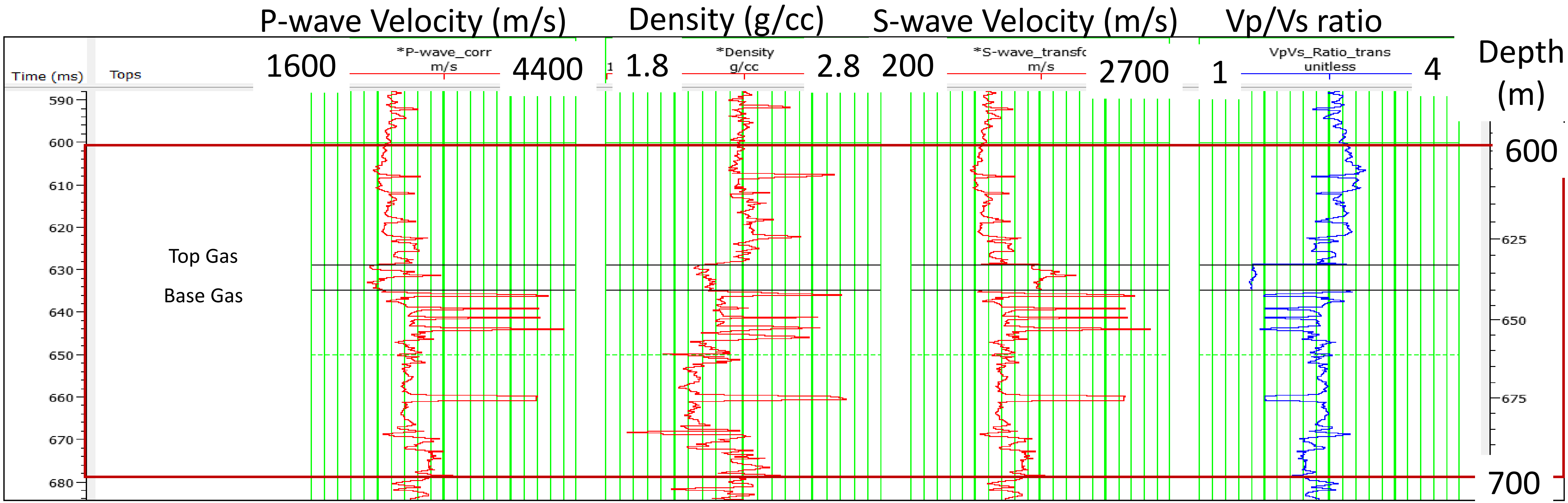
Rock physics, inversion and Bayesian classification

Brian Russell

Introduction

- Today, most geoscientists have an array of tools available to perform seismic reservoir characterization.
- However, the complexity of these tools increases year by year, and can be overwhelming at times.
- In this talk, I want to discuss some visualization tools that improve the user-friendliness of the reservoir characterization process.
- These tools will include both statistical methods and deterministic methods, and will combine both well log measurements and pre-stack inversion.
- I will illustrate the various methods with examples from a shallow gas sand in Alberta.

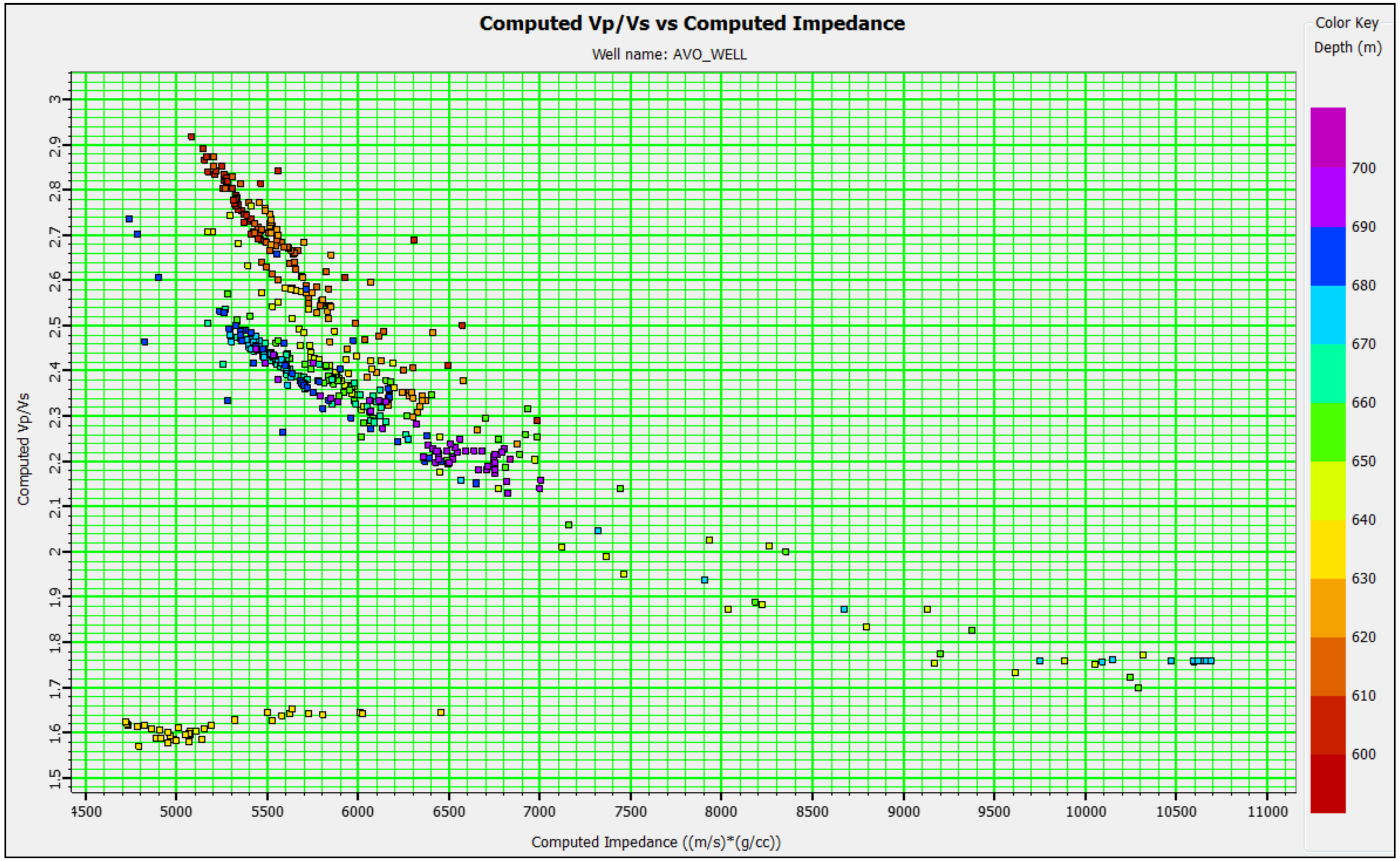
Gas Sand well log



- This figure shows the shallow gas sand used in this study.
- The P-wave sonic and density logs were recorded with wireline logs, the S-wave log was created using the Castagna equation and Gassmann fluid substitution.

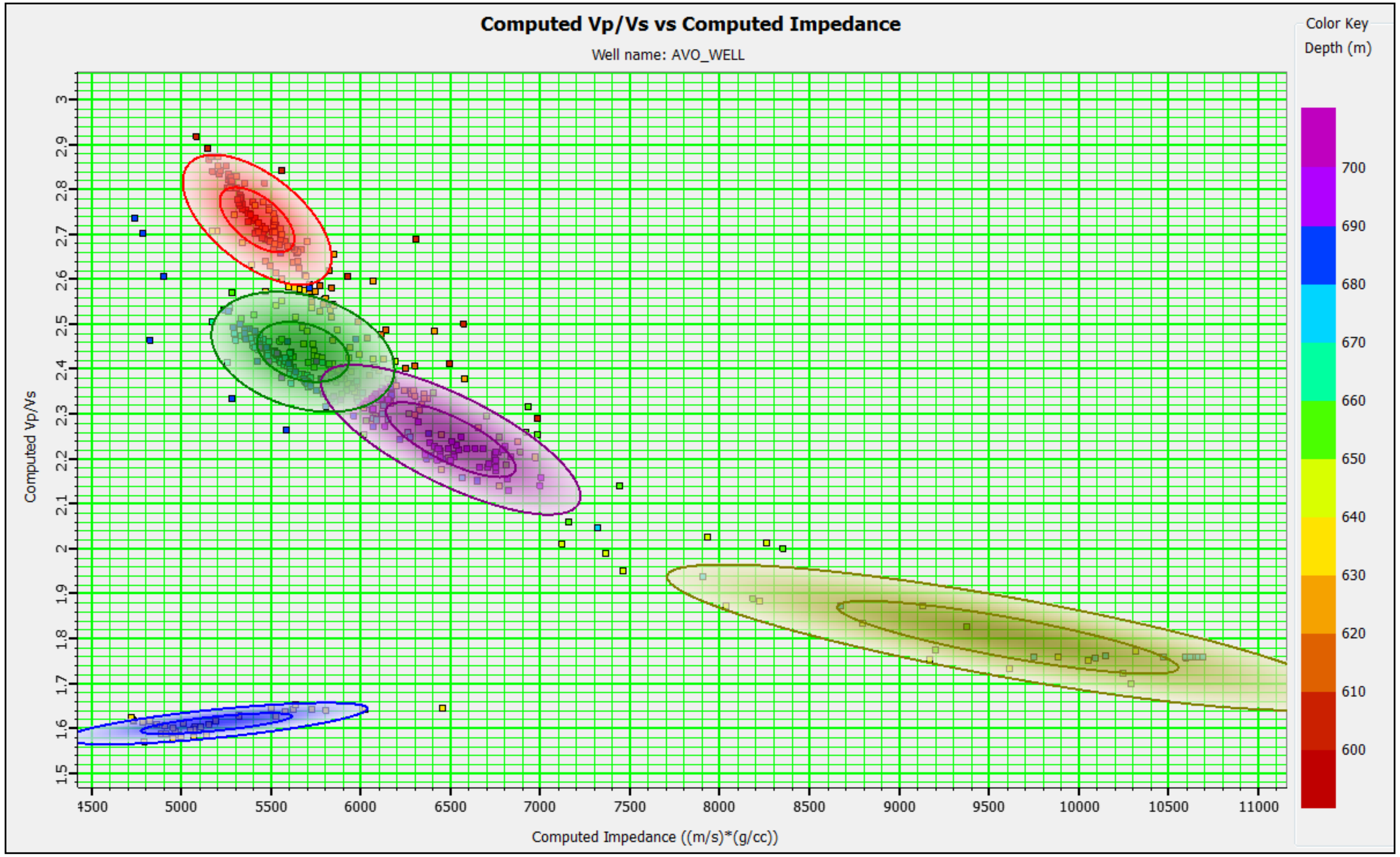
Well log crossplot

- This is a cross-plot of V_P/V_S ratio versus P-impedance (ρV_P) for the zone between 600 and 700 m around the gas sand.
- We can analyze this cross-plot either statistically or deterministically.
- I will start with statistical clustering and then use a deterministic approach to explain the clusters.



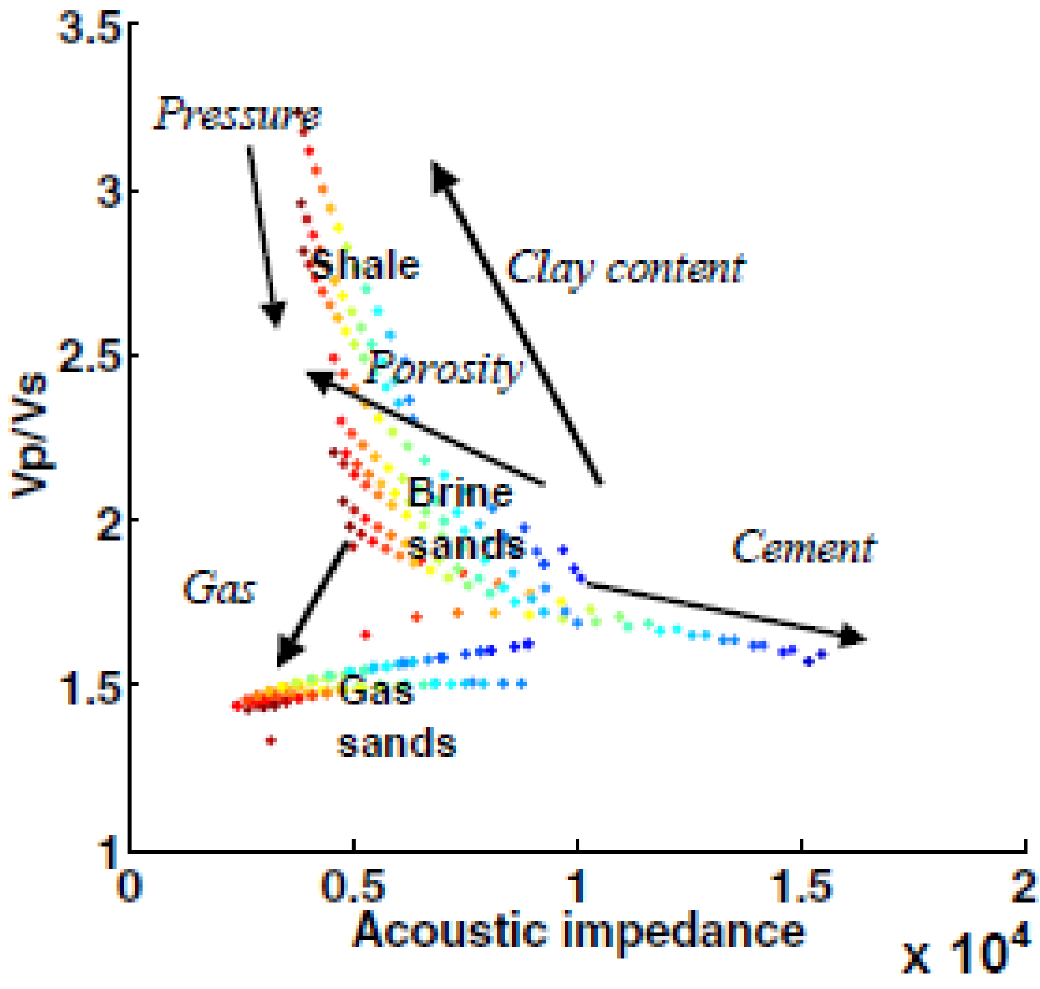
Automatic clustering

- The clusters on the crossplot have been identified using *K*-means clustering with a statistical distance algorithm.
- The key question is how to interpret these five clusters.
- I will next discuss a rock physics template method which allows us to perform such an interpretation.



The rock physics template (RPT)

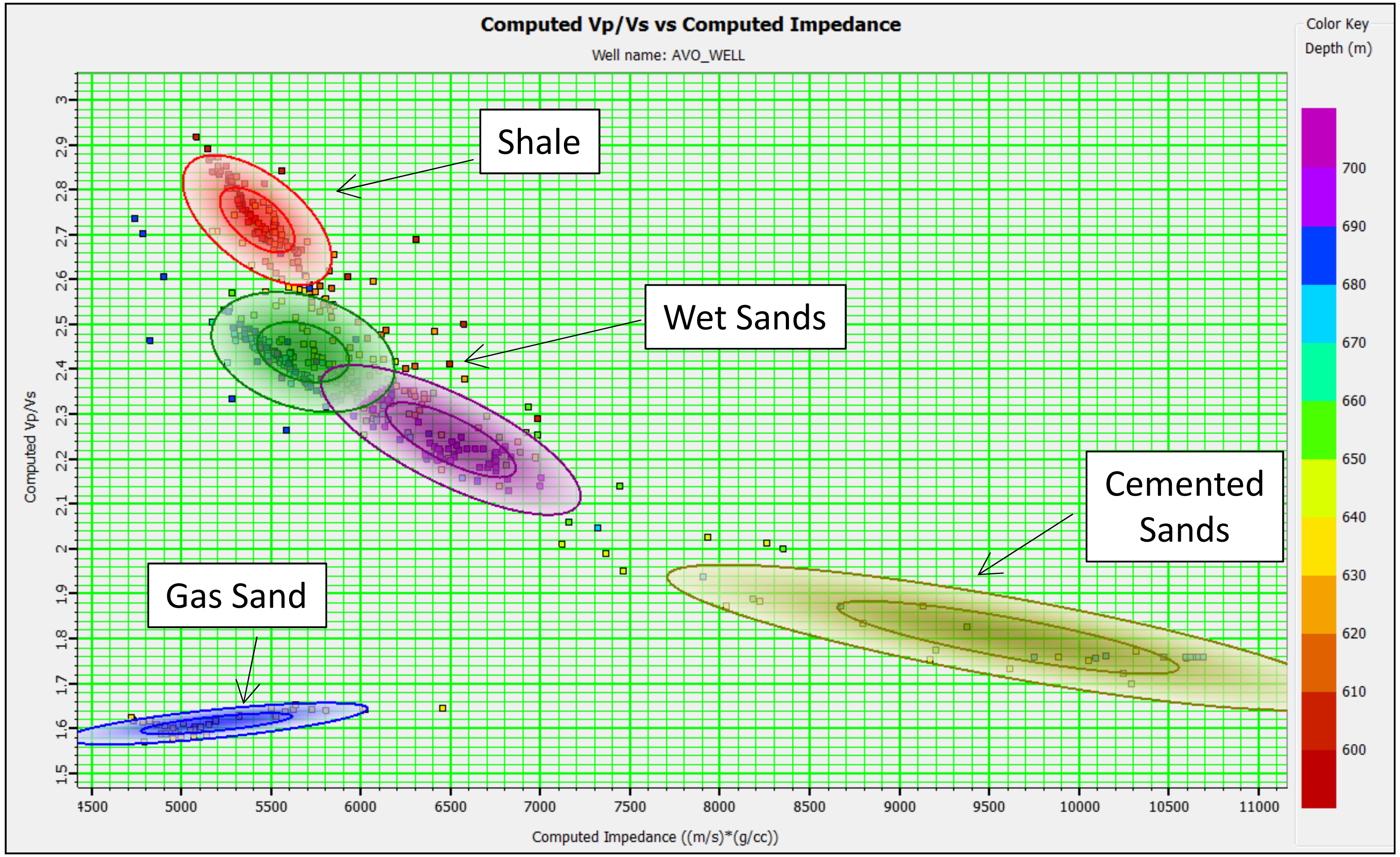
- Ødegaard and Avseth (2003) developed a rock physics template in which the fluid and mineralogical content of a reservoir could be estimated on a cross-plot of V_P/V_S ratio against acoustic impedance.
- The elastic constants are computed as a function of porosity, pressure and saturation using Hertz-Mindlin theory, the lower Hashin-Shtrikman bound and Gassmann fluid substitution.
- This cross-plot allows us to identify pressure, clay content, porosity, cement and fluid trends.



from Ødegaard and Avseth (2003)

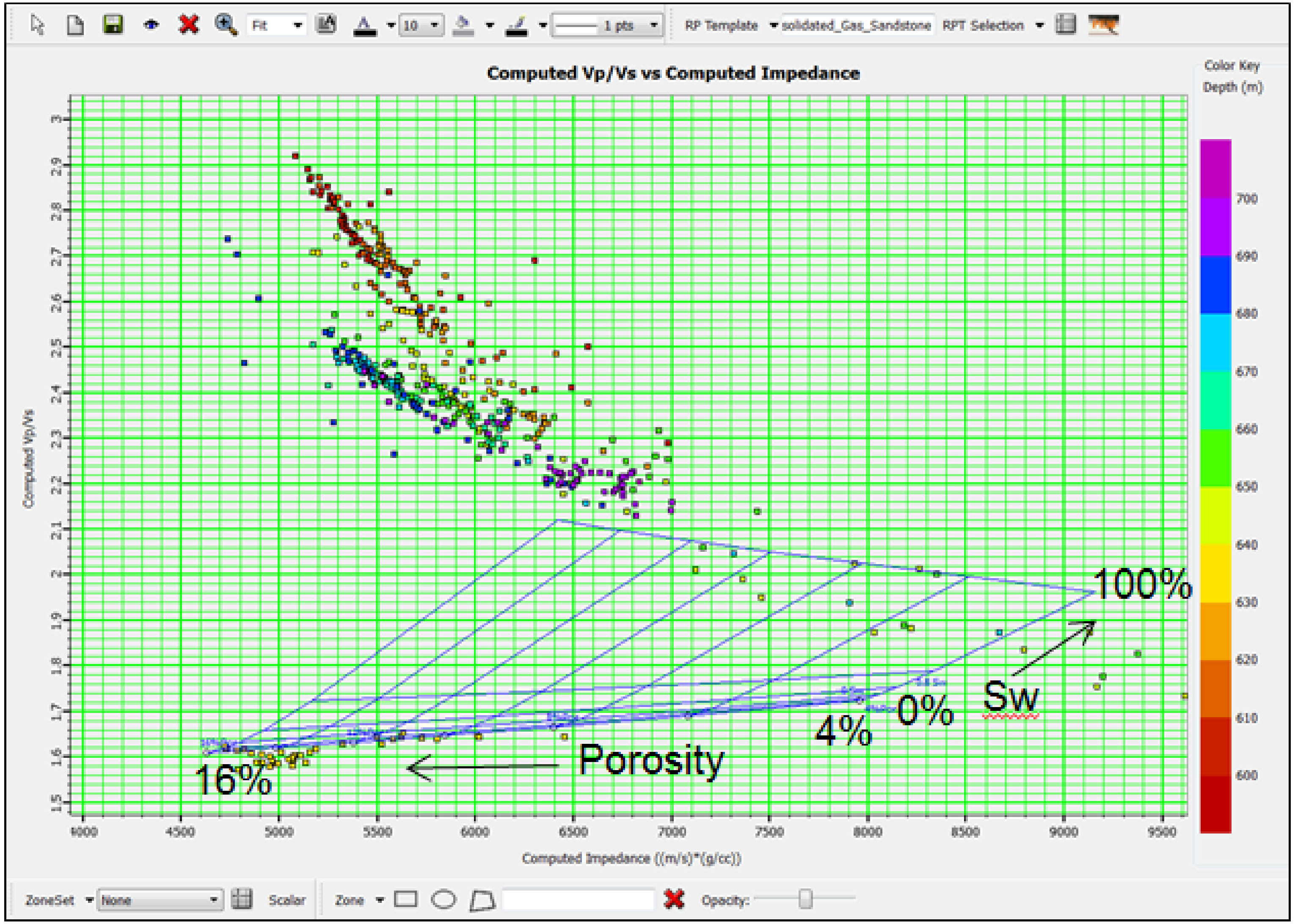
Interpreting the clusters

- The clusters from the previous plot can be interpreted as shown using the Ødegaard and Avseth RPT template.
- This is one use of the rock physics template.
- A second use, shown next, is to draw a set if curves on the cross-plot as a function of saturation and porosity, or any other two parameters.



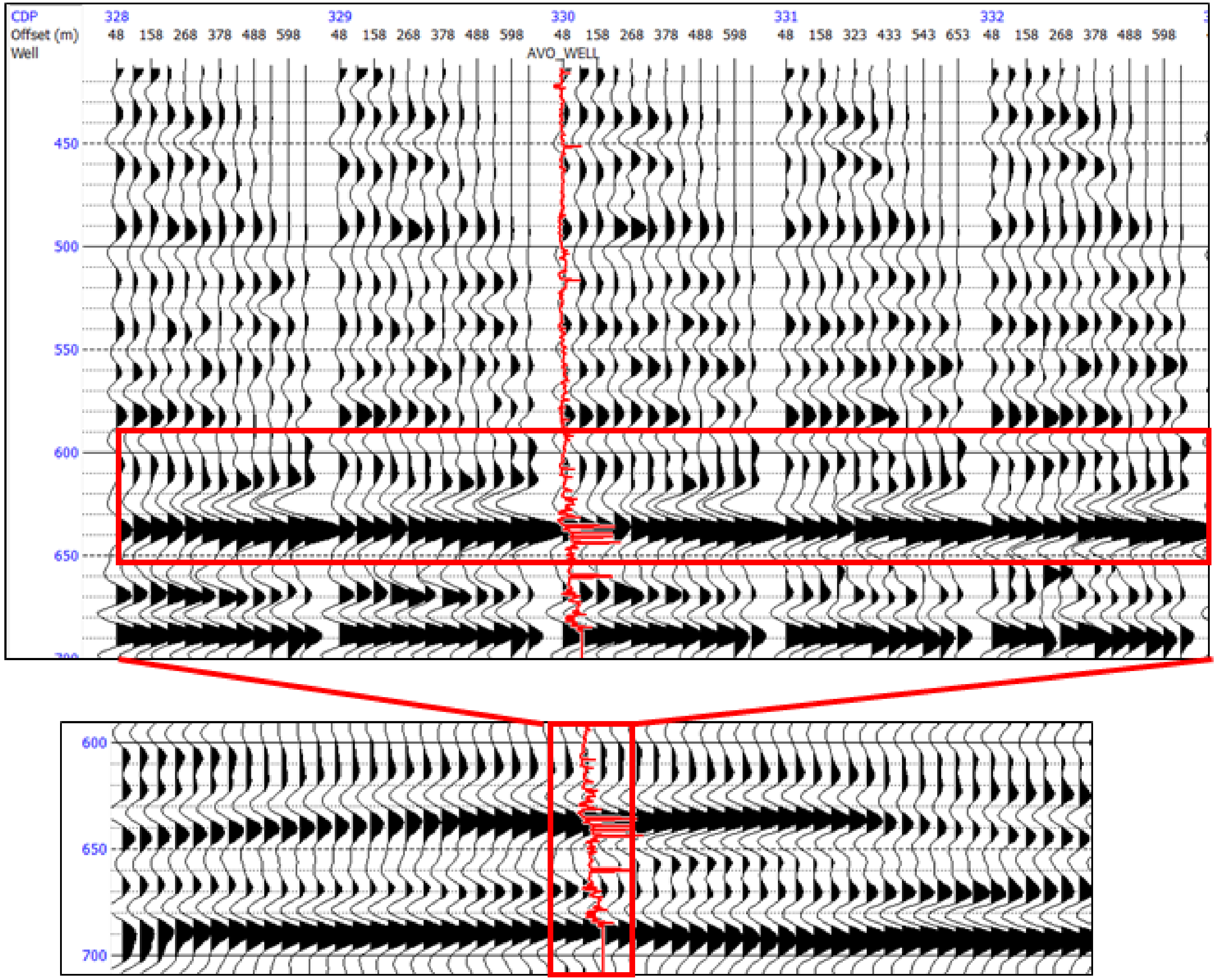
A porosity versus saturation template

- The rock physics template for the gas sand model is shown here, as a function of water saturation and porosity.
- Note that the template fits the gas sand well for low S_W and high porosity.
- Later, I will show how to colour-code this RPT and display the results on the seismic.



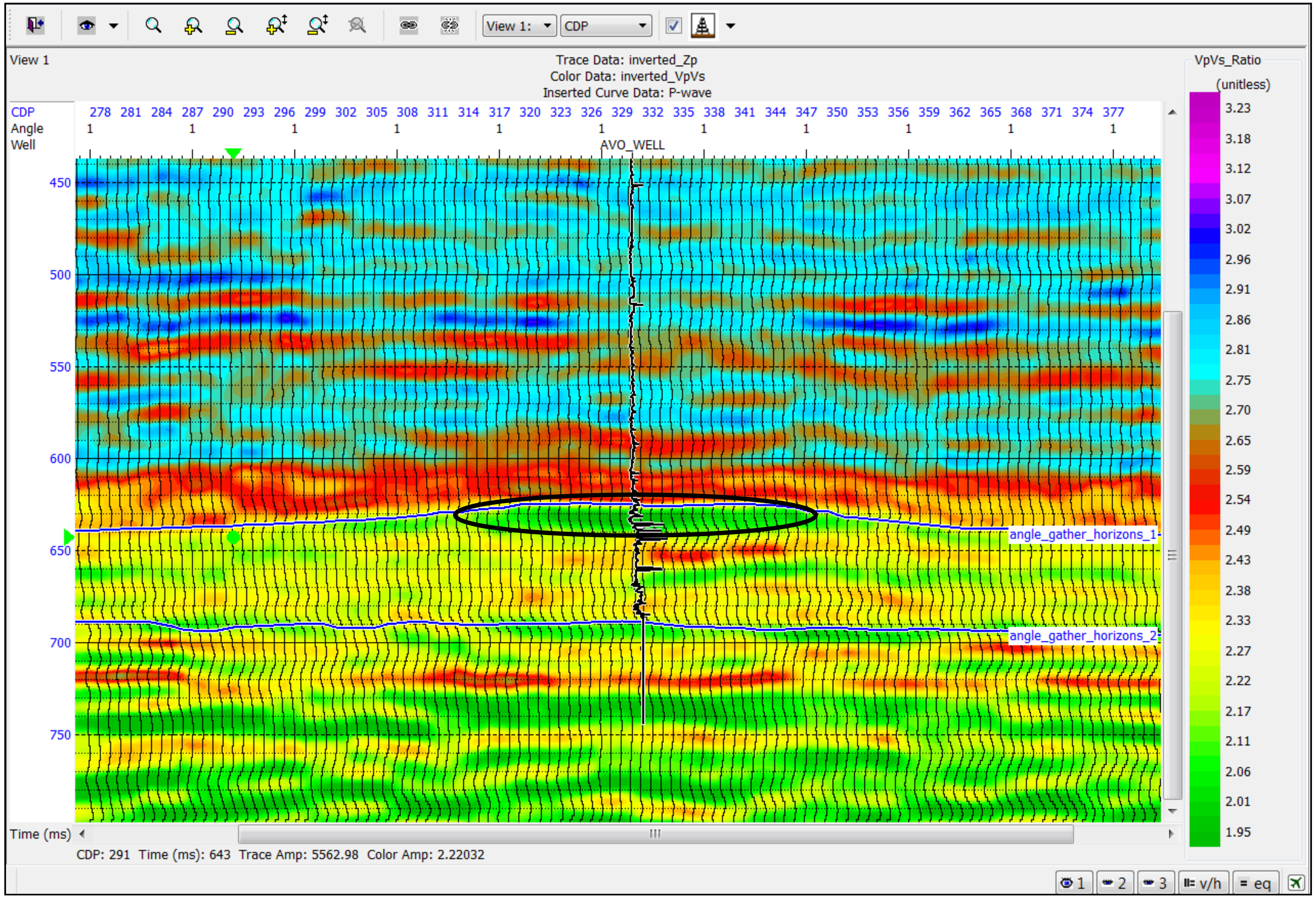
The seismic dataset

- The top figure shows CMP gathers over a seismic line that intersects our well.
- An AVO Class 3 anomaly is observed around the gas sand, created by a drop in P-impedance and V_P/V_S ratio.
- The bottom part of the figure shows the stack of these gathers, which forms part of an amplitude “bright spot”.



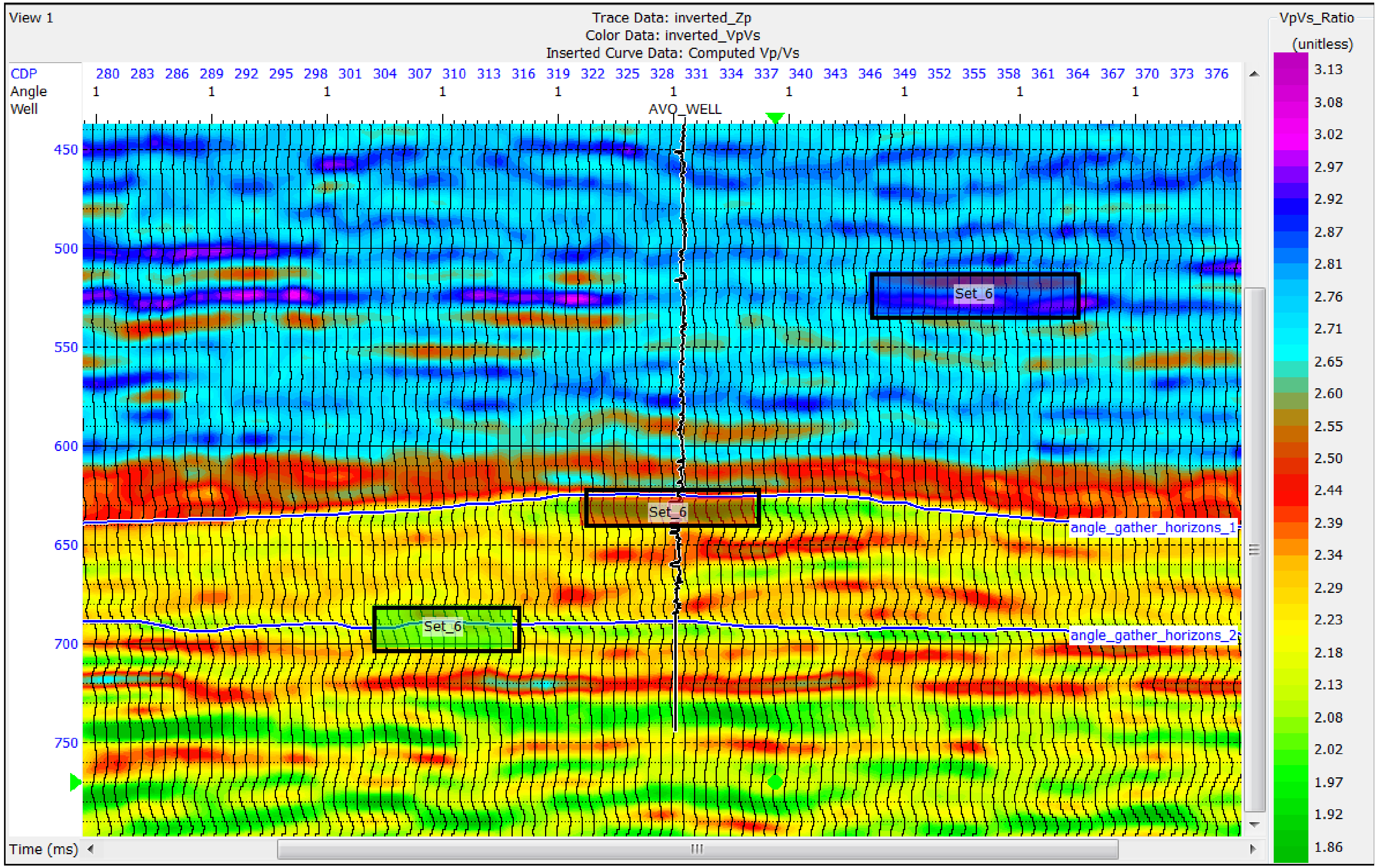
Simultaneous pre-stack inversion

- The simultaneous pre-stack inversion of the gathers on the previous slide, where colour shows V_P/V_S ratio and wiggle trace shows P-impedance.
- The gas sand displays a low V_P/V_S ratio.
- Above the gas sand is are Cretaceous sand/shales.
- Below the gas sand are cemented sands and carbonates.



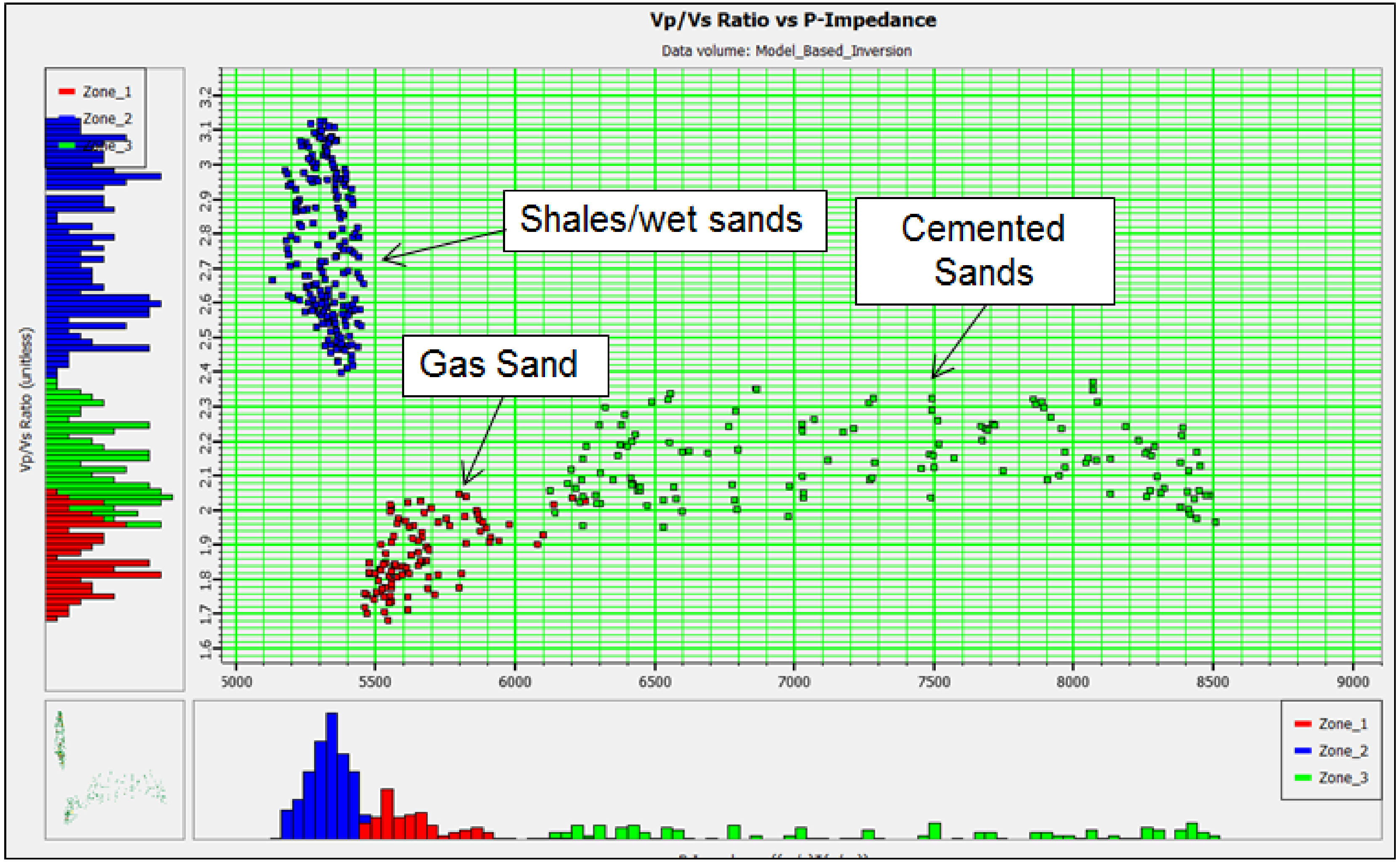
Interactively picked zones

- Three zones have been picked on the section: wet (blue), gas (red) and consolidated (green).
- We would hope that these zones would correspond to the RPT interpretation.
- The best way to test this is on a V_P/V_S ratio vs P-impedance X-plot.



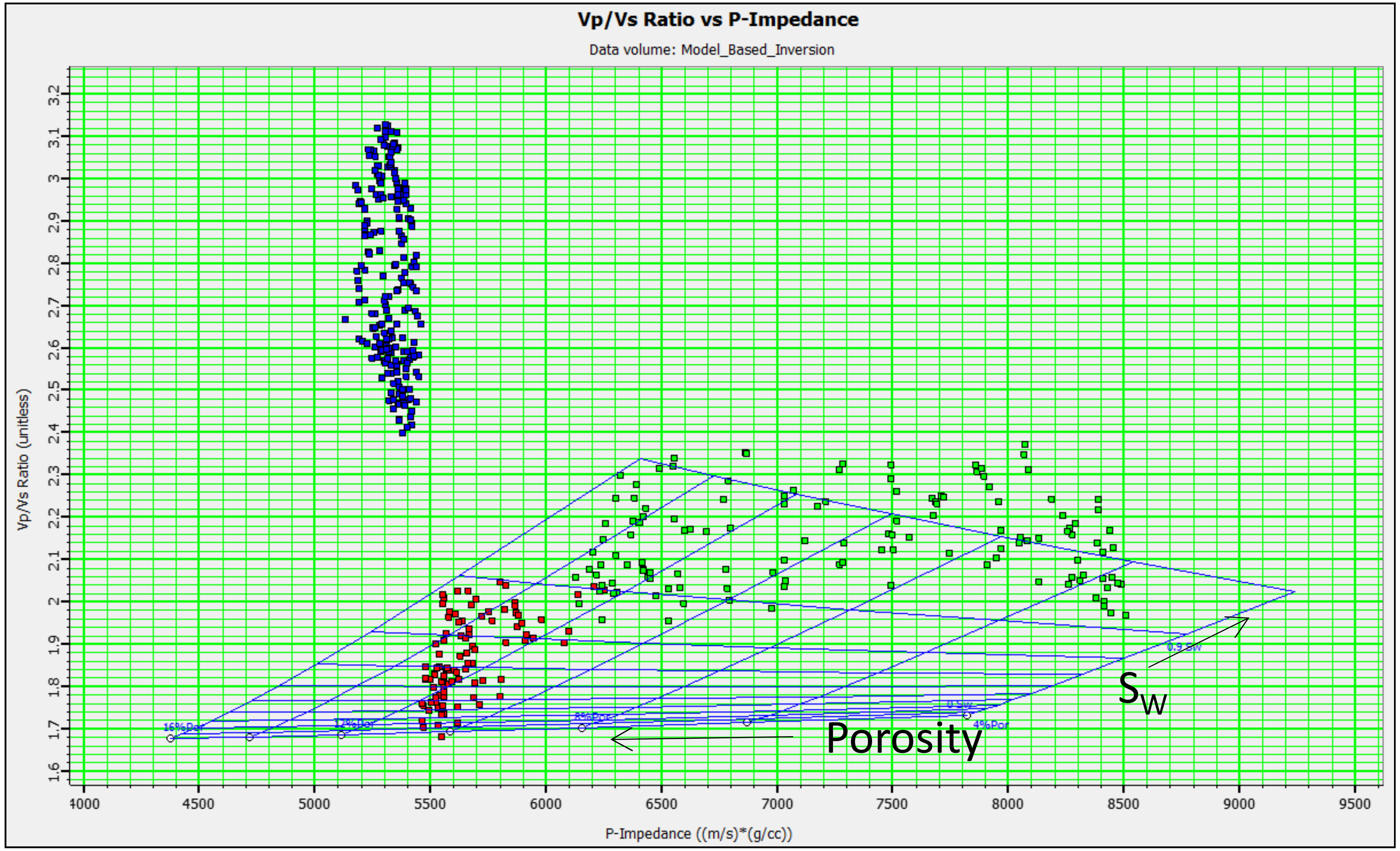
Interactive cross-plot

- Here are the three zones picked on the previous inverted section.
- The V_P/V_S ratio and acoustic impedance histograms of the three zones are also displayed.
- These zones show good correspondence to the zones seen on the well logs.



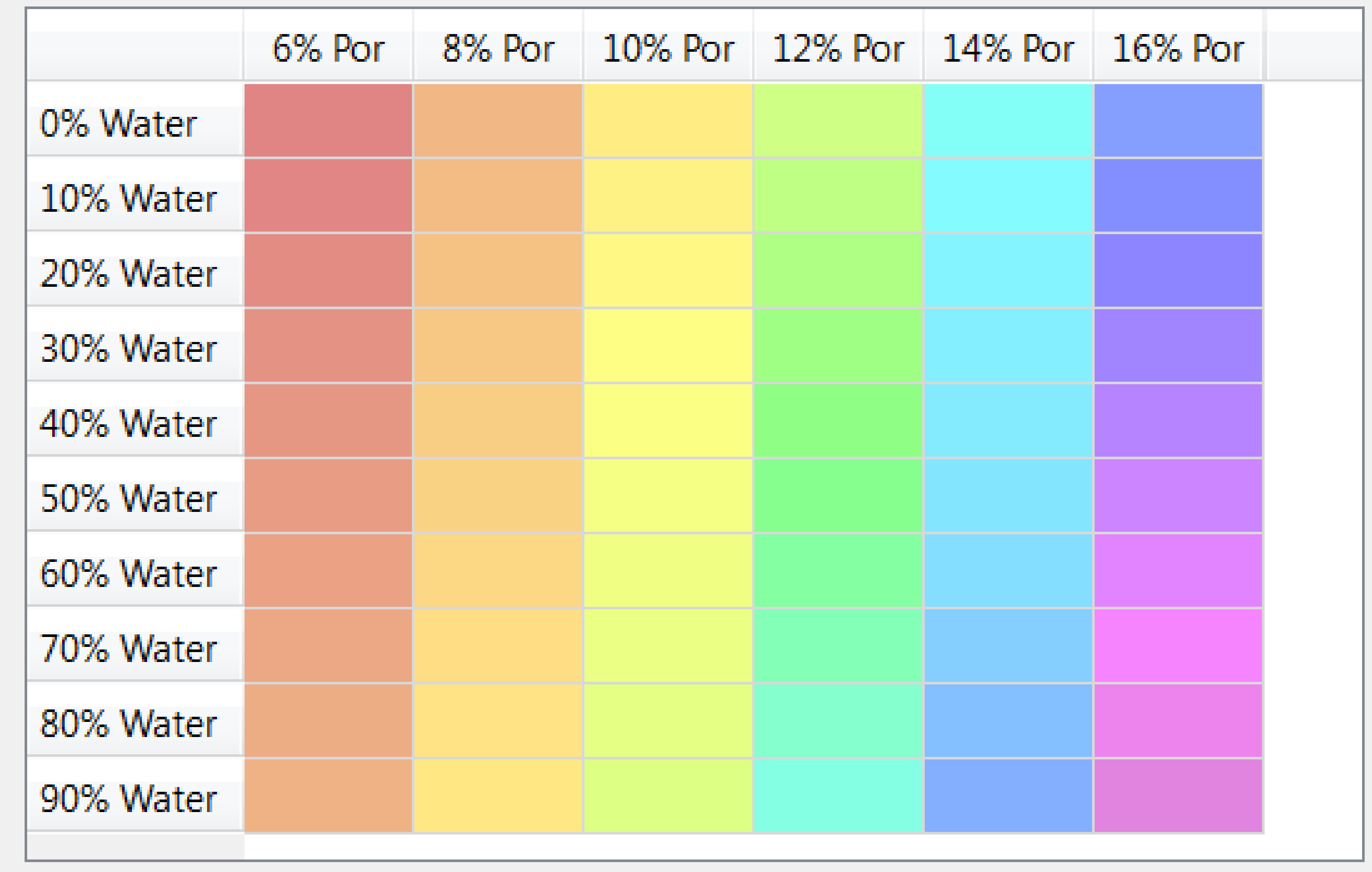
Superimposing a rock physics template

- This figure shows the superposition of a rock physics template of S_W vs Porosity on the seismic cross-plot, optimized by adjusting V_{shale} and pressure.
- Note that the red points from the gas sand show high porosity and low water saturation, as expected.



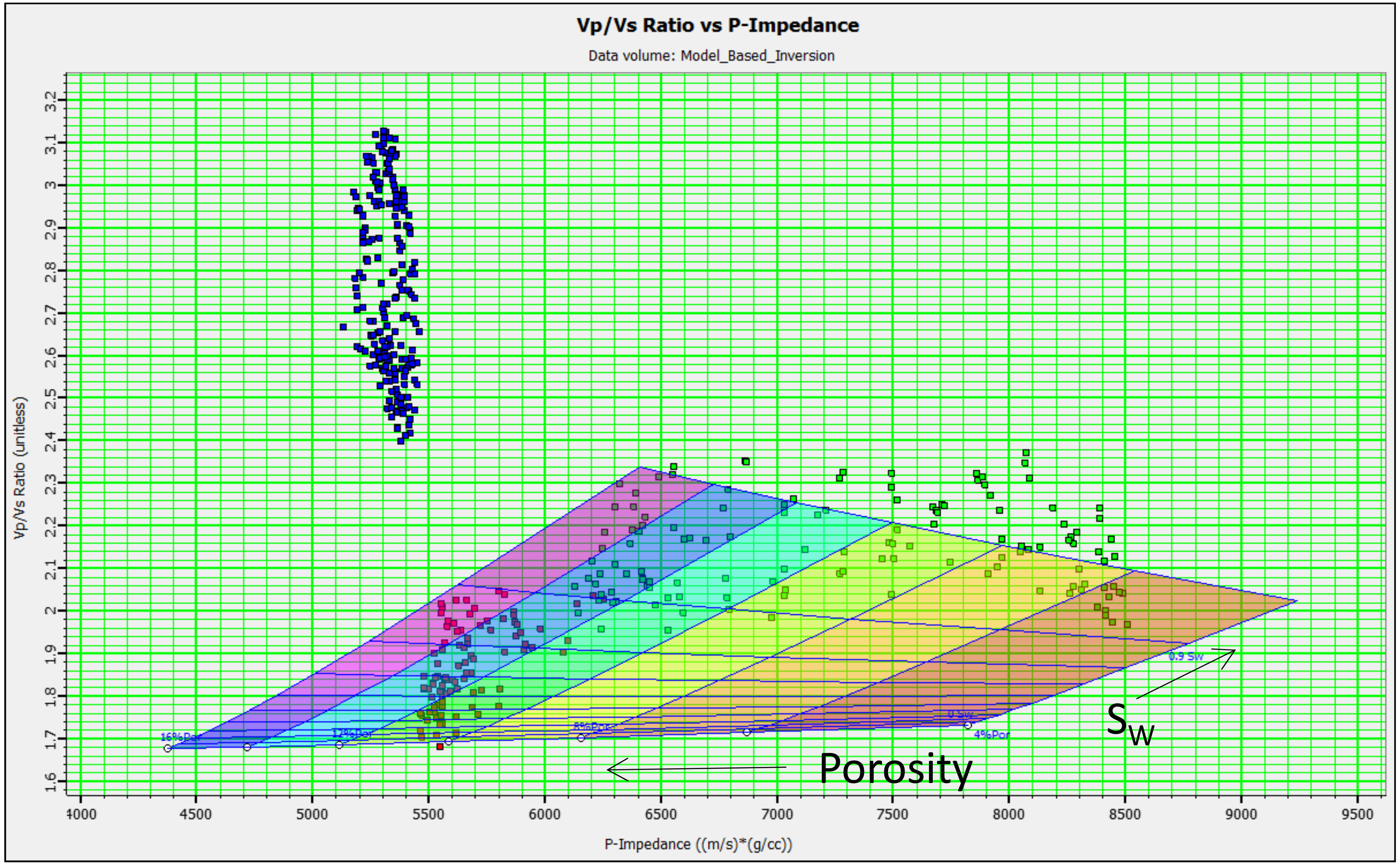
Colouring the rock physics template

- We can now fill in a colour template for the RPT.
- Note that each colour fills in a grid cell delineated by porosity and water saturation increments.



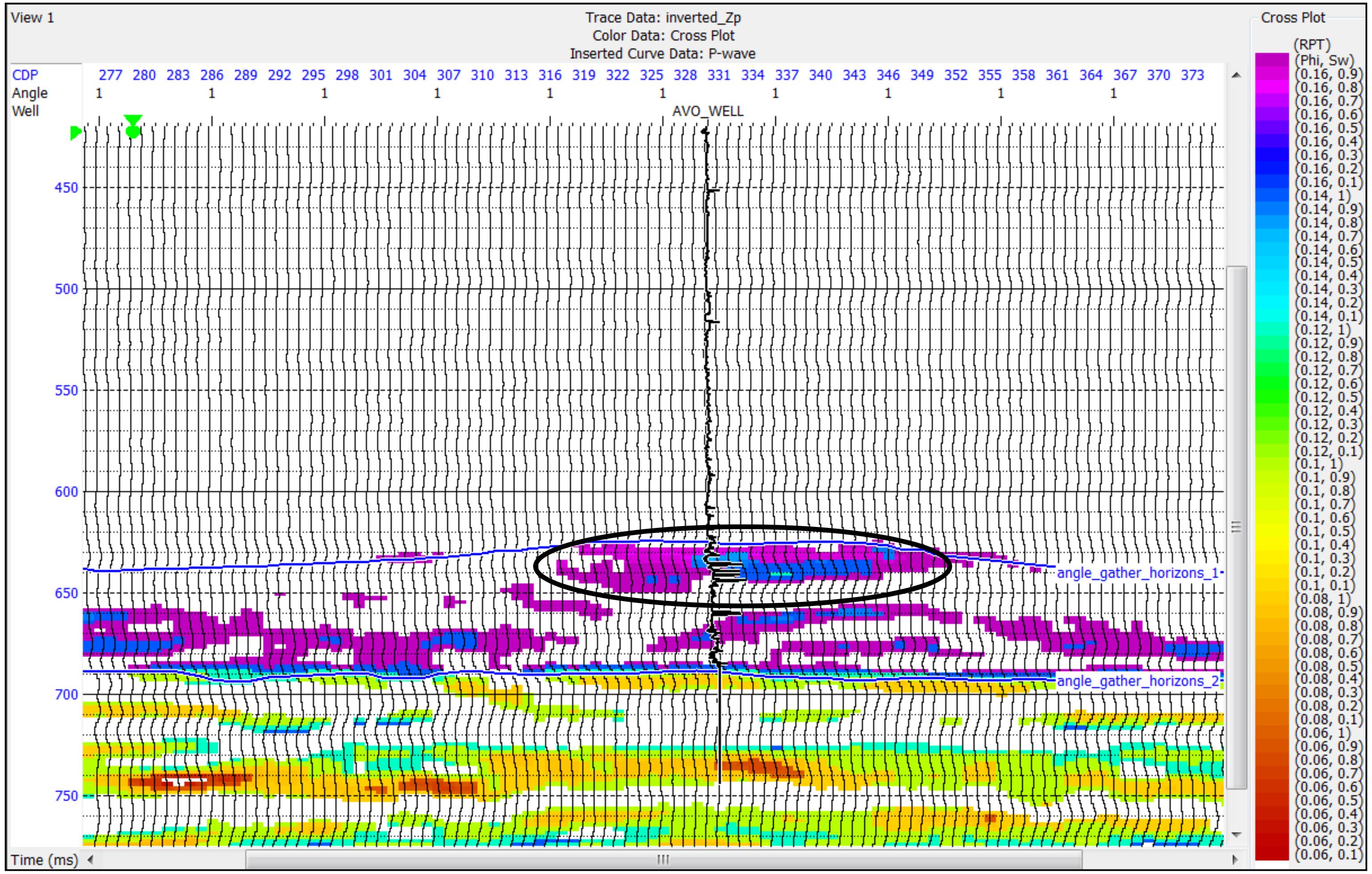
Superimposing the colours

- Here is the application of the colour palette with opacity turned on so we can still see the points.
- We can now superimpose these colours on the seismic data traces (wiggle trace only).



Seismic section coloured by the PEM

- Here is the superposition of the RPT colours on the seismic section.
- Although the gas sand shows up as the purple and blue colours, the other colours makes this display too “busy” to easily interpret.
- To improve this display, we can edit the colours.



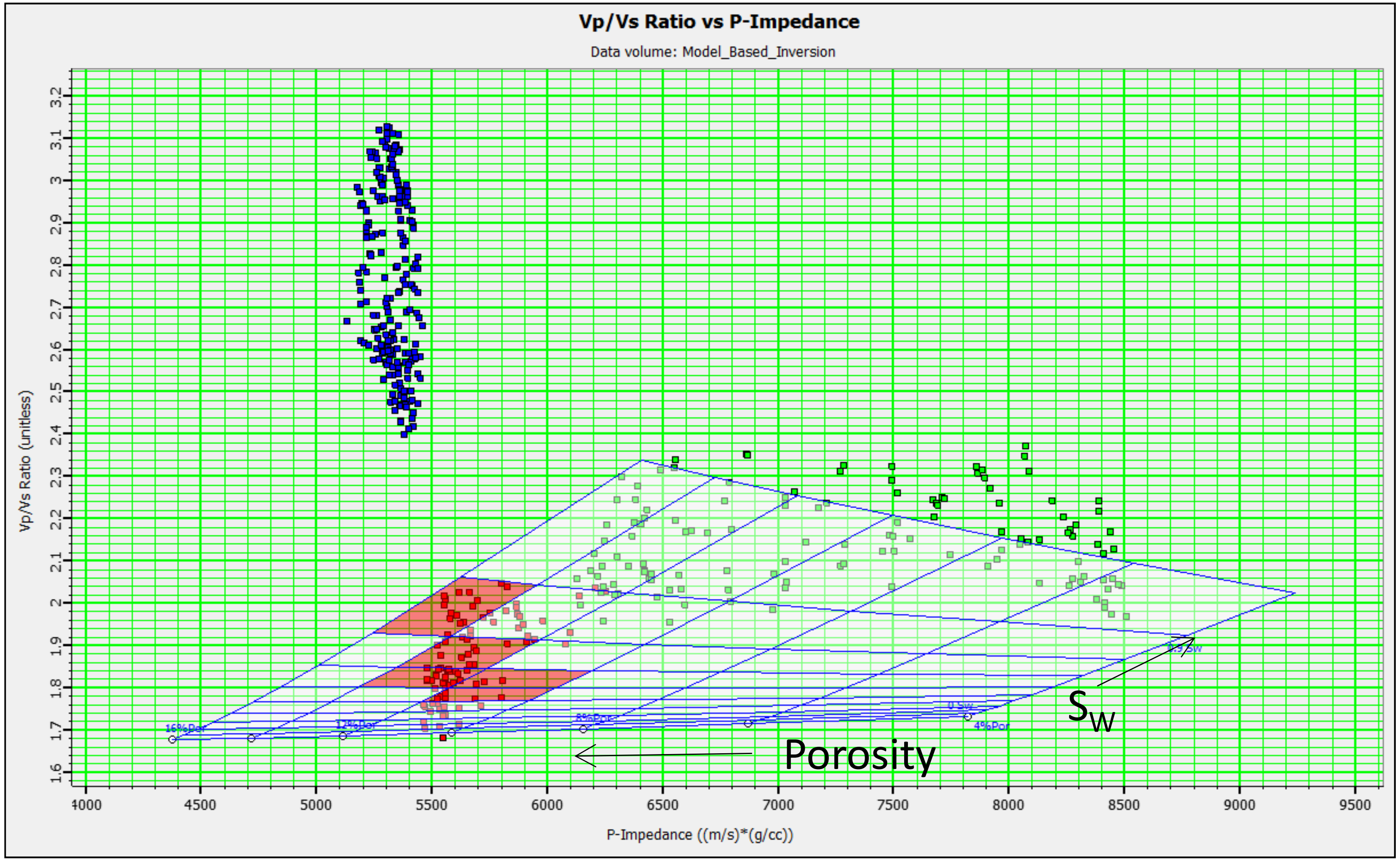
Re-colouring the rock physics template

- All the colours are initially set to white and then slowly filled in with red.
- Note that a region with moderate porosity and gas saturation has been highlighted.

	6% Por	8% Por	10% Por	12% Por	14% Por	16% Por
0% Water						
10% Water						
20% Water						
30% Water						
40% Water						
50% Water						
60% Water						
70% Water						
80% Water						
90% Water						

Superimposing the colours

- Here is the application of the new colour palette with opacity turned on so we can still see the points.
- We can now superimpose these new colours on the seismic data traces (wiggle trace only).



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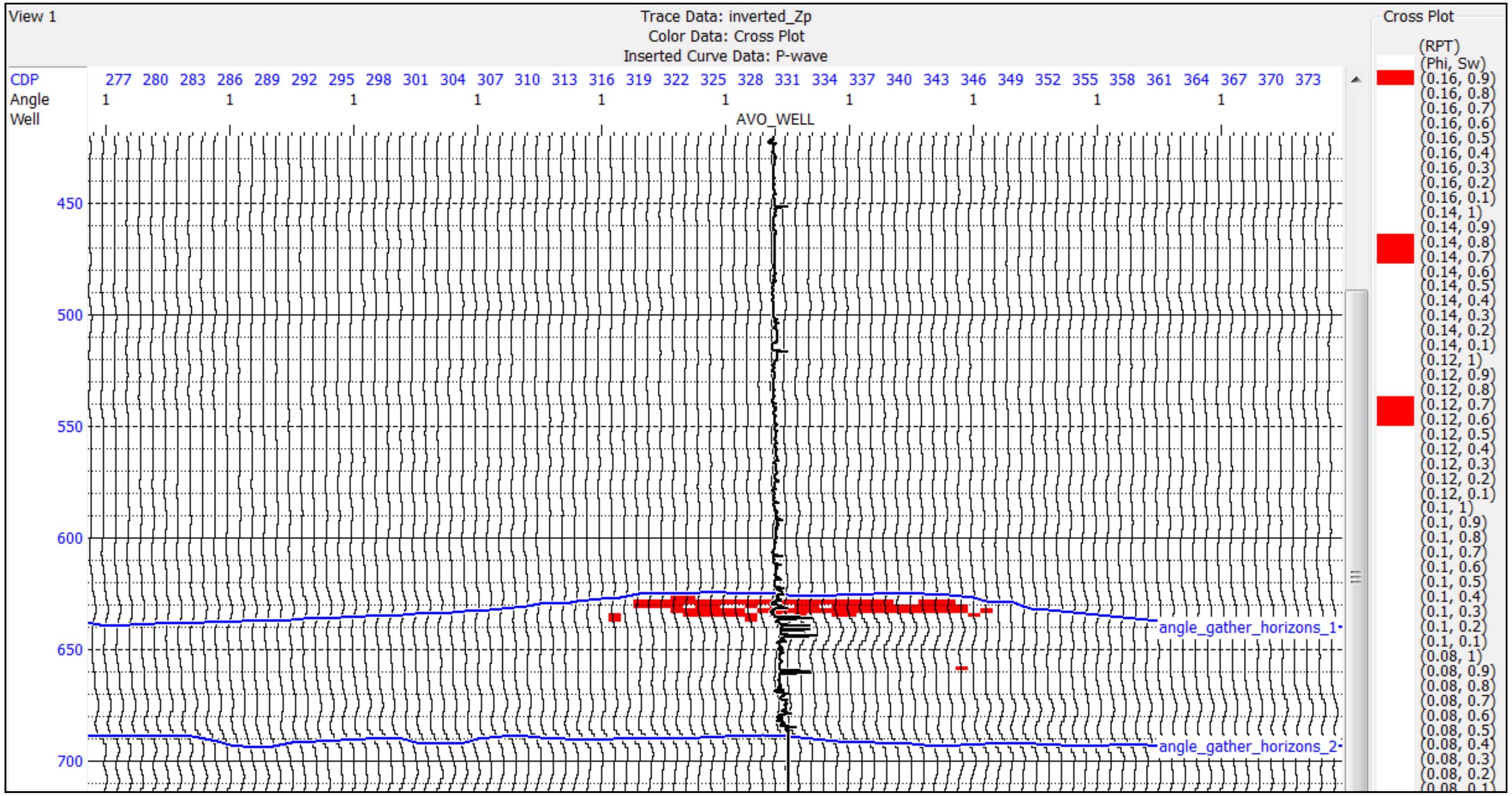
Seismic section coloured by the PEM

- Here is the new colour scheme superimposed on the seismic volume, clearly showing the gas sand.
- Although this is a 2D line, in a 3D volume the colour would be mapped throughout the entire volume.



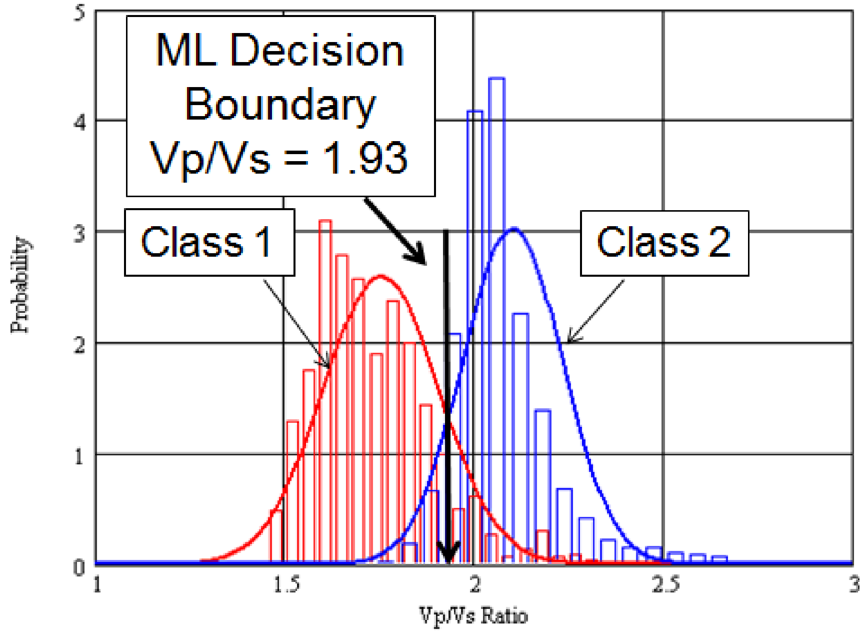
Bayesian Classification

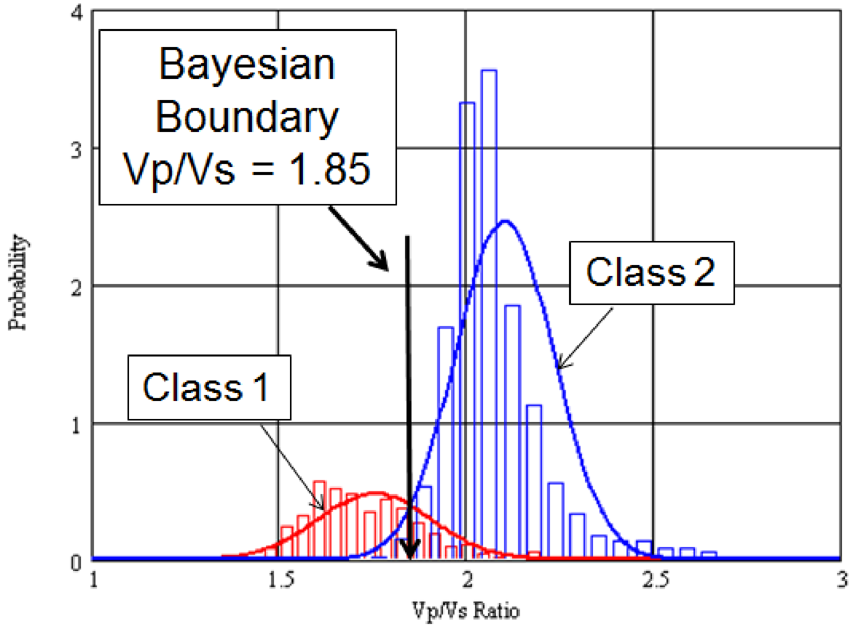
- Now that we have identified the clusters associated with gas, wet and cemented sands on the crossplot, we can assign a Bayesian probability classification scheme to the three clusters.
- For $\boldsymbol{K}$ clusters, the $\boldsymbol{k}^{th}$ cluster, or class, can be defined by the Gaussian pdf $\boldsymbol{f(x|c_k)}$.
- Note that $\boldsymbol{x}$ can be a single variable, in which case the pdf is a Gaussian curve, or a two-dimensional vector, in which case the pdf is an ellipse.
- We then compute the separation between the $\boldsymbol{i}^{th}$ and $\boldsymbol{j}^{th}$ clusters using the following Bayesian decision boundary:

$$f(x \mid c_i)p(c_i) = f(x \mid c_j)p(c_j), \text{ where } p(c_i) \text{ and } p(c_j) \text{ are the priors.}$$

Bayesian Classification

- The Bayesian priors are computed by adding the total number of points for all classes and dividing the number of points in each class by the total number of points.
- If the priors are set to equal values, the result is called maximum likelihood (ML) classification, rather than Bayesian classification.
- Here is an example from a 1D data set, where the figure on the left shows ML classification, and the one on the right shows Bayesian classification:





Two-Dimensional Classification

- Here are the statistics for the classification of the three 2D clusters seen on the previous inversion result and crossplot.

Parameters	Value
x mean	5658 m/s
y mean	1.87
x variance	29341
y variance	0.0091
covariance	9.316

Cluster 1 (Red)

Parameters	Value
x mean	5322 m/s
y mean	2.77
x variance	4825
y variance	0.043
covariance	-33.93

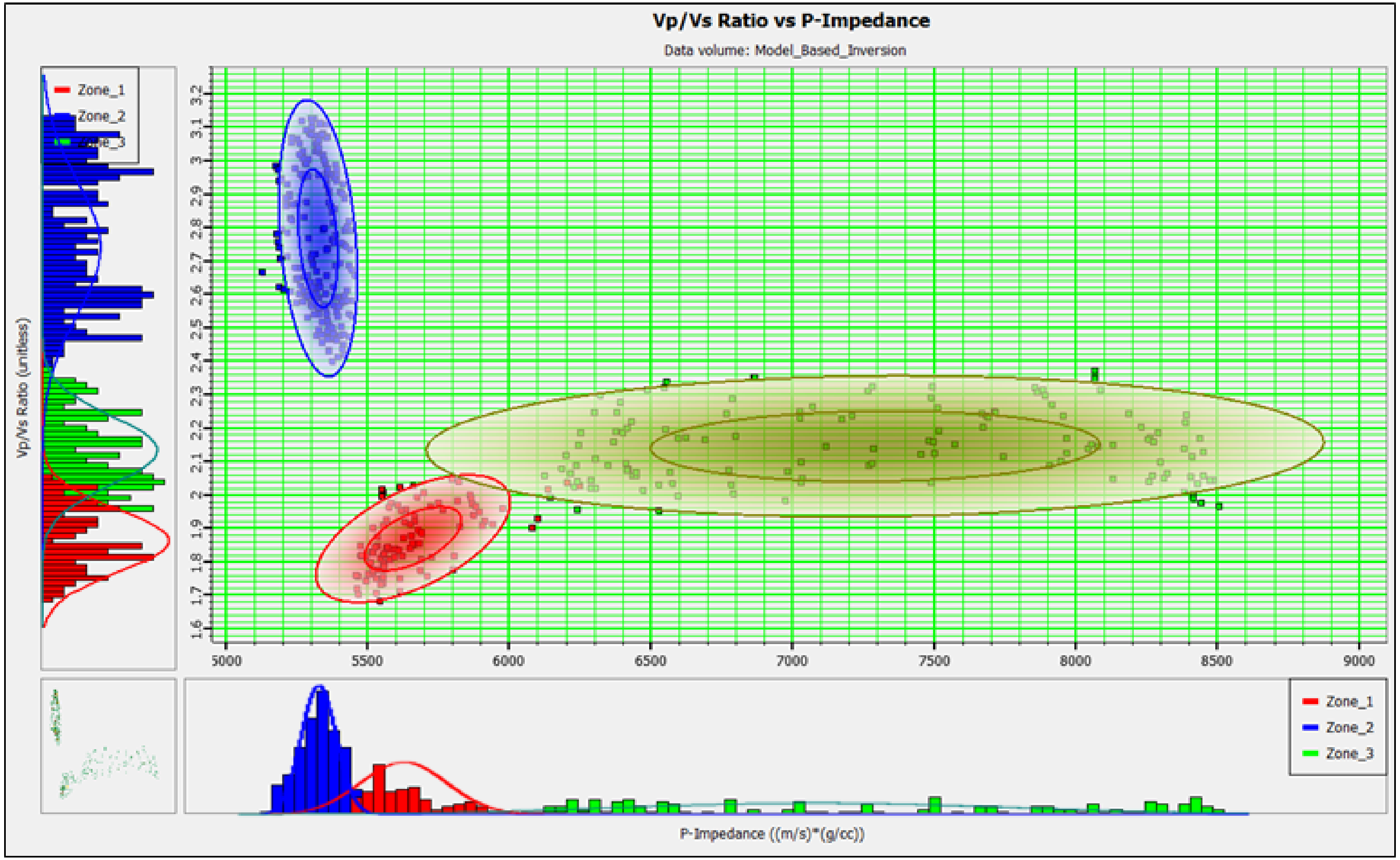
Cluster 2 (Blue)

Parameters	Value
x mean	7288 m/s
y mean	2.148
x variance	627481
y variance	0.011
covariance	5.402

Cluster 3 (Green)

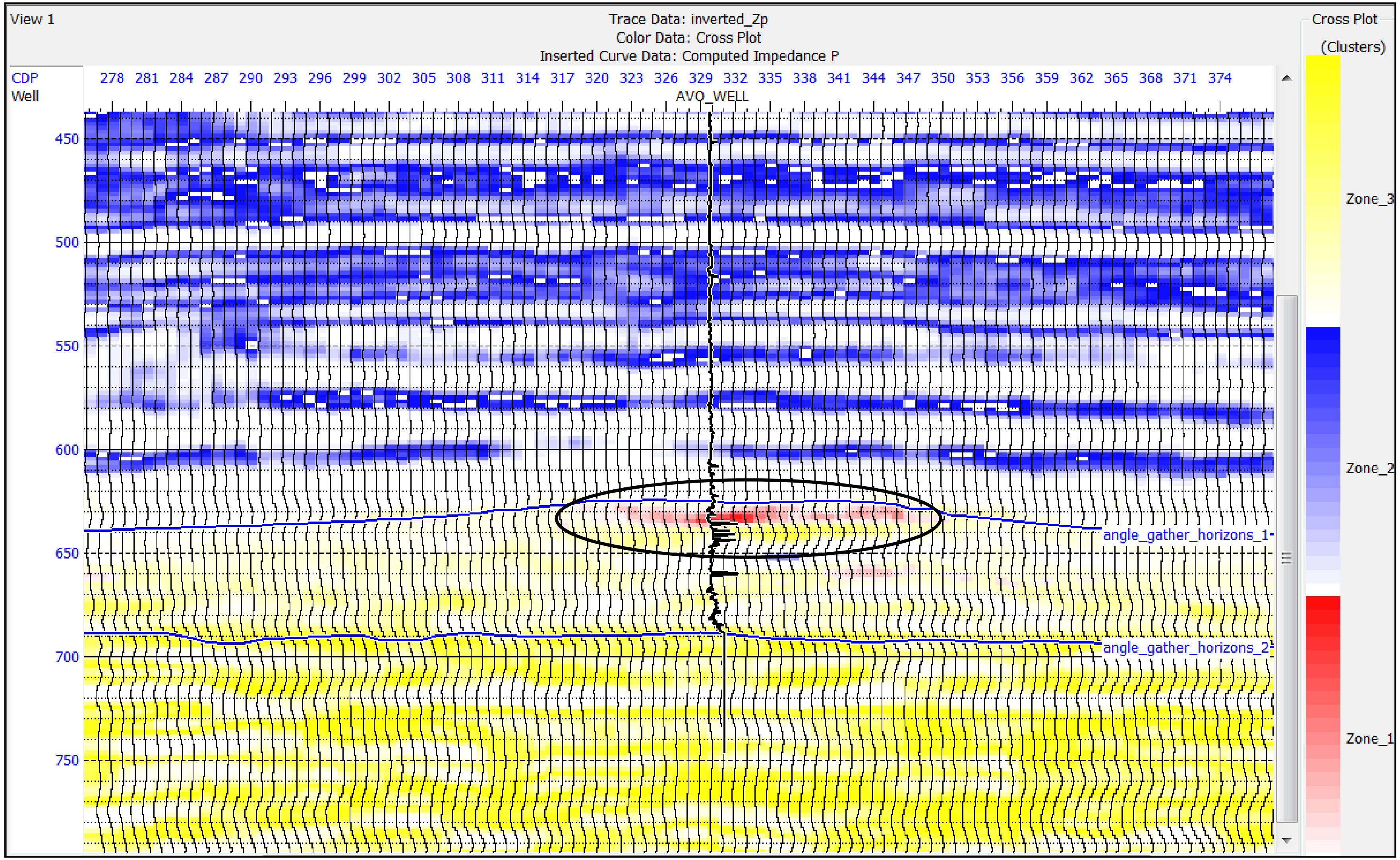
Bayesian Classification

- Here is the result of Bayesian classification of the three zones, with Gaussian PDFs.
- Since these zones were picked by the user, automatic clustering is not needed.
- Note that the univariate PDFs have been superimposed on the histograms.



Classification Results

- Classification results are then projected back onto the seismic data.
- The colour intensity indicates distance below the peak of the distribution.
- Now the gas sand and other lithologies are each assigned a probability.



Mixture Model Classification

- Next, we will extend our Bayesian analysis using the mixture model approach with Gaussian pdfs.
- In this approach, each cluster is modeled as the sum of J Gaussian pdf functions with weights w_j, given by:

$$p(x|c_k) = \sum_{j=1}^{J} w_j f(x|j), \text{ where :}$$

$$\sum_{j=1}^{J} w_j = 1.0 \text{ and } \iint_{x,y} f(x\,|\,j)\,dxdy = 1.0.$$

- That is, the sum of the weights and the area of the final pdf function both equal 1.0.

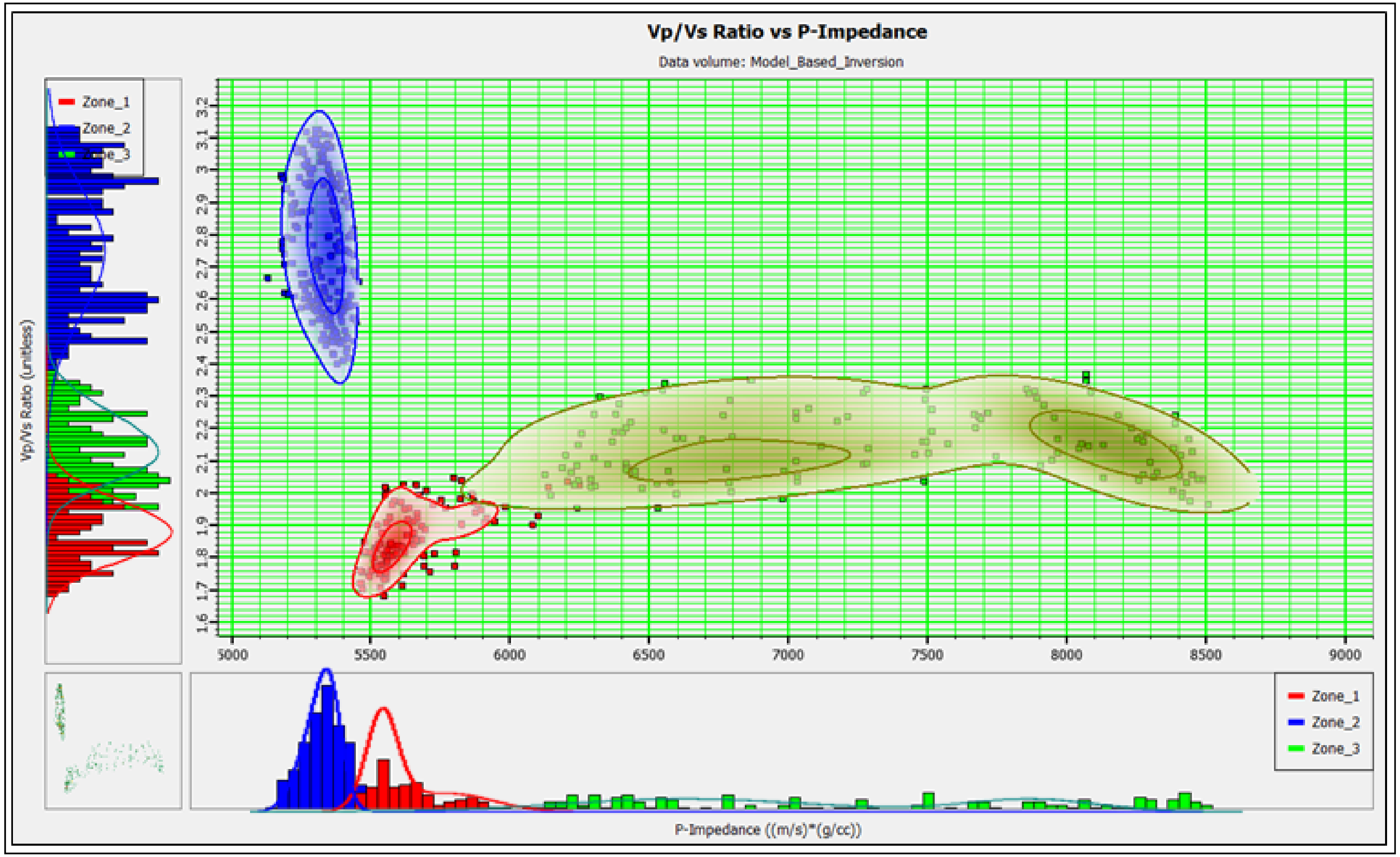
Mixture model classification

- Here are the statistics and weights for the first cluster (the other two clusters have a similar look):

	Mixture 1	Mixture 2	Mixture 3
Mixture weight	0.3298	0.3319	0.3382
x mean	5572	5565	5832
y mean	1.827	1.869	1.918
x variance	5815	3197	31002
y variance	0.0056	0.0098	0.0073
covariance	4.347	2.350	8.551

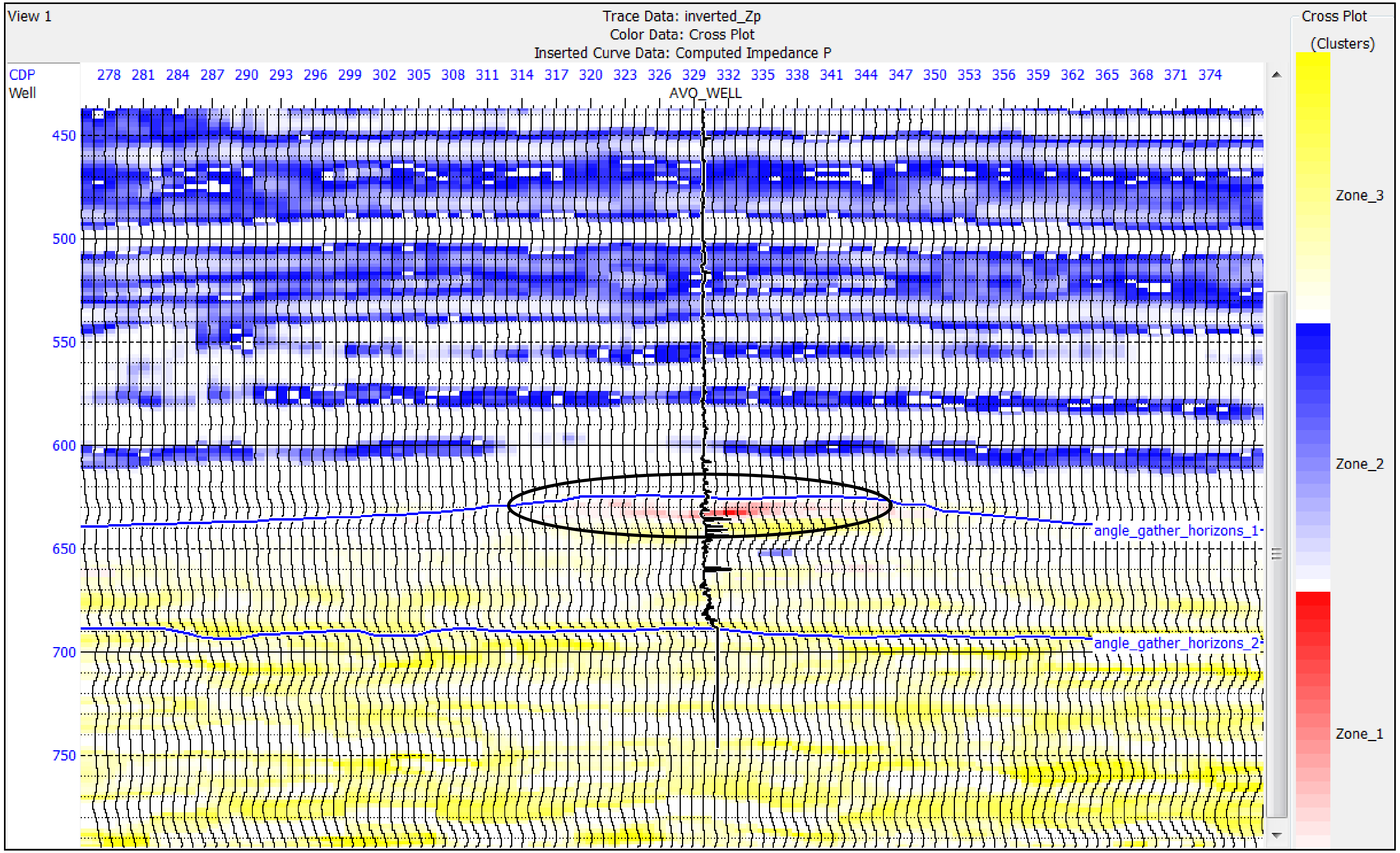
Mixture model classification

- Here is the result of mixture model classification of the three zones.
- Again, the univariate PDFs have been superimposed on the histograms.
- Note that the fit to the points is much tighter than in the single Gaussian approach.



Mixture model classification results

- The mixture model classification results are projected back onto the seismic data, as shown here.
- Again, the colour intensity indicates distance below the distribution peak.
- The gas sand extent has been decreased from the single Gaussian results.



Conclusions

- In this talk, I discussed two separate approaches to linking rock physics models to inverted seismic data: a deterministic and a statistical approach.
- In the deterministic approach, we built petro-elastic models and displayed the resulting rock physics templates (RPTs) on V_P/V_S versus P-impedance cross-plots.
- By connecting the RPT grid lines and assigning colours to the resulting grid cells, we then visualized the results on the seismic display.
- Our first statistical approach performed automatic clustering on the cross-plot and correlation with the deterministic RPT results.
- Our second statistical approach used Bayesian classification with single Gaussian pdfs.
- Finally, this was extended to a mixture model approach, in which multiple Gaussian pdfs were used to model each cluster.

Acknowledgements

- I wish to thank the CREWES sponsors and my colleagues at Hampson-Russell, CGG, and CREWES.
- In particular, I want to thank Dr. Qing Li and Kim Andersen for their efforts in implementing the ideas shown in this talk in the Hampson-Russell software platform.
- Also, I want to thank Dan Hampson and Jon Downton for their suggestions that improved this talk.

The Ødegaard/Avseth equations for the dry moduli

- Ødegaard and Avseth (2003) compute K_{dry} and μ_{dry} as a function of porosity and pressure using Hertz-Mindlin theory and the lower Hashin-Shtrikman bound:

$$K_{dry} = \left[\frac{\phi/\phi_c}{K_{HM} + (4/3)\mu_{HM}} + \frac{1-\phi/\phi_c}{K_m + (4/3)\mu_{HM}} \right]^{-1} - \frac{4}{3}\mu_{HM}$$

$$\mu_{dry} = \left[\frac{\phi/\phi_c}{\mu_{HM} + z} + \frac{1-\phi/\phi_c}{\mu_m + z} \right]^{-1} - z, \text{ where } z = \frac{\mu_{HM}}{6}\left(\frac{9K_{HM} + 8\mu_{HM}}{K_{HM} + 2\mu_{HM}}\right),$$

$$K_{HM} = \left[\frac{n^2(1-\phi_c)^2 \mu_m^2}{18\pi^2(1-\nu_m)^2} P \right]^{\frac{1}{3}}, \mu_{HM} = \frac{4-4\nu_m}{5(2-\nu_m)}\left[\frac{3n^2(1-\phi_c)^2 \mu_m^2}{2\pi^2(1-\nu_m)^2} P \right]^{\frac{1}{3}},$$

$P =$ confining pressure, $K_m, \mu_m =$ mineral bulk and shear modulus, $n =$ contacts per grain, $\nu_m =$ mineral Poisson's ratio, $\phi =$ porosity, and $\phi_c =$ critical porosity.

Fluid substitution with the Gassmann equation

- The Gassmann (1951) equation is then used for fluid substitution for the saturated bulk modulus:

$$\frac{K_{sat}}{K_m - K_{sat}} = \frac{K_{dry}}{K_m - K_{dry}} + \frac{K_f}{\phi(K_m - K_f)}, \text{ where}: K_{sat} = \text{saturated bulk modulus,}$$

$$\frac{1}{K_f} = \frac{S_w}{K_w} + \frac{1 - S_w}{K_{hc}}, K_f = \text{fluid bulk modulus,} K_w = \text{water bulk modulus,}$$

$$K_{hc} = \text{hydrocarbon bulk modulus, and } S_w = \text{water saturation.}$$

- Note that Gassmann shows that there is no change in the shear modulus, meaning that:

$$\mu_{sat} = \mu_{dry}$$

Bayesian Classification

- For a single variable with K clusters, the k^{th} cluster, or class, can be defined by the following Gaussian pdf:

$$f(x \mid c_k) = \frac{1}{\sigma_k \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{x - \mu_k}{\sigma_k} \right)^2 \right], \text{ where}$$

$$\mu_k = \frac{1}{N_k} \sum_{i=1}^{N_k} x_{ki}, \text{ and } \sigma_k^2 = \frac{1}{N_k} \sum_{i=1}^{N_k} (x_{ki} - \mu_k)^2.$$

- We then compute the separation between the i^{th} and j^{th} clusters using the following Bayesian decision boundary:

$$f(x \mid c_i) p(c_i) = f(x \mid c_j) p(c_j), \text{ where } p(c_i) \text{ and } p(c_j) \text{ are the priors.}$$

Two-Dimensional Classification

- For an two-dimensional variable with ***K*** clusters, the ***k^th^*** cluster can be defined by the following two-dimensional Gaussian pdf:

$$f(z|c_k) = \frac{1}{2\pi|\Sigma_k|^{1/2}} \exp\left[-\frac{1}{2}(z-\mu_k)^T \Sigma_k^{-1}(z-\mu_k)\right]$$

$$\text{where}: z = \begin{bmatrix} x \\ y \end{bmatrix}, \mu_k = \begin{bmatrix} \mu_{kx} \\ \mu_{ky} \end{bmatrix}, \Sigma_k = \begin{bmatrix} \sigma_{kxx} & \sigma_{kxy} \\ \sigma_{kxy} & \sigma_{kyy} \end{bmatrix}, \sigma_{kxx} = \sigma_x^2,$$

$$\sigma_{kyy} = \sigma_y^2 \text{ and } \sigma_{kxy} = \frac{1}{N_k - 1}\sum_{i=1}^{N_k}\left[(x_i - \mu_{kx})(y_i - \mu_{ky})\right]$$

Mixture Model Classification

- We can extend our Bayesian analysis using the mixture model approach with Gaussian pdfs.
- In this approach, each cluster is modeled as the sum of $\boldsymbol{J}$ Gaussian pdf functions with weights $\boldsymbol{w_j}$, given by:

$$p(z|c_k) = \sum_{j=1}^{J} w_j f(z|j), \text{ where :}$$

$$\sum_{j=1}^{J} w_j = 1.0 \text{ and } \iint_{x,y} f(z \mid j) dxdy = 1.0.$$

- Note that the sum of the weights and the area of the final pdf function both equal 1.0.