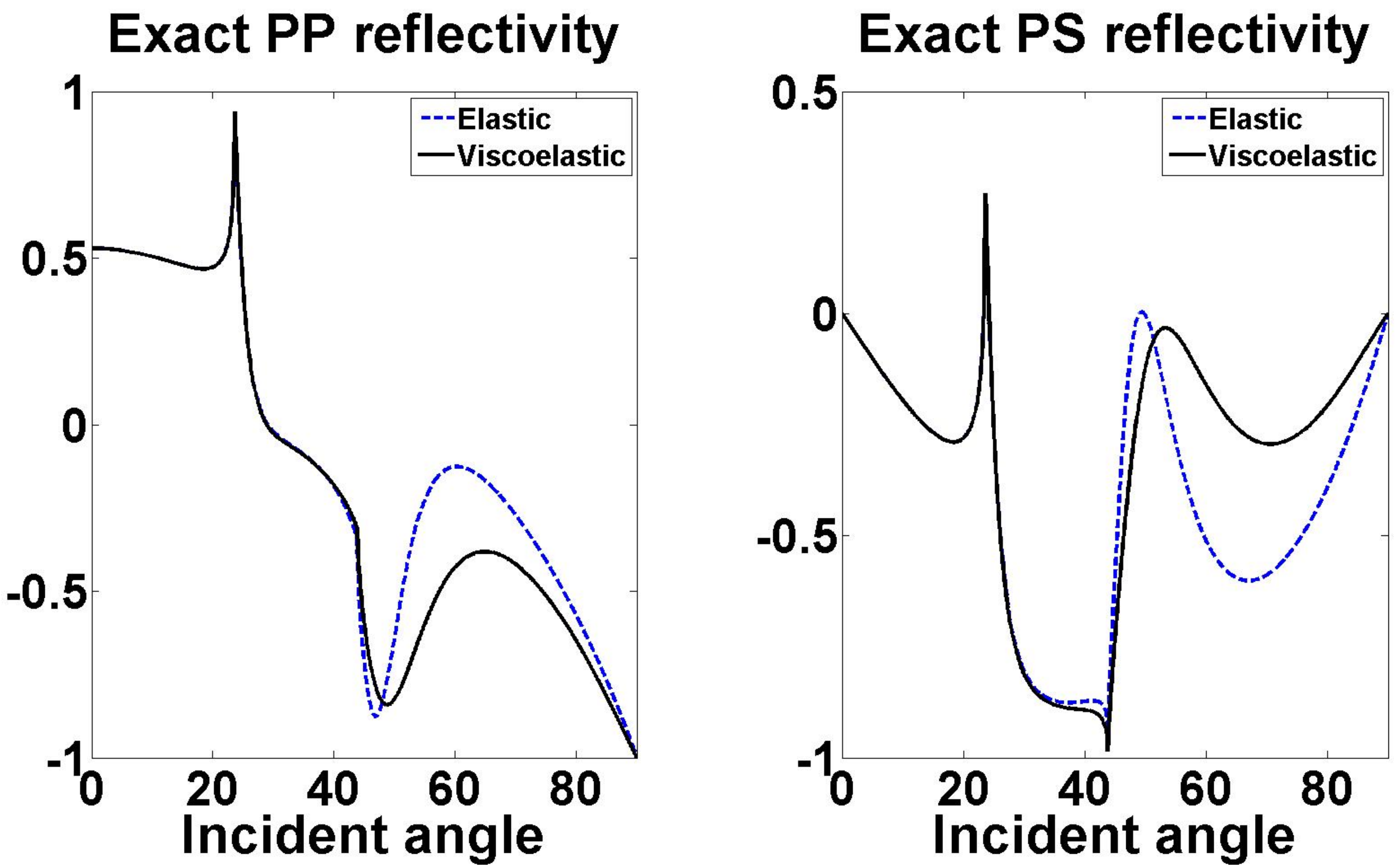
Viscoelastic Zoeppritz equations

Short offset



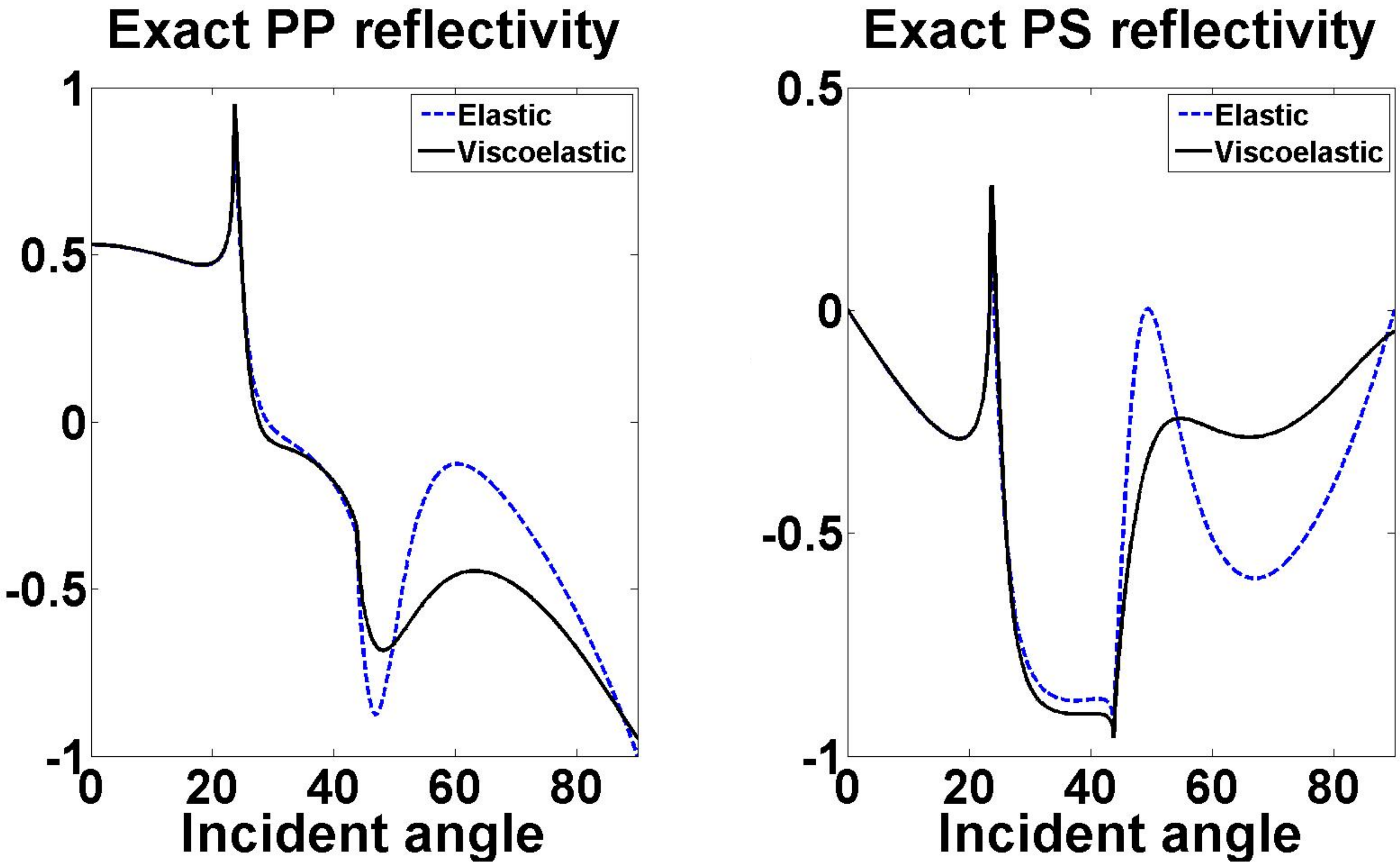
Viscoelastic Zoeppritz equations

Homogeneous wave ($\delta_P = 0^{\circ}$)



Viscoelastic Zoeppritz equations

Inhomogeneous wave ($\delta_P = 45^\circ$)



Complex Snell's law

Ray parameter is complex so the Snell's law has real and imaginary part

Real part of Snell's law

- Incident angle is equal to the reflected angle
- Change in the phase angle across the boundary is obtained by linearization of the real part of Snell's law

Imaginary part of Snell's law

- Incident attenuation angle is equal to the reflected attenuation angle
- Transmitted attenuation angle can be expressed in terms of incident phase and attenuation angle
- Change in the attenuation angle across the boundary is obtained by linearization of the imaginary part of the Snell's law

Complex Snell's law

Complex ray parameter is

$$\mathrm{p} = \frac{1}{V_{\mathrm{P1}}}\left[\sin\theta_{\mathrm{P1}} - \frac{i}{2}Q_{\mathrm{P1}}^{-1}(\sin\theta_{\mathrm{P1}} - \cos\theta_{\mathrm{P1}}\tan\delta_{\mathrm{P1}})\right]$$
$$= \frac{1}{V_{\mathrm{P2}}}\left[\sin\theta_{\mathrm{P2}} - \frac{i}{2}Q_{\mathrm{P2}}^{-1}(\sin\theta_{\mathrm{P2}} - \cos\theta_{\mathrm{P2}}\tan\delta_{\mathrm{P2}})\right]$$

Real

Imaginary

$$\sin\theta_{\mathrm{P2}} = \frac{V_{\mathrm{P2}}}{V_{\mathrm{P1}}}\sin\theta_{\mathrm{P1}}$$

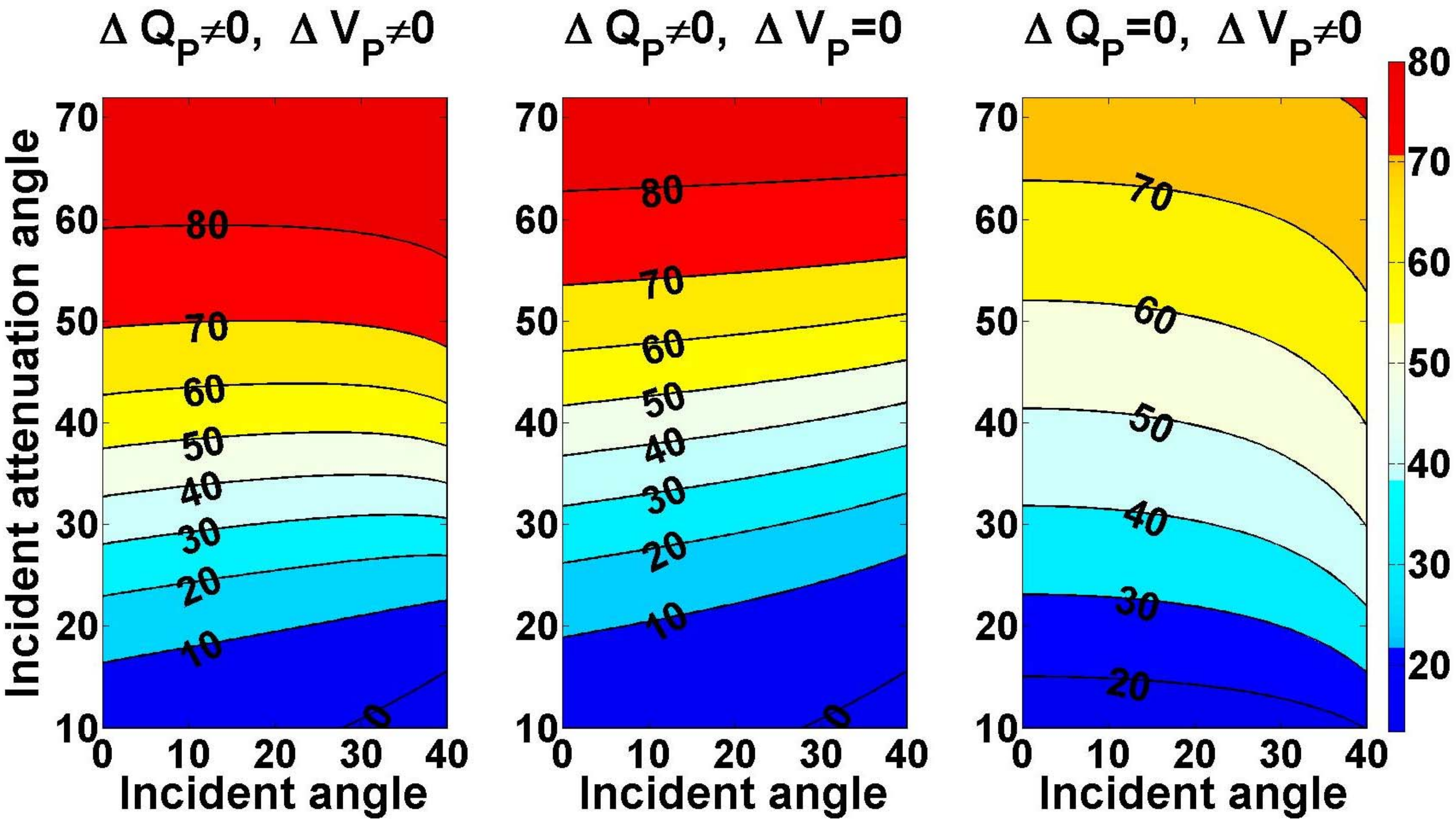
Incident attenuation angle

$$\tan\delta_{\mathrm{P2}} = \left(\frac{V_{\mathrm{P2}}}{V_{\mathrm{P1}}}\right)\frac{\sin\theta_{\mathrm{P1}} - \frac{Q_{\mathrm{P2}}}{Q_{\mathrm{P1}}}\left[\sin\theta_{\mathrm{P1}} - \cos\theta_{\mathrm{P1}}\tan\delta_{\mathrm{P1}}\right]}{\sqrt{1-\left(\frac{V_{\mathrm{P2}}}{V_{\mathrm{P1}}}\right)^2\sin^2\theta_{\mathrm{P1}}}}$$

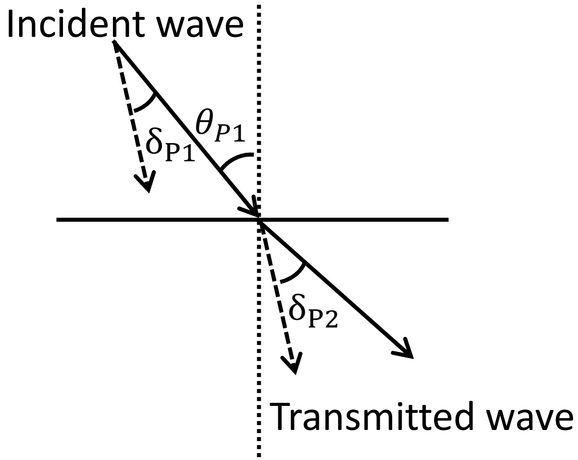
Transmitted attenuation angle

Complex Snell's law

Transmitted attenuation angle in terms of incident phase and attenuation angles

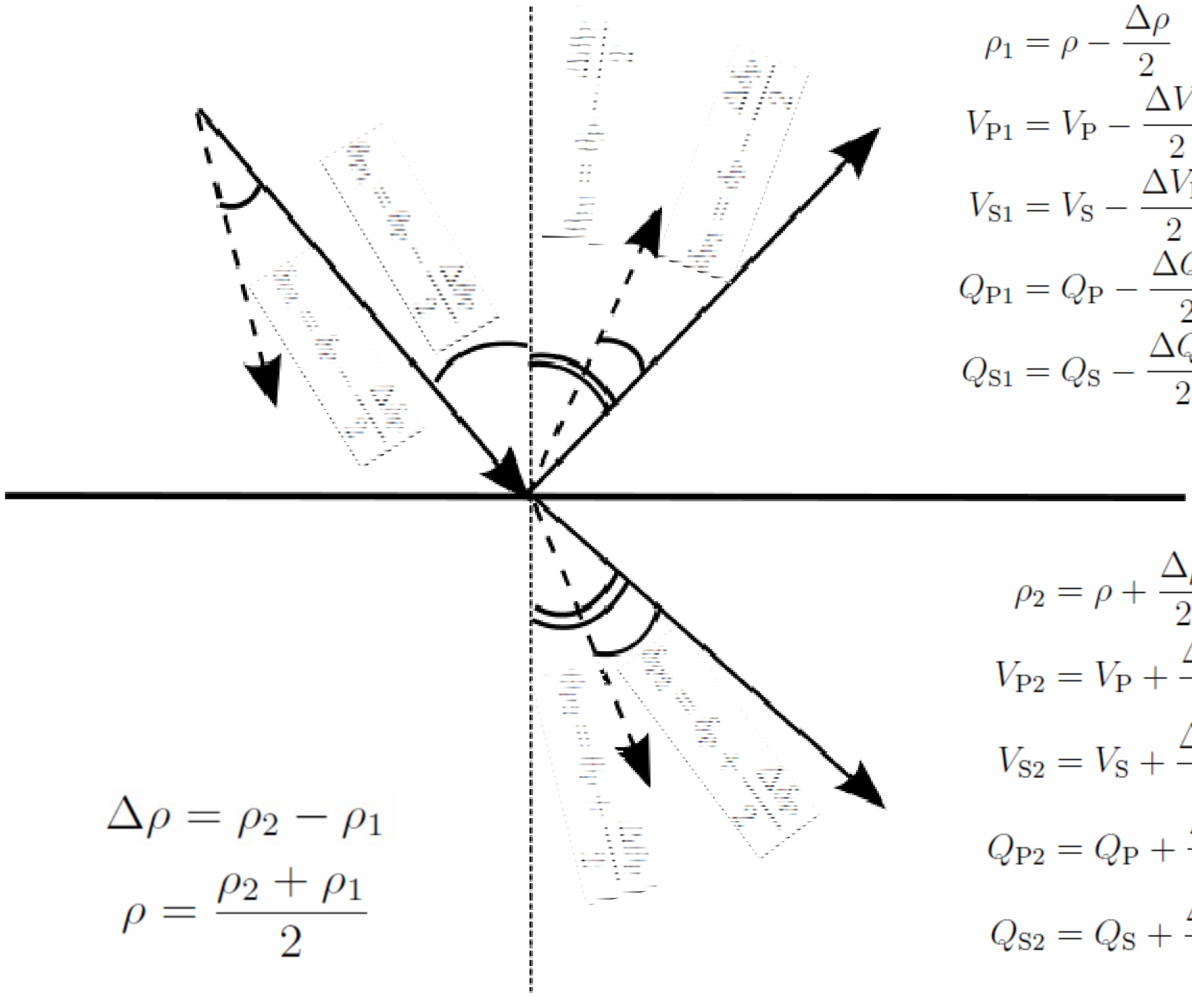


Contrast in velocity with constant quality factor can cause the change in incident attenuation angle



Linearization of Snell’s law

To calculate the approximate reflectivities for a low contrast model, and to write the physical quantities in medium 1, and medium 2 in terms of fractional perturbations, we must express the phase and attenuation angles in perturbed form. This in turn requires us to linearize the generalized Snell's law.



$$\rho_1 = \rho - \frac{\Delta\rho}{2}$$
$$V_{P1} = V_P - \frac{\Delta V_P}{2}$$
$$V_{S1} = V_S - \frac{\Delta V_P}{2}$$
$$Q_{P1} = Q_P - \frac{\Delta Q_P}{2}$$
$$Q_{S1} = Q_S - \frac{\Delta Q_S}{2}$$

$$\rho_2 = \rho + \frac{\Delta\rho}{2}$$
$$V_{P2} = V_P + \frac{\Delta V_P}{2}$$
$$V_{S2} = V_S + \frac{\Delta V_P}{2}$$
$$Q_{P2} = Q_P + \frac{\Delta Q_P}{2}$$
$$Q_{S2} = Q_S + \frac{\Delta Q_S}{2}$$

$$\Delta\rho = \rho_2 - \rho_1$$
$$\rho = \frac{\rho_2 + \rho_1}{2}$$

Linearization of Snell's law

Real part of Snell's law

$$\frac{\sin\theta_{P1}}{V_{P1}} = \frac{\sin\theta_{P2}}{V_{P2}}$$

$$\sin\theta_P\left(1 - \frac{1}{2}\frac{\Delta\theta_P}{\tan\theta_P}\right)$$

$$\sin\theta_P\left(1 + \frac{1}{2}\frac{\Delta\theta_P}{\tan\theta_P}\right)$$

$$V_P\left(1 + \frac{1}{2}\frac{\Delta V_P}{V_P}\right)$$

$$V_P\left(1 - \frac{1}{2}\frac{\Delta V_P}{V_P}\right)$$

$$\Delta\theta_P = \tan\theta_P\frac{\Delta V_P}{V_P}$$

Linearization of Snell's law

Imaginary part of Snell's law

$$\frac{Q_{\mathrm{P1}}^{-1}}{2V_{\mathrm{P1}}}(\sin\theta_{\mathrm{P1}} - \cos\theta_{\mathrm{P1}}\tan\delta_{\mathrm{P1}}) = \frac{Q_{\mathrm{P2}}^{-1}}{2V_{\mathrm{P2}}}(\sin\theta_{\mathrm{P2}} - \cos\theta_{\mathrm{P2}}\tan\delta_{\mathrm{P2}})$$

$$\Big\downarrow \text{Linearization}$$

$$\Delta\delta_{\mathrm{P}} = \frac{1}{2}\sin 2\delta_{\mathrm{P}}\left\{\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}\frac{1}{\cos^2\theta_{\mathrm{P}}} + \left(1 - \frac{\tan\theta_{\mathrm{P}}}{\tan\delta_{\mathrm{P}}}\right)\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}}\right\}$$

AVO equations

P-to-P reflection coefficient

$$\mathrm{R}_{\mathrm{PP}}(\theta_{\mathrm{P}}, \delta_{\mathrm{P}}) = \mathrm{R}^{\mathrm{E}}_{\mathrm{PP}}(\theta_{\mathrm{P}}) + i\mathrm{R}^{\mathrm{AH}}_{\mathrm{PP}}(\theta_{\mathrm{P}}) + i\mathrm{R}^{\mathrm{AIH}}_{\mathrm{PP}}(\theta_{\mathrm{P}}, \delta_{\mathrm{P}})$$

Elastic term: function of fractional changes in elastic properties

Homogenous term: function of fractional changes in elastic and anelastic properties

Inhomogeneous term: function of fractional changes in elastic properties and attenuation angle

AVO equations

Elastic term

$$\mathrm{R}^{\mathrm{E}}_{\mathrm{PP}} = A^{\mathrm{E}}_{\mathrm{PP}} + B^{\mathrm{E}}_{\mathrm{PP}} \sin^2\theta_P + C^{\mathrm{E}}_{\mathrm{PP}} (\tan^2\theta_P - \sin^2\theta_P)$$

Zero offset term

$$\frac{1}{2}\left[\frac{\Delta\rho}{\rho} + \frac{\Delta V_{PE}}{V_{PE}}\right]$$

$$\frac{1}{2}\frac{\Delta V_{PE}}{V_{PE}}$$

AVO gradient

$$-2\left(\frac{V_{SE}}{V_{PE}}\right)^2 \left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{SE}}{V_{SE}}\right] + \left[\frac{1}{2}\frac{\Delta V_{PE}}{V_{PE}}\right]$$

AVO equations

Homogenous term

$$\mathrm{R}^{\mathrm{AH}}_{\mathrm{PP}}(\theta_{\mathrm{P}}) = A^{\mathrm{A}}_{\mathrm{PP}} + B^{\mathrm{A}}_{\mathrm{PP}} \sin^2\theta_P + A^{\mathrm{A}}_{\mathrm{PP}}(\tan^2\theta_P - \sin^2\theta_P)$$

Zero offset term

$$-\frac{1}{4}Q_P^{-1}\frac{\Delta Q_P}{Q_P}$$

$$-\frac{1}{4}Q_P^{-1}\frac{\Delta Q_P}{Q_P}$$

AVO gradient

$$-2\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2\left[\left(Q_S^{-1} - Q_P^{-1}\right)\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right) - Q_S^{-1}\frac{\Delta Q_S}{Q_S}\right] - \frac{1}{4}Q_P^{-1}\frac{\Delta Q_P}{Q_P}$$

Inhomogeneous term

$$R_{\mathrm{PP}}^{\mathrm{AIH}}(\theta_{\mathrm{P}},\delta_{\mathrm{P}}) = C_{\mathrm{PP}}^{\mathrm{A}}(\tan\theta_P + \tan^3\theta_P) + F_{\mathrm{PP}}^{\mathrm{A}}\sin(2\theta_P)$$

$$\frac{1}{2}\tan\delta_P Q_P^{-1}\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}$$

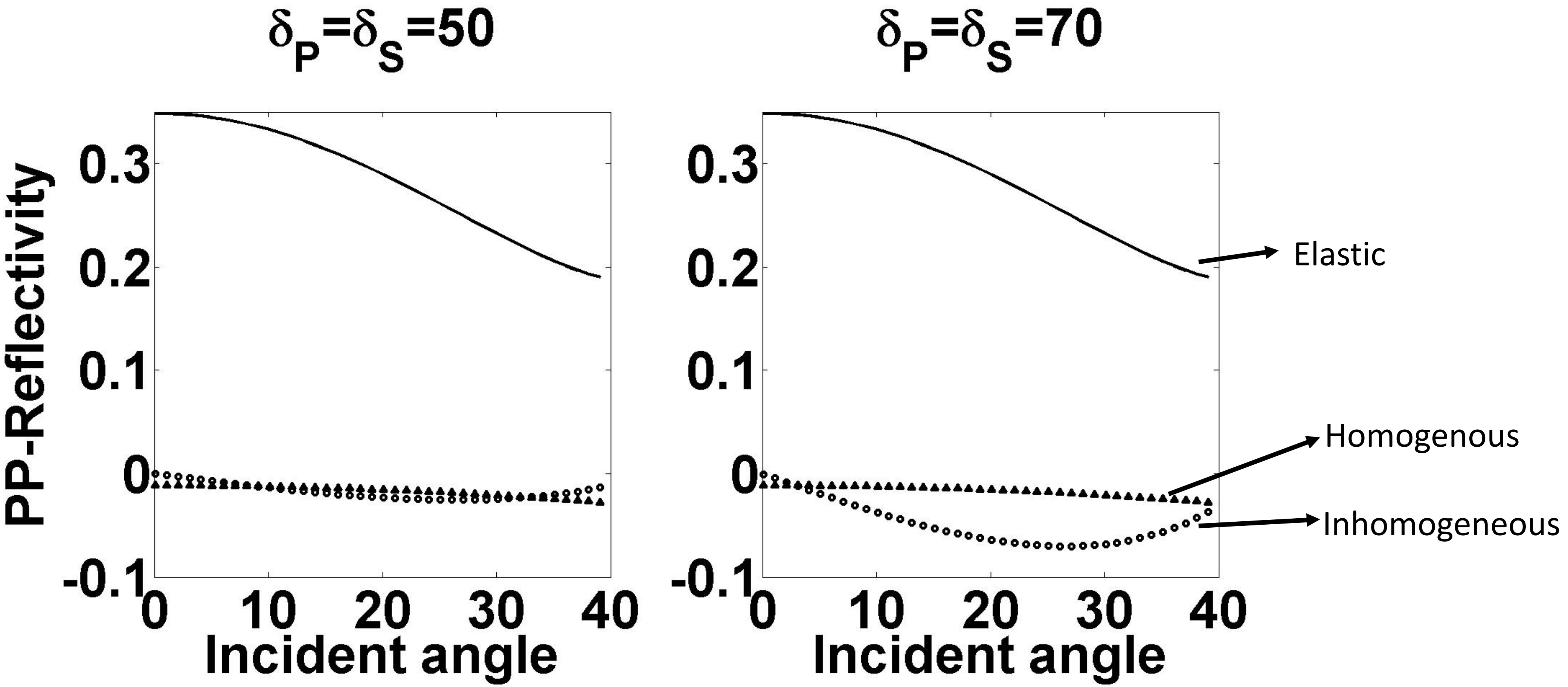
$$-\tan\delta_P\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2 Q_P^{-1}\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right]$$

AVO equations

$$\frac{V_{P2}}{V_{P1}} = \frac{V_{S2}}{V_{S1}} = 1.5$$

$$\frac{Q_{P2}}{Q_{P1}} = \frac{Q_{S2}}{Q_{S1}} = 1.3$$

$$\frac{\rho_2}{\rho_1} = 1.3$$

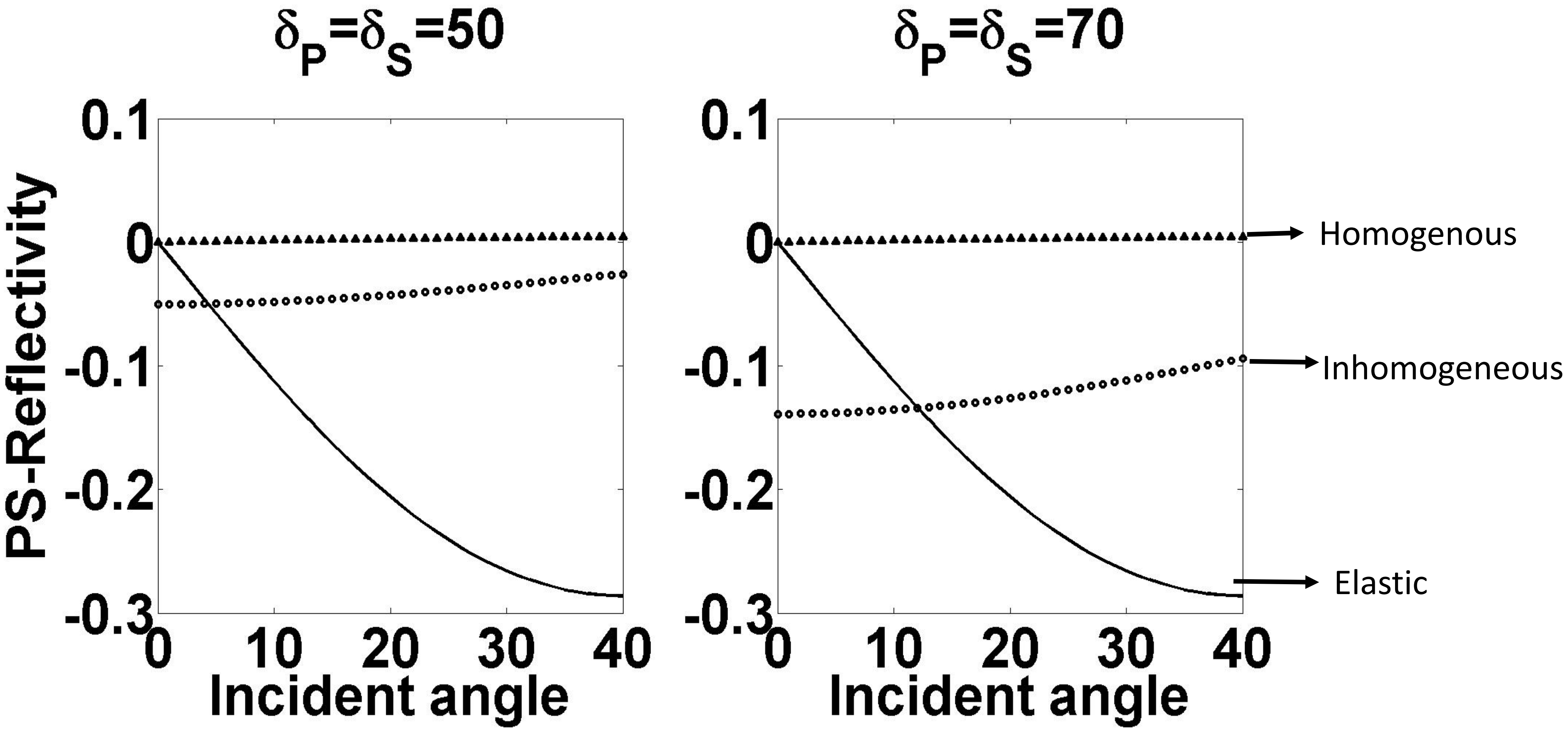


AVO equations

$$\frac{V_{P2}}{V_{P1}} = \frac{V_{S2}}{V_{S1}} = 1.5$$

$$\frac{Q_{P2}}{Q_{P1}} = \frac{Q_{S2}}{Q_{S1}} = 1.3$$

$$\frac{\rho_2}{\rho_1} = 1.3$$



AVO inversion

For small angles the reflectiviteis for homogeneous waves can be written as

Normalized PS reflectivity to generate the linearity at small incident angles

Introduction to petroleum seismology
LT Ikelle, L Amundsen - 2005 - library.seg.org

$$\mathrm{R}^{\mathrm{E}}_{\mathrm{PP}}(\theta_{\mathrm{P}}) = A^{\mathrm{E}}_{\mathrm{PP}} + B^{\mathrm{E}}_{\mathrm{PP}} \sin^2\theta_P$$

$$\mathrm{R}^{\mathrm{AH}}_{\mathrm{PP}}(\theta_{\mathrm{P}}) = A^{\mathrm{A}}_{\mathrm{PP}} + B^{\mathrm{A}}_{\mathrm{PP}} \sin^2\theta_P$$

$$\mathbb{R}^{\mathrm{E}}_{\mathrm{PS}}(\theta_{\mathrm{P}}) = \frac{\mathrm{R}^{\mathrm{E}}_{\mathrm{PS}}(\theta_{\mathrm{P}})}{\sin\theta_{\mathrm{P}}} = A^{\mathrm{E}}_{\mathrm{PS}} + B^{\mathrm{E}}_{\mathrm{PS}} \sin^2\theta_{\mathrm{P}}$$

$$\mathbb{R}^{\mathrm{AH}}_{\mathrm{PS}}(\theta_{\mathrm{P}}) = \frac{\mathrm{R}^{\mathrm{A}}_{\mathrm{PS}}(\theta_{\mathrm{P}})}{\sin\theta_{\mathrm{P}}} = A^{\mathrm{A}}_{\mathrm{PS}} + B^{\mathrm{A}}_{\mathrm{PS}} \sin^2\theta_{\mathrm{P}}$$

AVO inversion

Shale/Sand(unconsolidated)

$\rho_1 = 2.16 \ \mathrm{g/cm^3}$

$V_{P1} = 2.057 \ \mathrm{Km/s^2}$

$V_{S1} = 0.489 \ \mathrm{Km/s^2}$

$\rho_2 = 2.08$

$V_{P2} = 2.134$

$V_{S2} = 0.969$

Shale/Salt

$\rho_1 = 2.40$

$V_{P1} = 3.811$

$V_{S1} = 2.263$

$\rho_2 = 2.05$

$V_{P2} = 4.573$

$V_{S2} = 2.729$

Shale/Limestone(gas)

$\rho_1 = 2.40$

$V_{P1} = 3.811$

$V_{S1} = 2.263$

$\rho_2 = 2.49$

$V_{P2} = 5.043$

$V_{S2} = 2.957$

Limestone/Salt

$\rho_1 = 2.65$

$V_{P1} = 5.335$

$V_{S1} = 2.957$

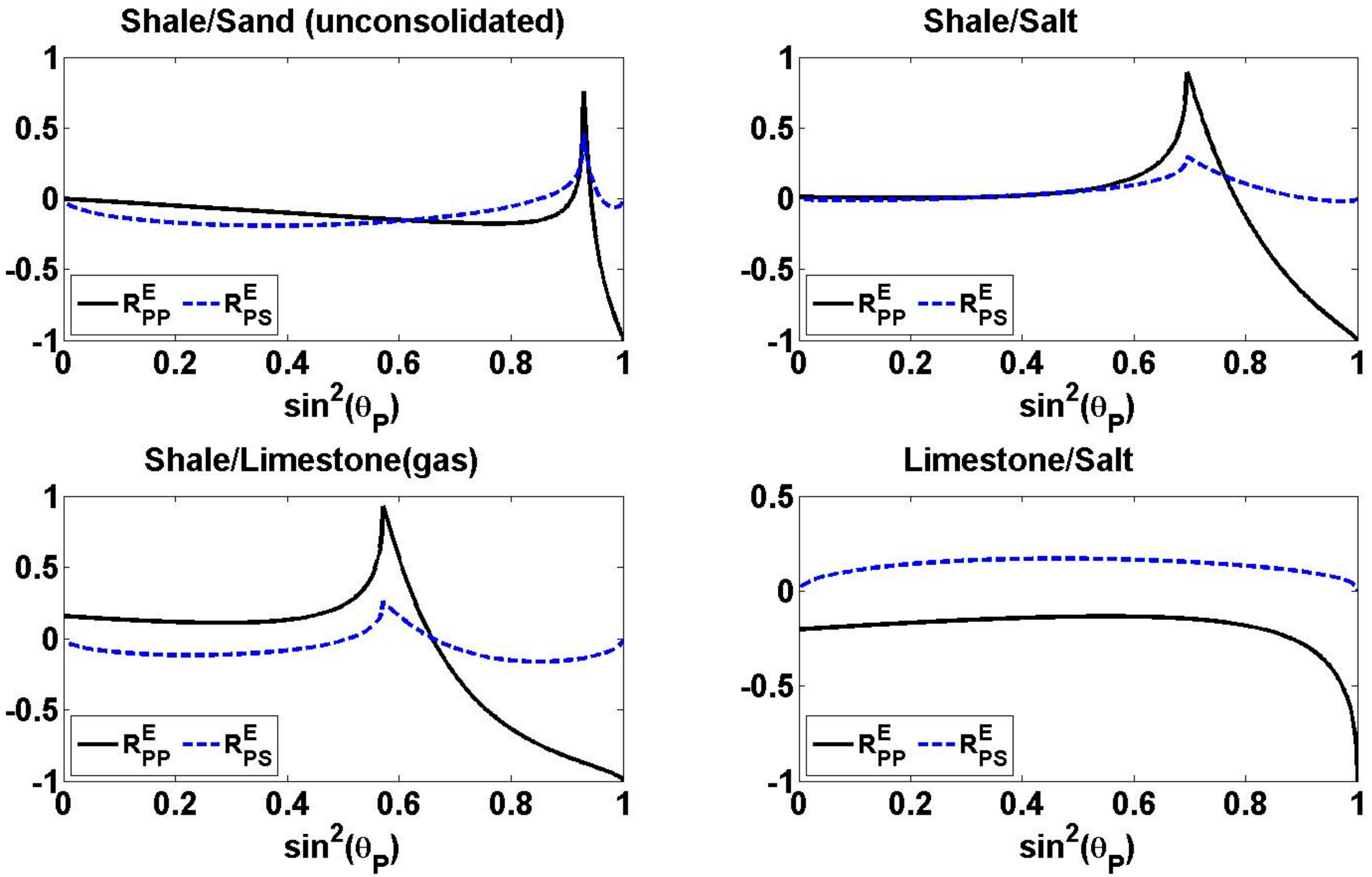
$\rho_2 = 2.05$

$V_{P2} = 4.573$

$V_{S2} = 2.729$

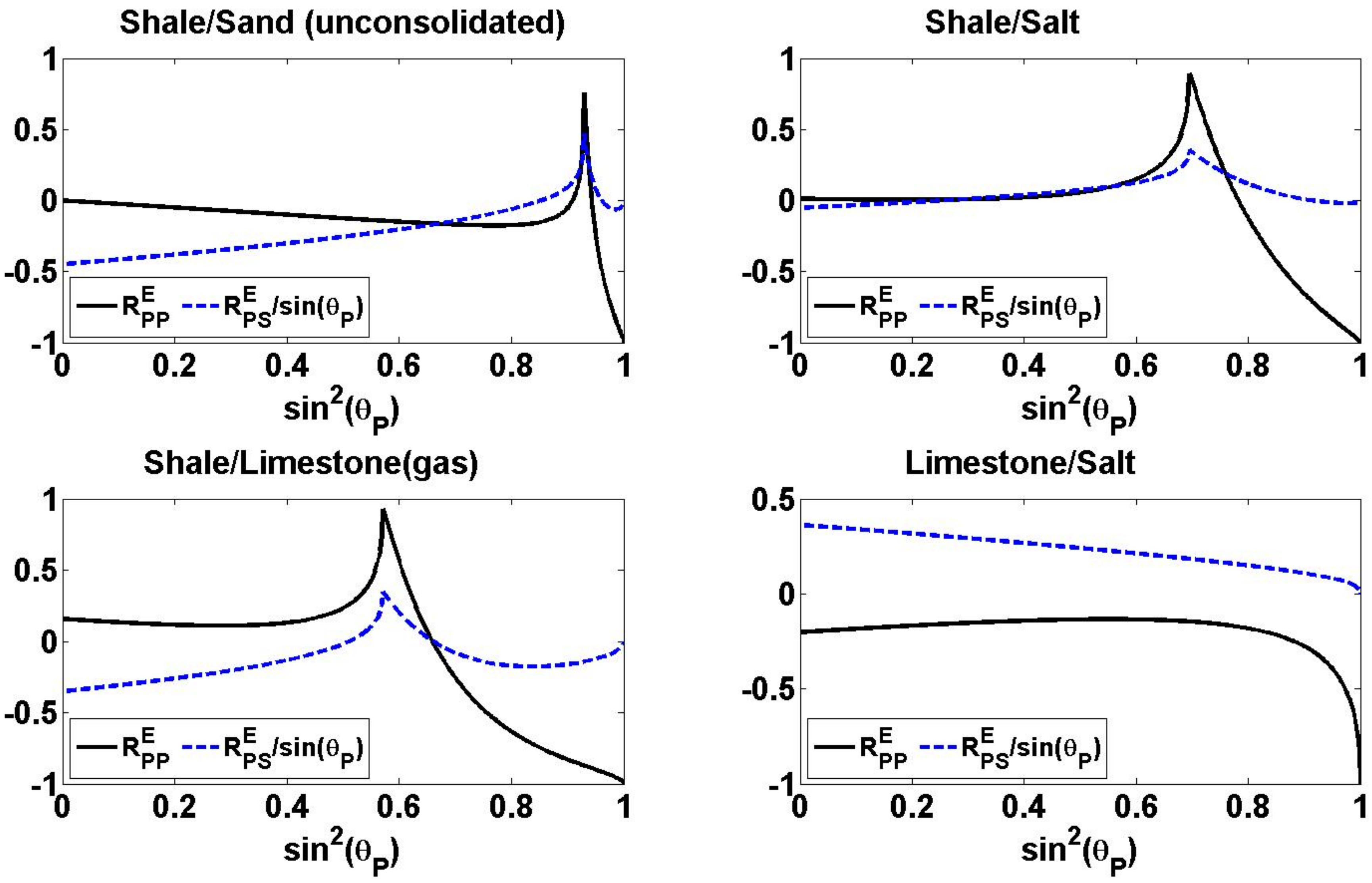
AVO inversion

Elastic PP-reflectivity
 vs
Elastic PS-reflectivity



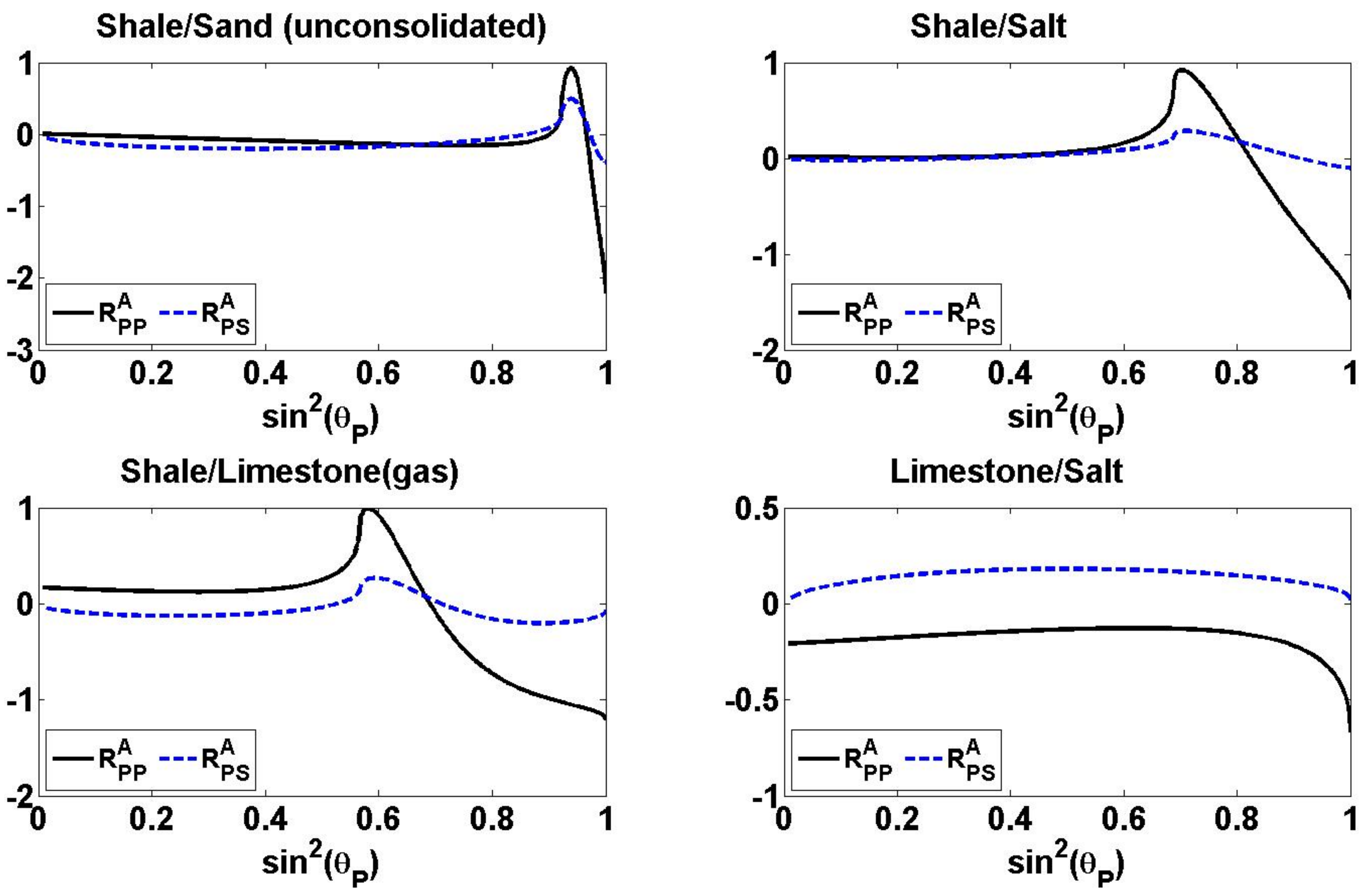
AVO inversion

Elastic PP-reflectivity
vs
Elastic Normalized
PS-reflectivity



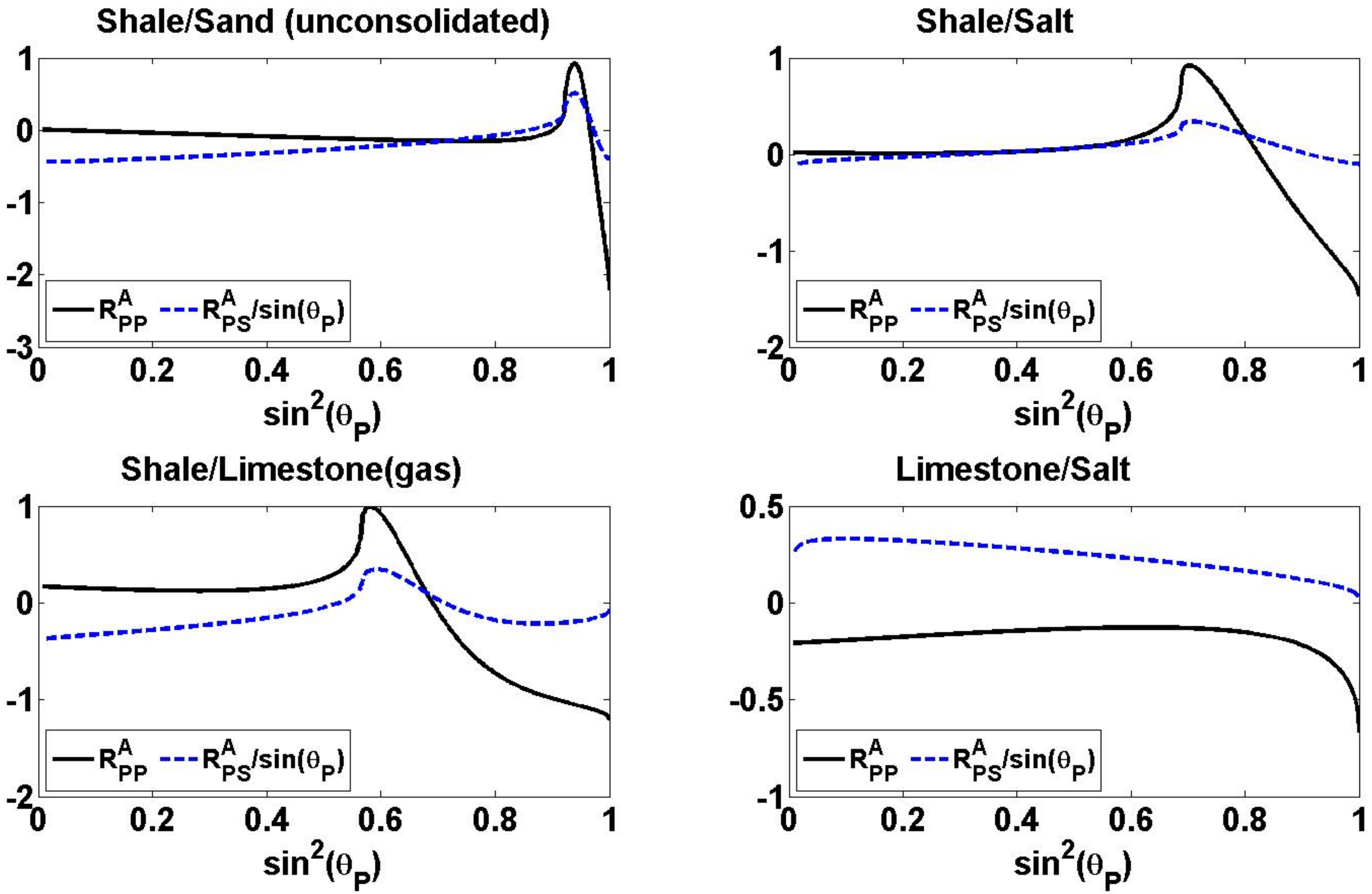
AVO inversion

Anlastic PP-reflectivity
 vs
Anelastic PS-reflectivity



AVO inversion

Anlastic PP-reflectivity
vs
Anelastic Normalized
PS-reflectivity



AVO inversion

Elastic intercepts and gradients

$$A_{\mathrm{PP}}^{\mathrm{E}} = \frac{1}{2}(x_1 + x_3)$$

$$B_{\mathrm{PP}}^{\mathrm{E}} = \frac{1}{2}x_1 - 2x_4^2(x_3 + 2x_2)$$

$$A_{\mathrm{PS}}^{\mathrm{E}} = -\frac{1}{2}x_3 - x_4\,(x_3 + 2x_2)$$

$$B_{\mathrm{PS}}^{\mathrm{E}} = -\frac{1}{4}x_4^2 x_3 + x_4\left(\frac{1}{2} + x_4\right)(x_3 + 2x_2)$$

Anelastic intercepts and gradients

$$A_{\mathrm{PP}}^{\mathrm{A}} = -\frac{1}{4}x_7 x_5$$

$$B_{\mathrm{PP}}^{\mathrm{A}} = -2x_4^2\left[(x_8 - x_7)\,(x_3 + 2x_2) - x_8 x_6\right] - \frac{1}{4}x_7 x_5$$

$$A_{\mathrm{PS}}^{\mathrm{A}} = -\frac{1}{2}x_4(x_8 - x_7)\,(x_3 + 2x_2) + x_4 x_8 x_6$$

$$B_{\mathrm{PS}}^{\mathrm{A}} = -\frac{1}{4}(x_8 - x_7)x_3 + x_4\left[-x_8 x_6\left(\frac{1}{2} + x_4\right) + (x_8 - x_7)\left(\frac{1}{4} + x_4\right)(x_3 + 2x_2)\right]$$

$$x_1 = \frac{\Delta V_P}{V_P}$$

$$x_2 = \frac{\Delta V_S}{V_S}$$

$$x_3 = \frac{\Delta \rho}{\rho}$$

$$x_4 = \frac{V_S}{V_P}$$

$$x_5 = \frac{\Delta Q_P}{Q_P}$$

$$x_6 = \frac{\Delta Q_S}{Q_S}$$

$$x_7 = Q_P$$

$$x_8 = Q_S$$

Summary and conclusion

- Based on the Snell's law in viscoelastic medium the incident and transmitted attenuation angles are not the same.
- Perturbation in attenuation angle can be expressed in terms of perturbations in elastic velocity and quality factor.
- Reflectivity not only depends upon the perturbations in elastic properties,

  but also on perturbations in quality factors for P- and S-waves.
- Inhomogeneity of the wave does not have any influence on the reflection coefficient for vertically incident waves.
- If there is no contrast in Qp and Qs across the interface, the inhomogeneity of the wave does make a contributions in the reflectivity.
- Major contribution in the anelastic part of the reflectivity caused by the inhomogeneity of the wave.
- By normalization of the elastic and anelastic parts of the PS reflectivity and using the nonlinear inversion, 5 fractional changes in medium properties, Vs/Vp ratio and averages in P- and S-wave quality factors can be inverted.

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Thank you