A geometrical model of DAS

angle dependence and vector/tensor estimation

Kris Innanen

CREWES Annual Meeting
Banff AB Canada
Dec 1 2016

Acknowledgments

Don Lawton, CaMI

CREWES industrial sponsors

NSERC-CRD (CRDPJ 461179-13)

Outline

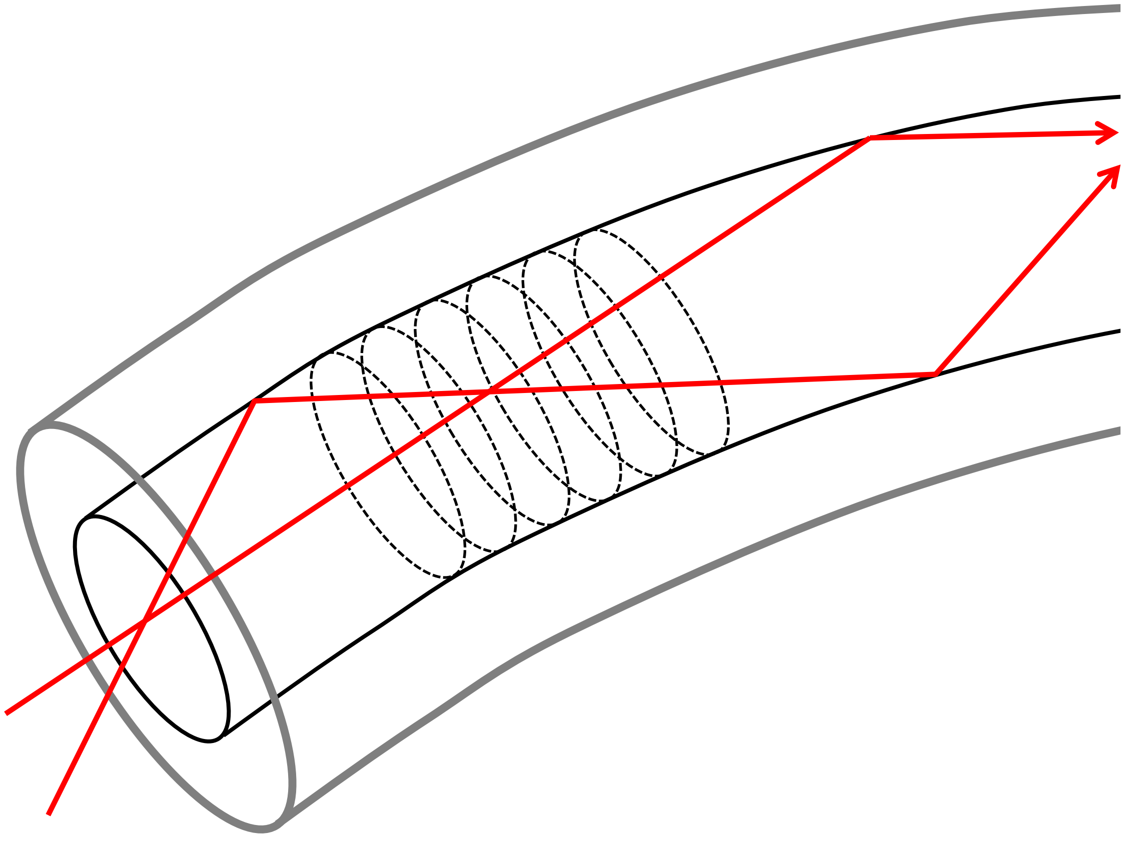
1. Background: aspects of DAS technology
    a) a key issue with the state of the art
    b) what is in and what is out of the model
2. Geometrical model of helical fibre / deviated cable
    a) axis of a deviated cable
    b) adding in the helix
3. Applications
    a) Embedding fibre in an elastic wavefield snapshot
    b) Estimating vector displacement
    c) Estimating tensor strain
    d) General characterization of directionality

Background

Brooks AB FRS

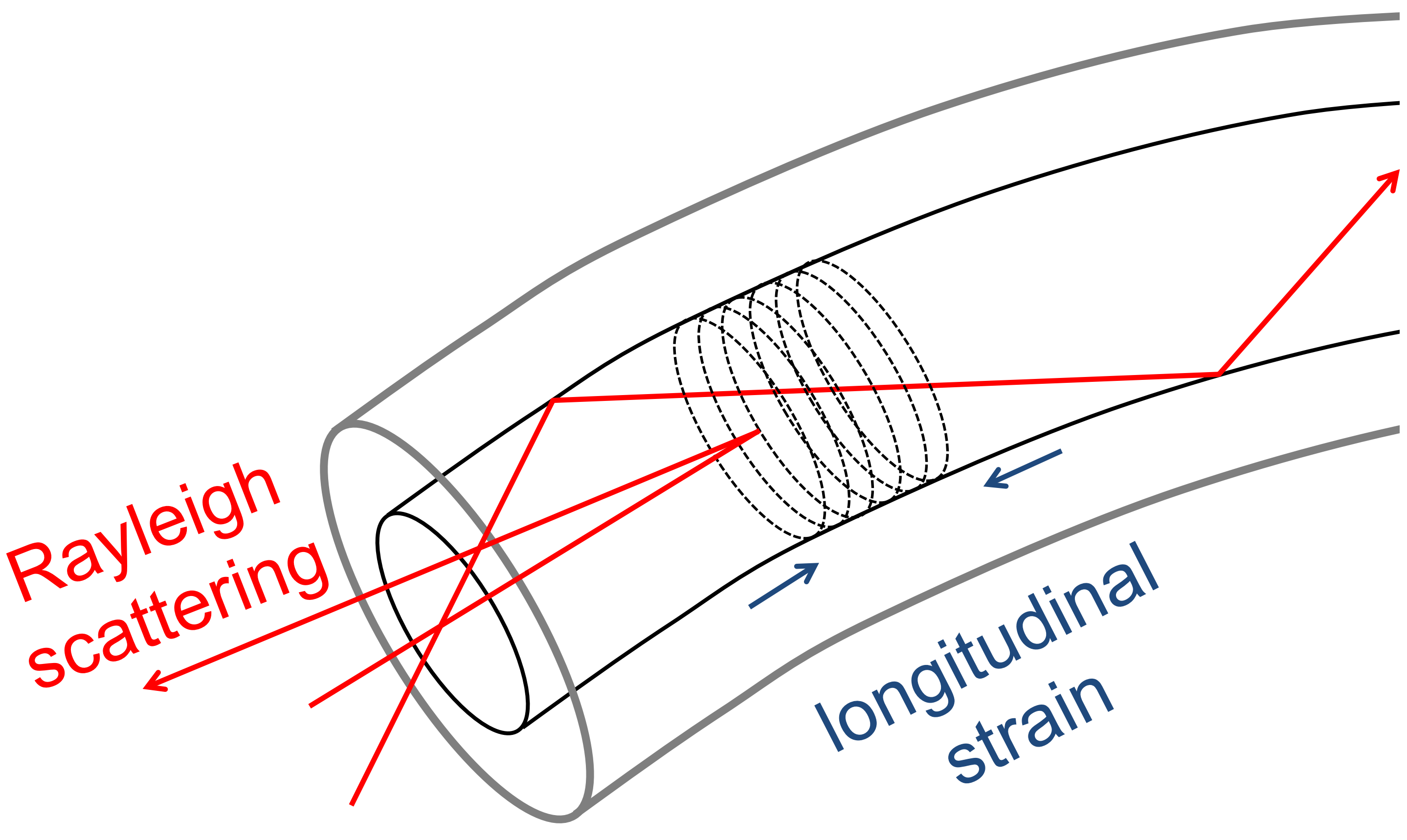
Background

Fibre-optic acoustic sensing



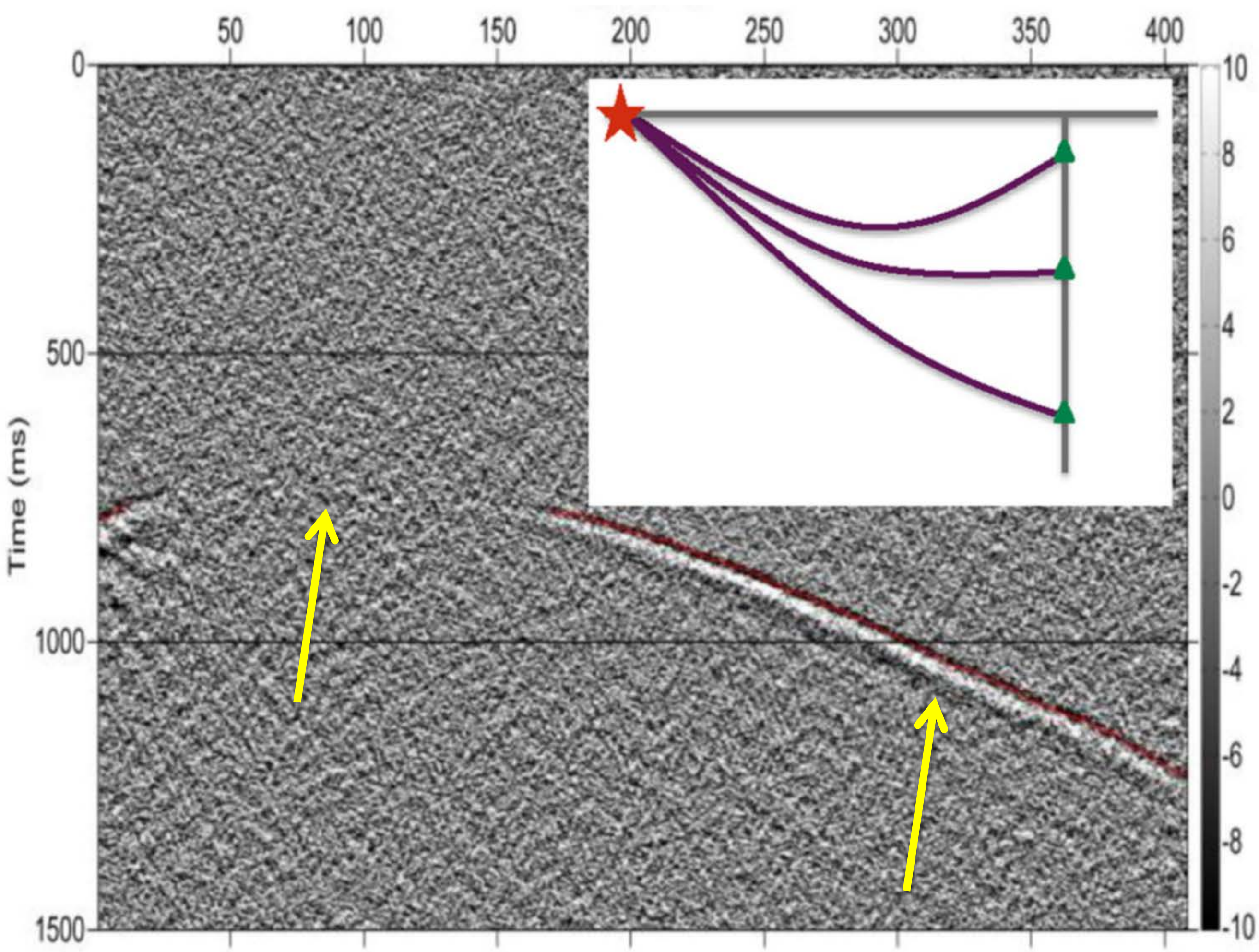
Background

Fibre-optic acoustic sensing



Background

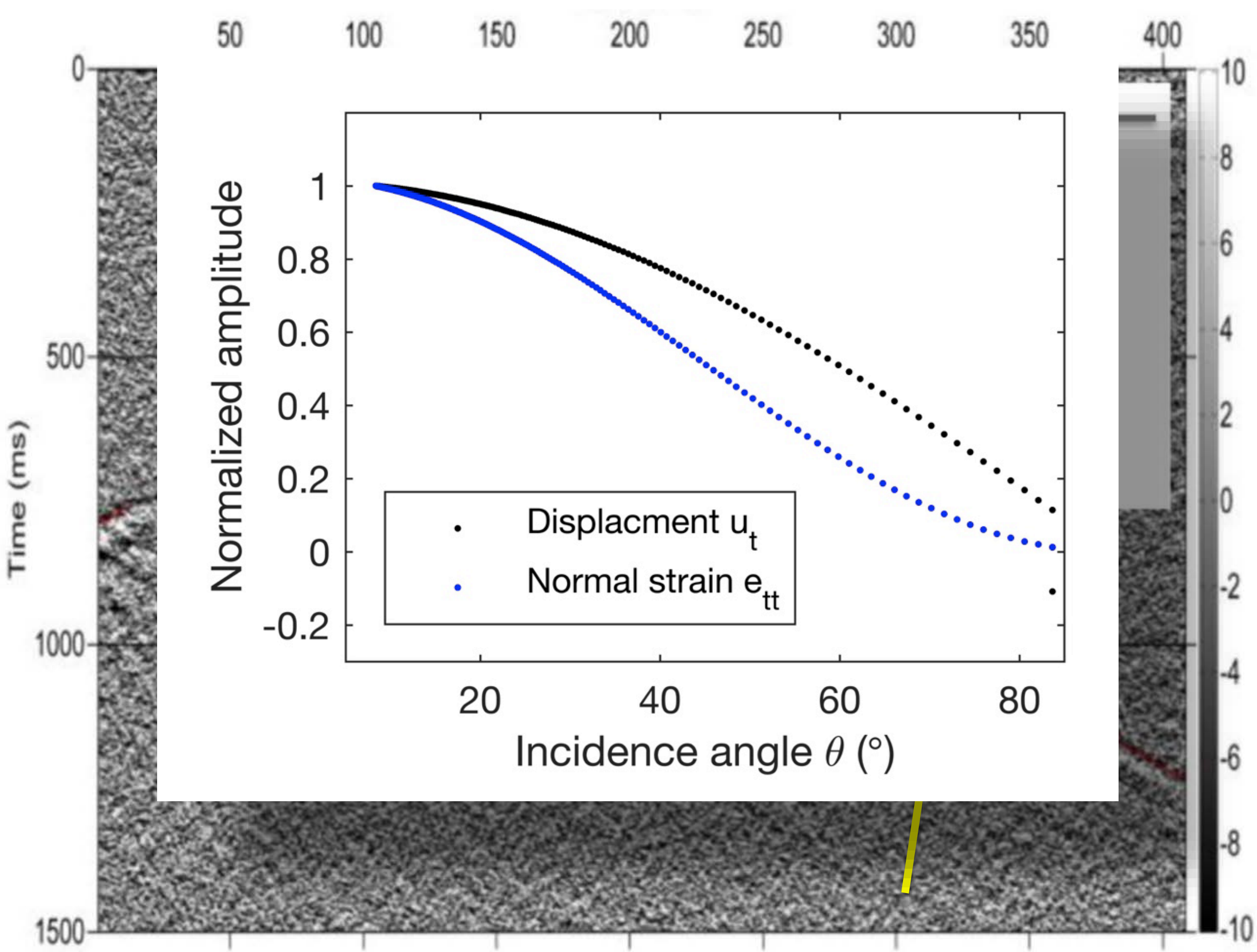
A key issue with the state of the art



Mateeva et al., 2014

Background

A key issue with the state of the art



Mateeva et al., 2014

Background

Possible solutions

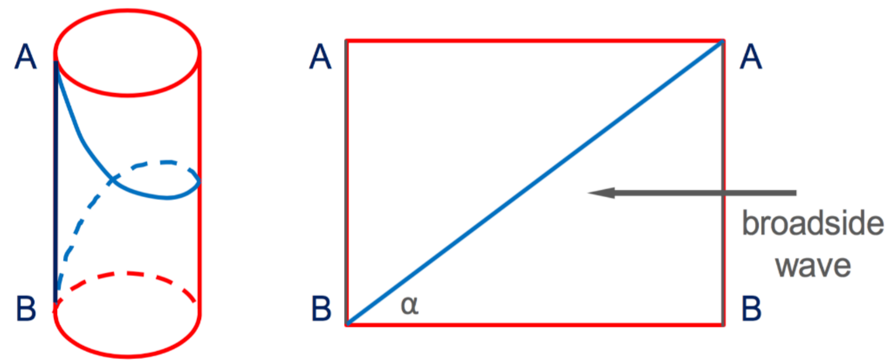


Figure 11 Helically wrapped fibre for a broadside sensitive cable (HWC) – Left: fibre (blue) in the cable (red). Right: cable cut along line AB and flattened to obtain a surface in which the wrapping angle α is defined.

Helically-wound fibre

"Twisted Strip" cable

Figure 12 The Twisted Strip Cable – another broad-side sensitive cable based on the "shaped fibre" concept. A sinusoid is made of fibre in one plane (the strip) and then twisted to achieve sensitivity in all directions.

Mateeva et al., 2014

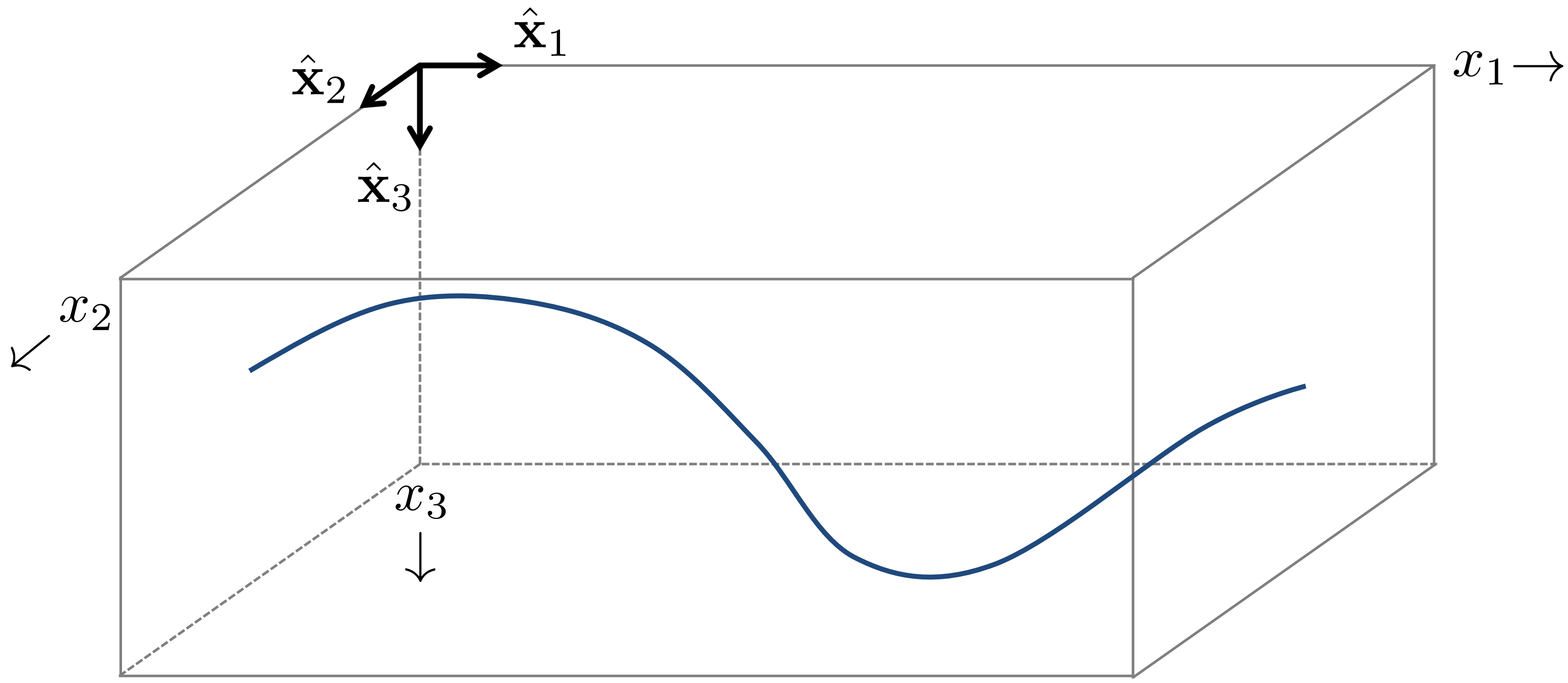
Background

Today: goals

1. General model of fibre embedded in a 3D volume
   a) helix + cable axis with arbitrary curvature
   b) enumerate all tangents sensed by this fibre
   c) formulate estimation of vector / tensor fields
   d) generalize $\cos^2\theta$ directionality rules for incident P-wave
2. What is left out of the model
   a) detailed calibration from strain to displacement
   b) ground-to-casing-to-fibre coupling
   c) SNR issues etc. in analyzing interrogator signal (except incorporation of gauge-length)

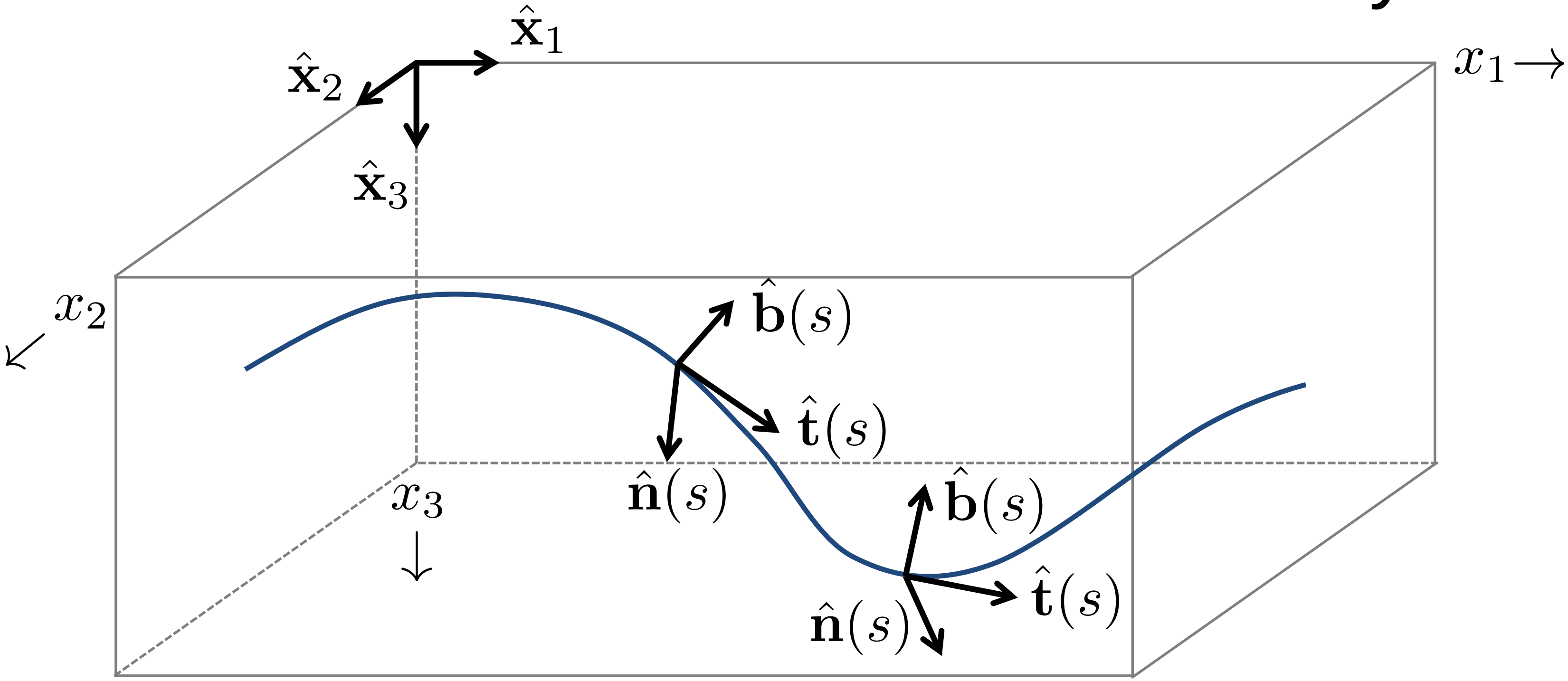
Geometrical model

of a helical fibre wound round an arbitrary cable



Geometrical model

of a helical fibre wound round an arbitrary cable

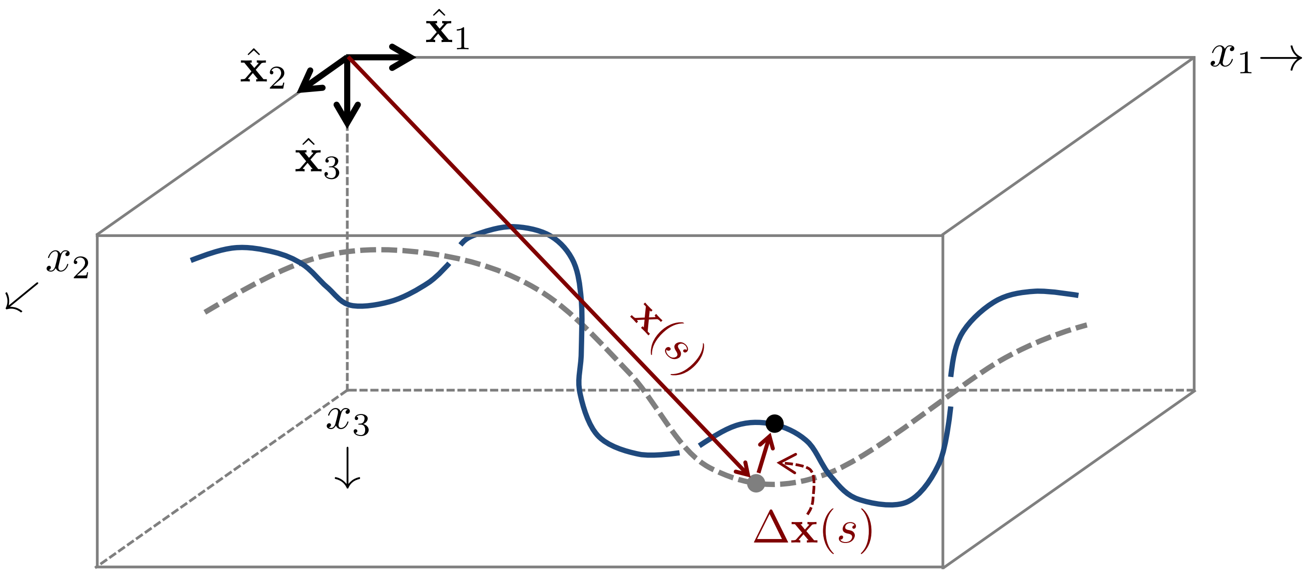


By similar arguments the derivative of the tangent must lie in the plane perpendicular, therefore

$$\hat{\mathbf{n}}(s) = \frac{\mathbf{n}(s)}{|\mathbf{n}(s)|}, \quad \mathbf{n}(s) = \frac{d\hat{\mathbf{t}}(s)}{ds} \qquad \hat{\mathbf{b}}(s) = \hat{\mathbf{t}}(s) \times \hat{\mathbf{n}}(s)$$

Geometrical model

of a helical fibre wound round an arbitrary cable



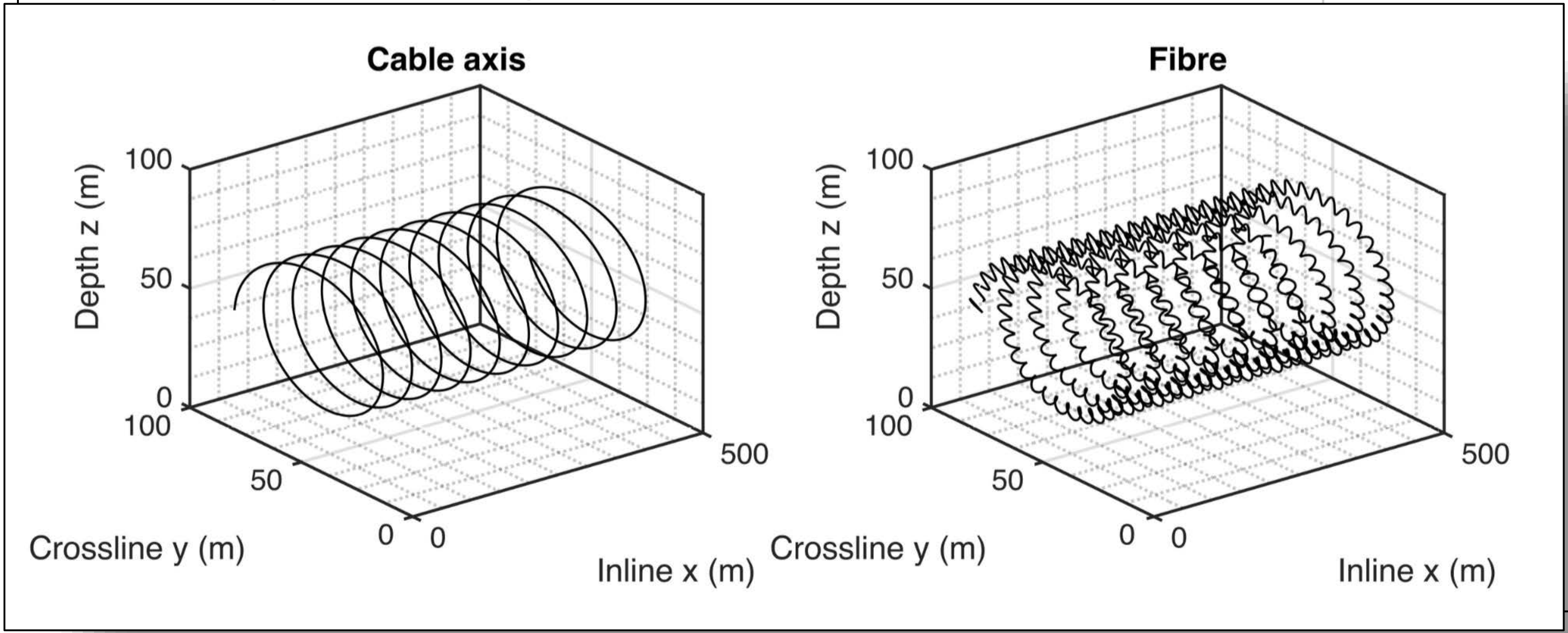
$$\begin{bmatrix} \tilde{x}_1(s) \\ \tilde{x}_2(s) \\ \tilde{x}_3(s) \end{bmatrix} = \begin{bmatrix} x_1(s) \\ x_2(s) \\ x_3(s) \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{x}}_1 \cdot \hat{\mathbf{t}}(s) & \hat{\mathbf{x}}_1 \cdot \hat{\mathbf{n}}(s) & \hat{\mathbf{x}}_1 \cdot \hat{\mathbf{b}}(s) \\ \hat{\mathbf{x}}_2 \cdot \hat{\mathbf{t}}(s) & \hat{\mathbf{x}}_2 \cdot \hat{\mathbf{n}}(s) & \hat{\mathbf{x}}_2 \cdot \hat{\mathbf{b}}(s) \\ \hat{\mathbf{x}}_3 \cdot \hat{\mathbf{t}}(s) & \hat{\mathbf{x}}_3 \cdot \hat{\mathbf{n}}(s) & \hat{\mathbf{x}}_3 \cdot \hat{\mathbf{b}}(s) \end{bmatrix} \begin{bmatrix} 0 \\ r\cos s/c(\gamma) \\ r\sin s/c(\gamma) \end{bmatrix}$$

Numerical modeling

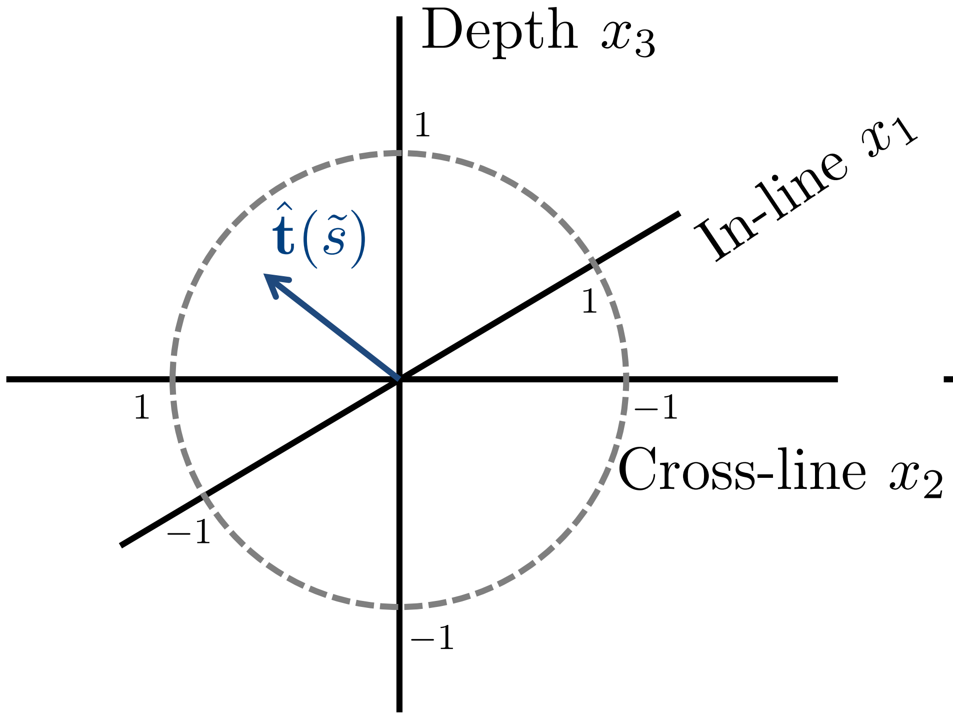
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Cable and fibre parameters & equations

fac = 4; Np = 512*fac;
dx1 = 2.0/fac; tol = 8*10e-5;

% Parameters
bigV = 200; bigR = 30;
littleV = 2; littleR = 4;
x2c = 50; x3c = 50;

% Cable axis curve equations
x10 = 1:Np; x10 = (x10-1)*dx1; x20 = x2c + bigR*cos(x10/bigV); x30 = x3c*ones(size(x10));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate geometry, fibre vectors, fibre tangent

[t0,n0,b0,ss] = curvegeometry(x10,x20,x30,'y',tol);
[x1,x2,x3] = addhelix(t0,n0,b0,x10,x20,x30,littleR,ss,littleV);
[t1,n1,b1,ss1] = curvegeometry(x1,x2,x3,'n',tol);
```

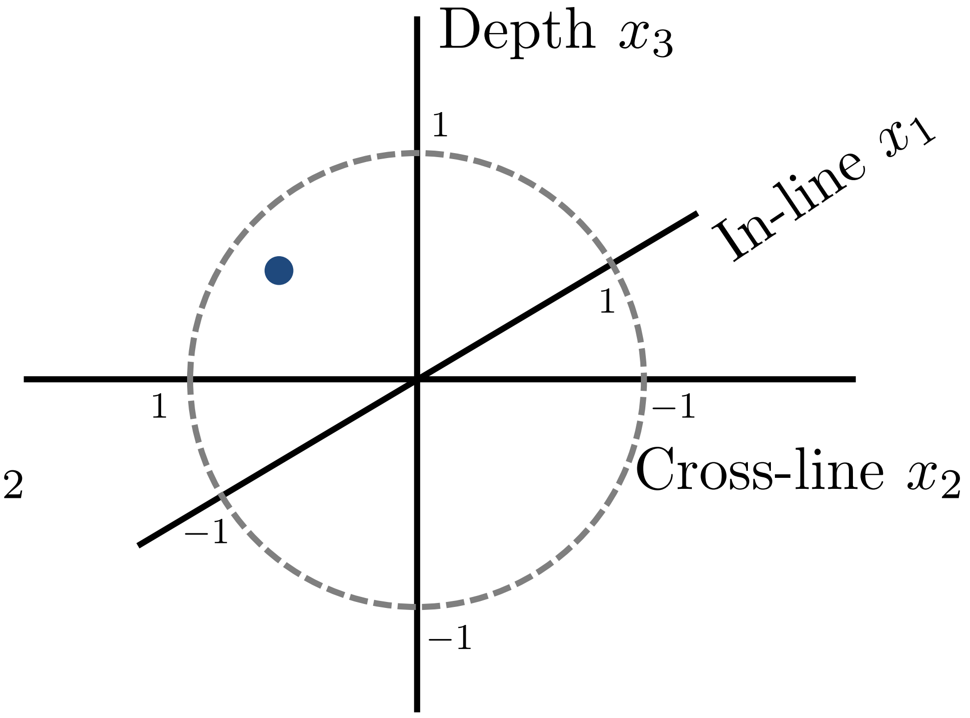
Numerical modeling

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Cable and fibre parameters & equations

fac = 4; Np = 512*fac;
dx1 = 2.0/fac; tol = 8*10e-5;
```



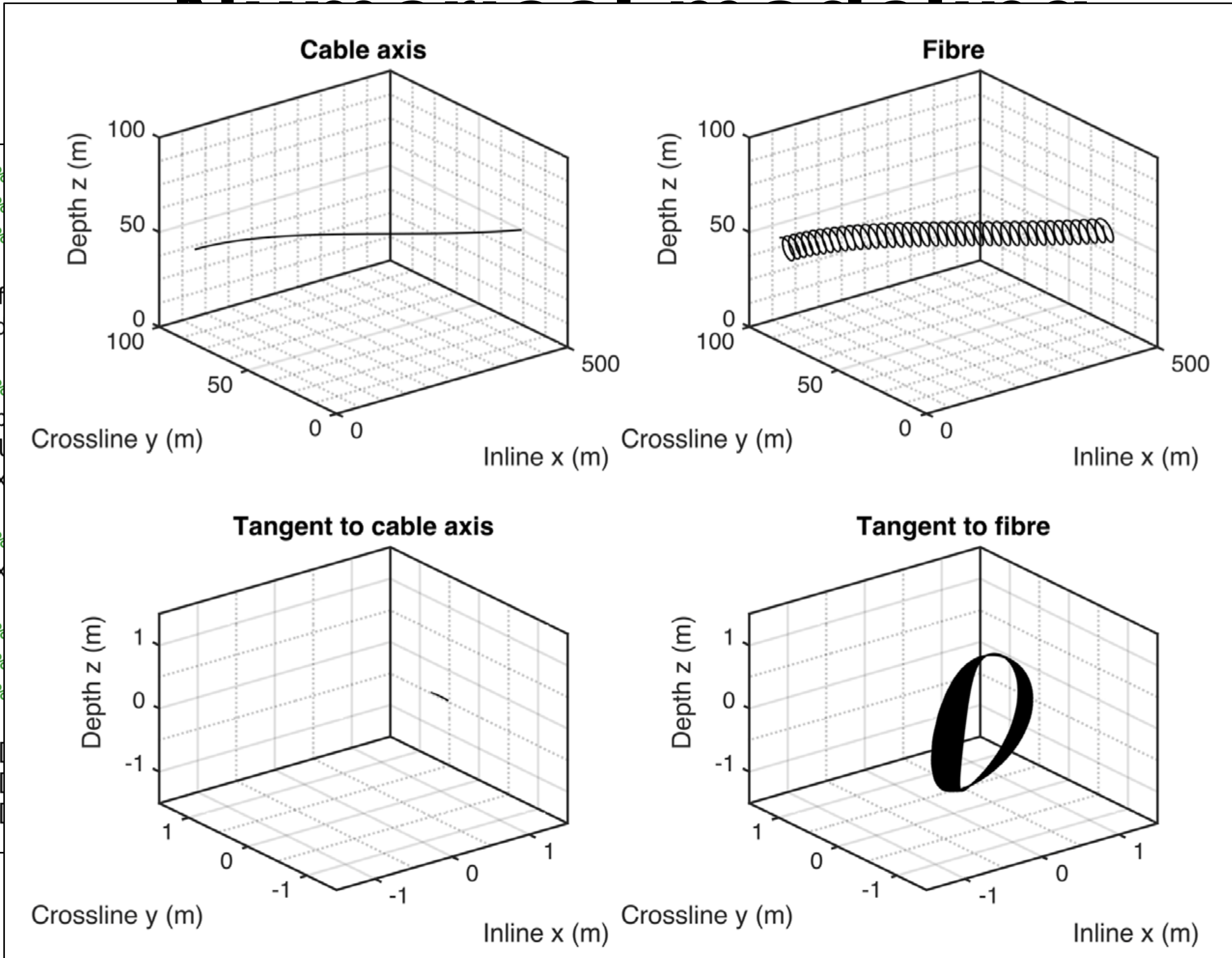
Numerical modeling





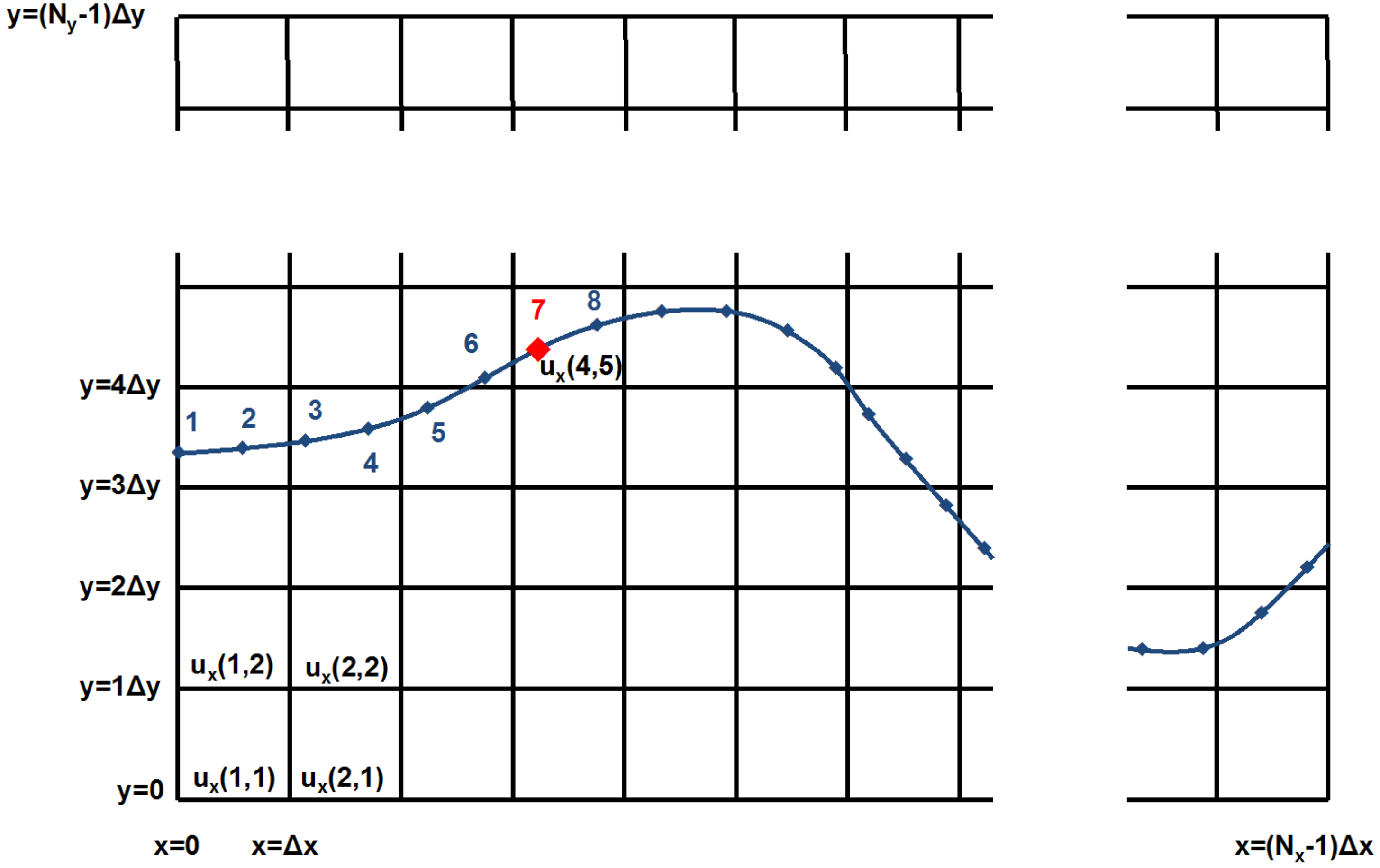
Numerical modeling

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Cable and fibre parameters & equations

fac = 4; Np = 512*fac;
dx1 = 2.0/fac; tol = 8*10e-5;

% Parameters
bigV = 200; bigR = 30;
littleV = 2; littleR = 4;
x2c = 50; x3c = 50;

% Cable axis curve equations
x10 = 1:Np; x10 = (x10-1)*dx1; x20 = x2c + bigR*cos(x10/bigV); x30 = x3c*ones(size(x10));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate geometry, fibre vectors, fibre tangent

[t0,n0,b0,ss] = curvegeometry(x10,x20,x30,'y',tol);
[x1,x2,x3] = addhelix(t0,n0,b0,x10,x20,x30,littleR,ss,littleV);
[t1,n1,b1,ss1] = curvegeometry(x1,x2,x3,'n',tol);
```


Cable axis
Fibre
Tangent to cable axis
Tangent to fibre
Depth z (m)
Crossline y (m)
Inline x (m)
100
50
0
500
1
-1

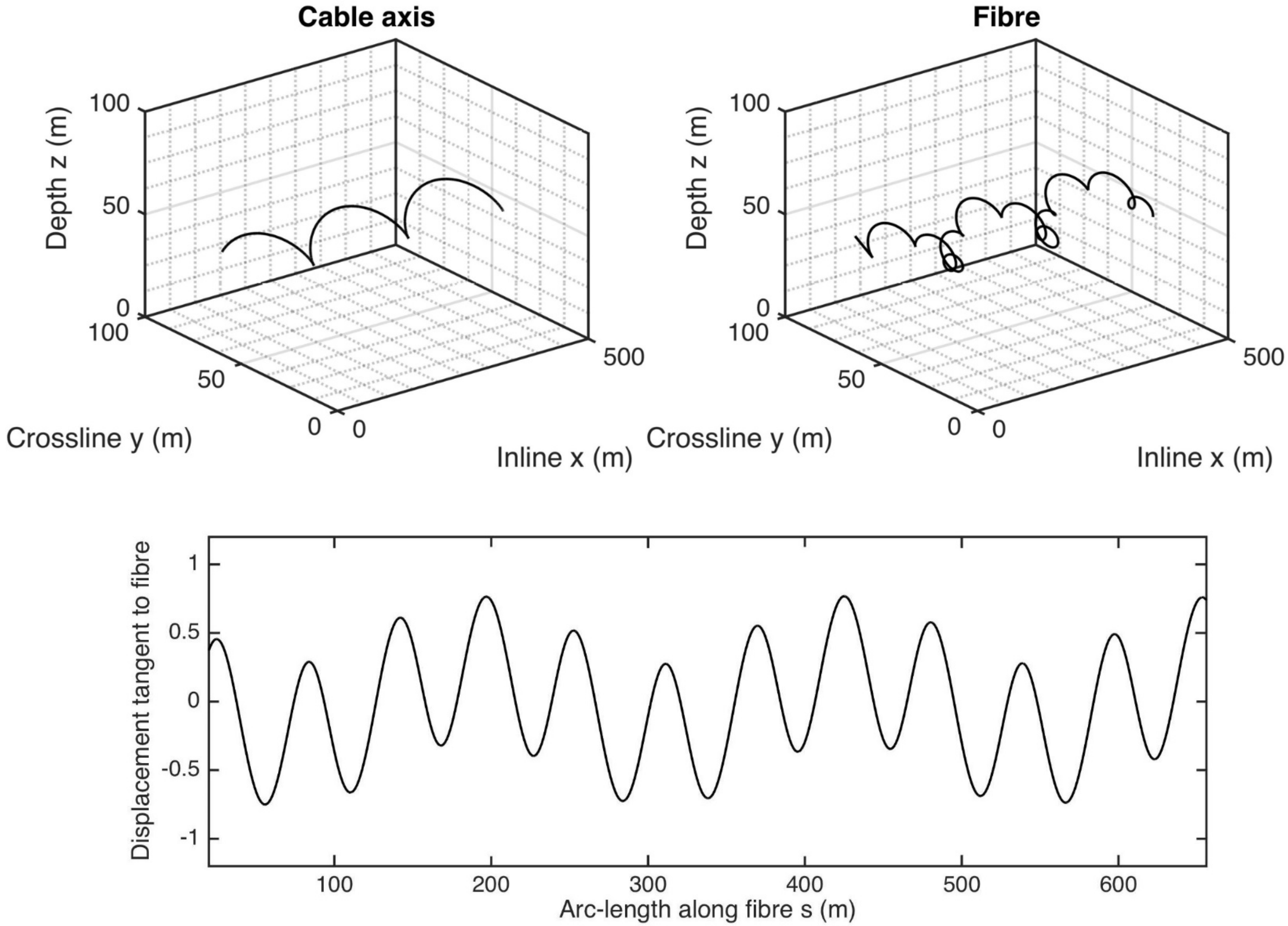
Applications

I. Embedding fibre in a given vector wave field



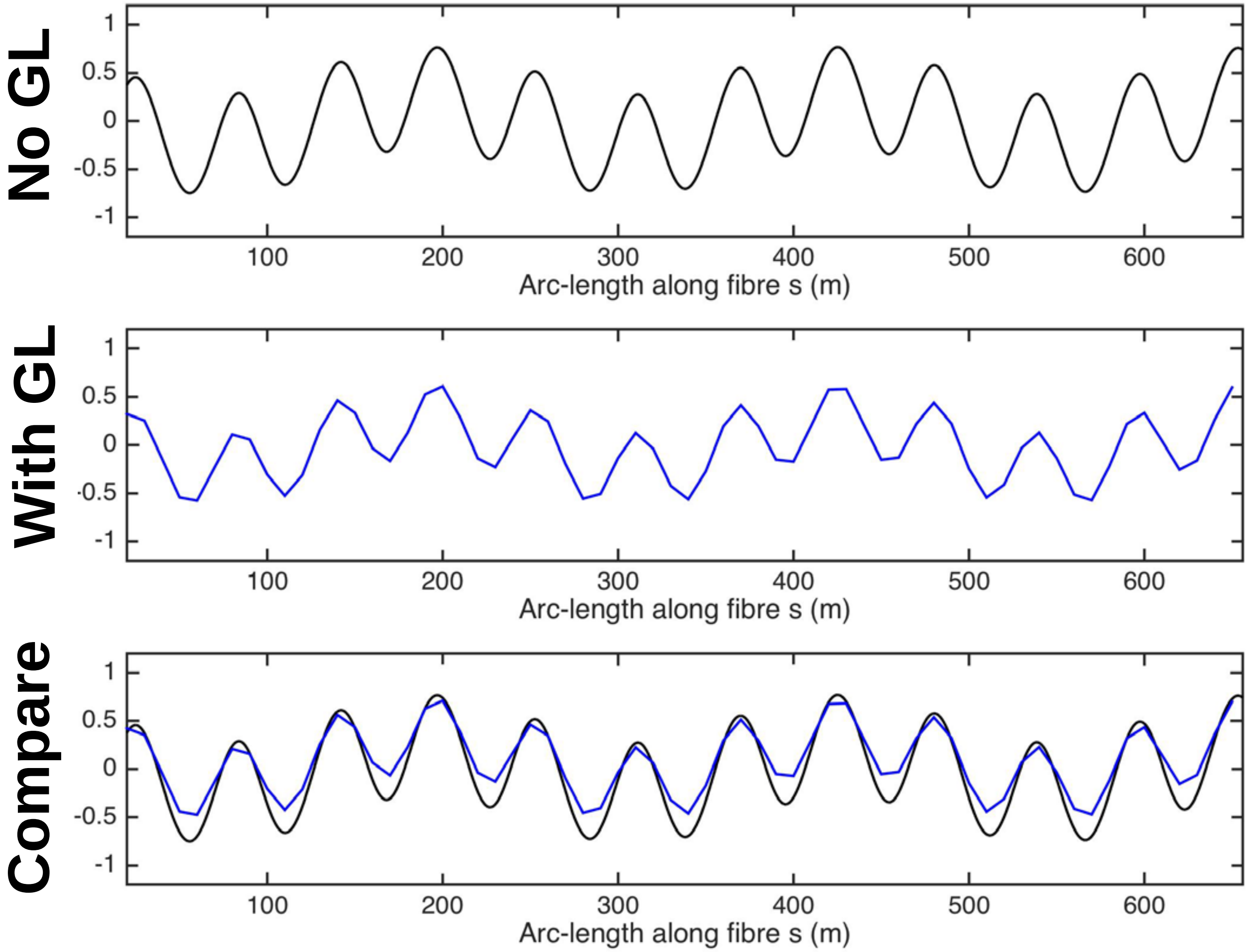
Applications

I. Embedding fibre in a given vector wave field



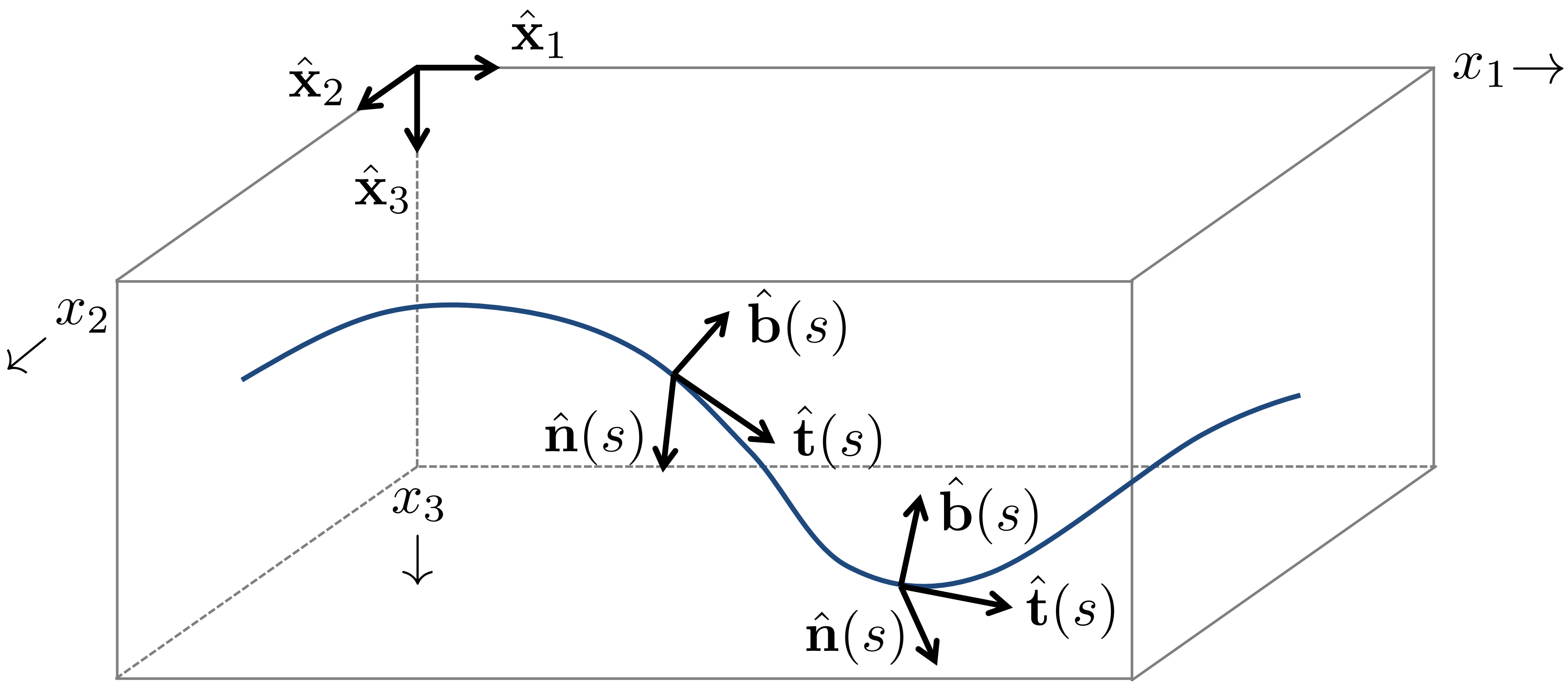
Applications

I. Embedding fibre in a given vector wave field



Applications

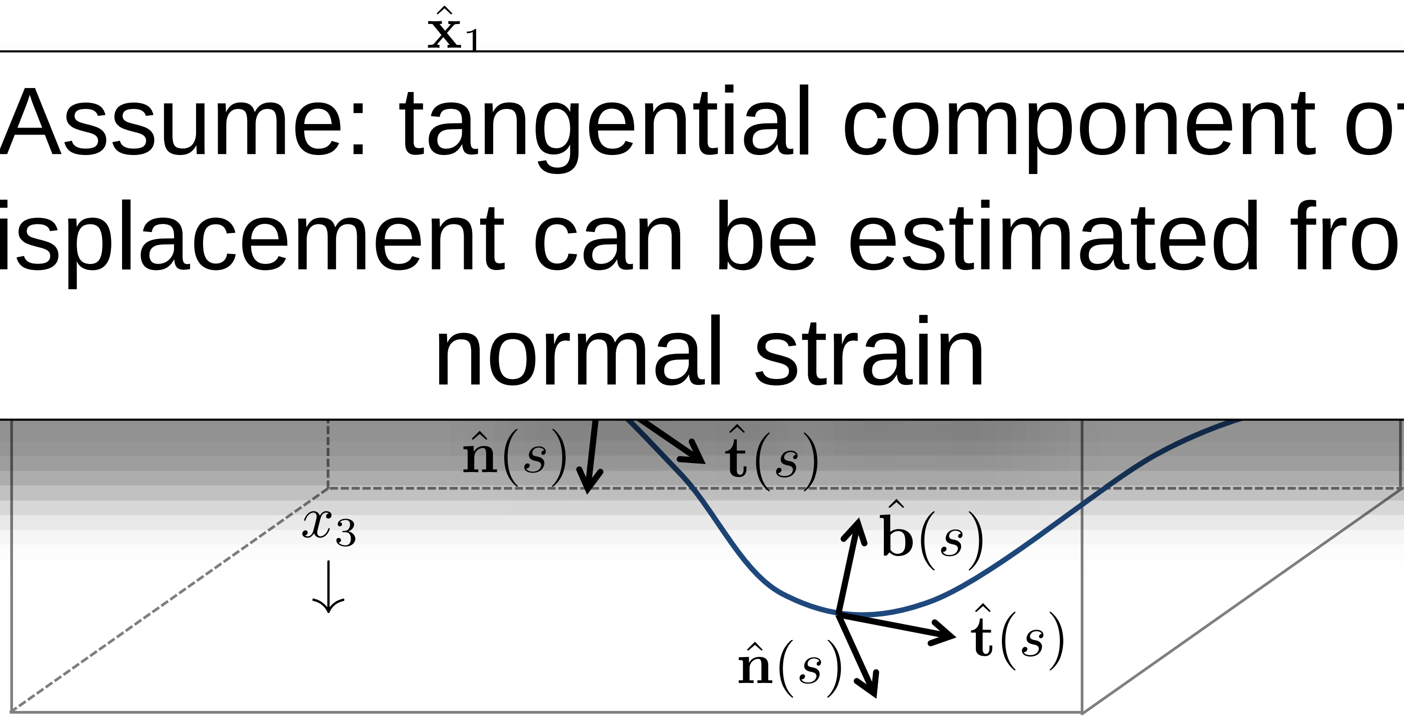
II. Estimating 3C of **u** from u_t



Input: $u_t(s) = \mathbf{u}^{\mathsf{T}}\mathbf{t}(s)$

Applications

II. Estimating 3C of **u** from u_t

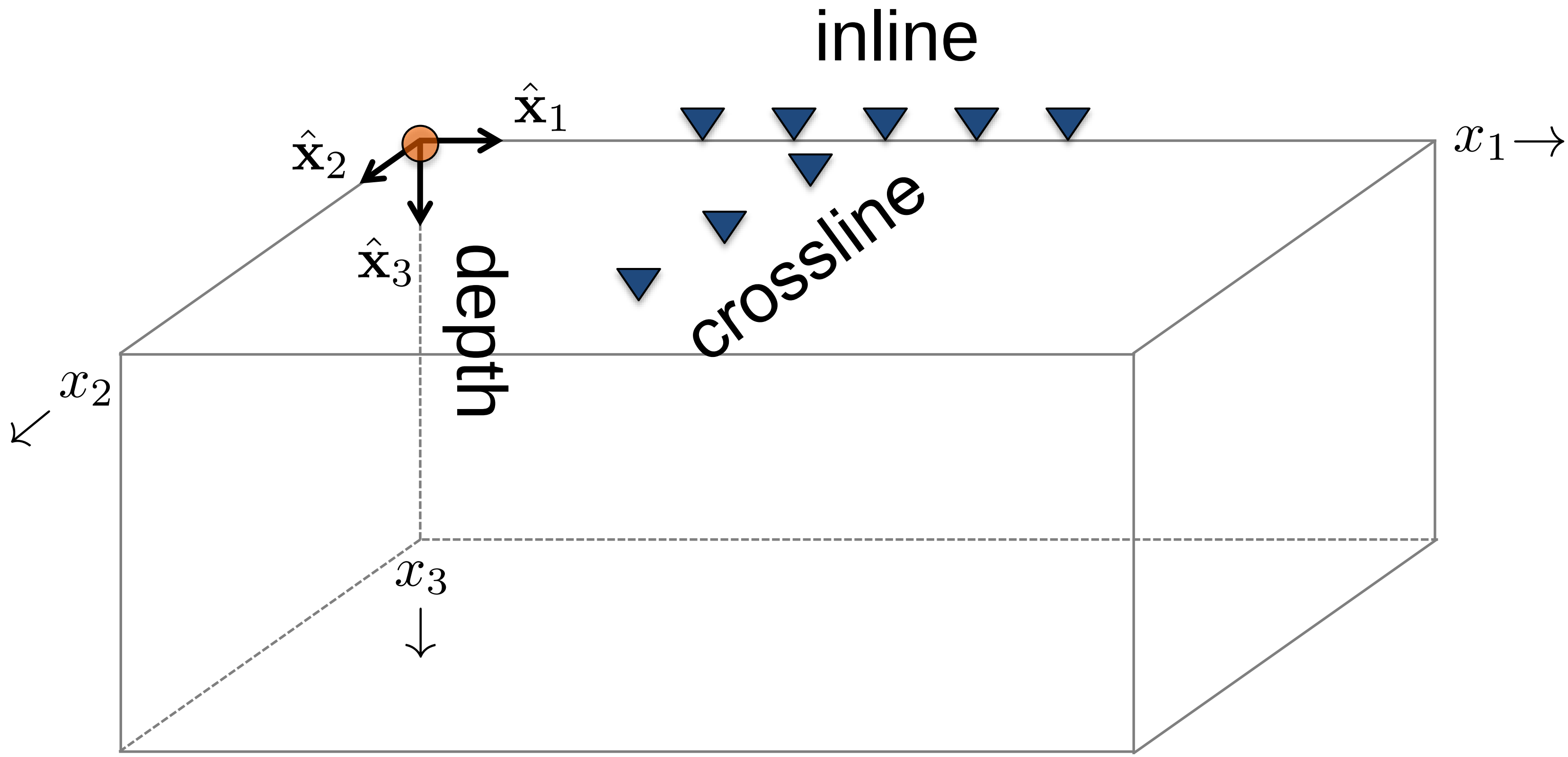


Assume: tangential component of displacement can be estimated from normal strain

Input: $u_t(s) = \mathbf{u}^T\mathbf{t}(s)$

Applications

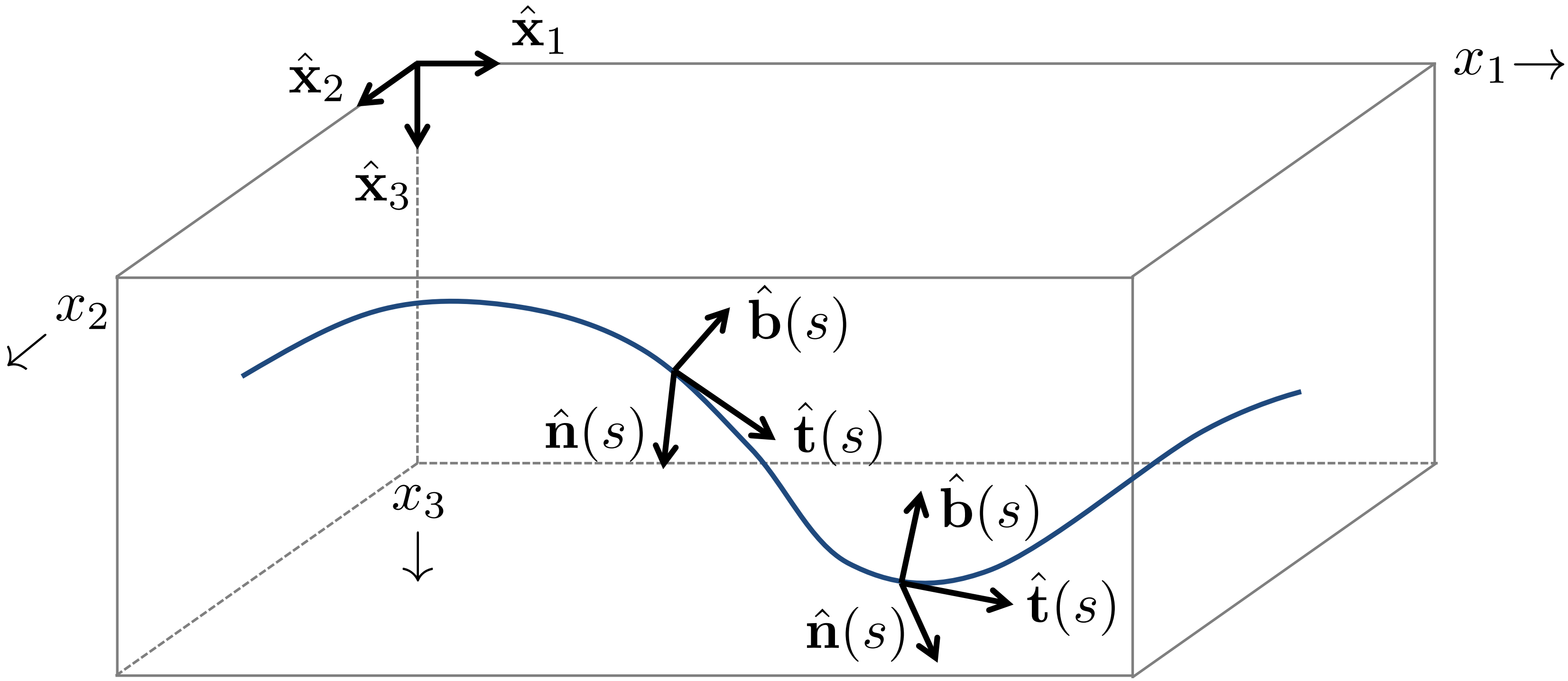
II. Estimating 3C of **u** from u_t



Desired output: **u** = $[u_1, u_2, u_3]^T$ in “field system”

Applications

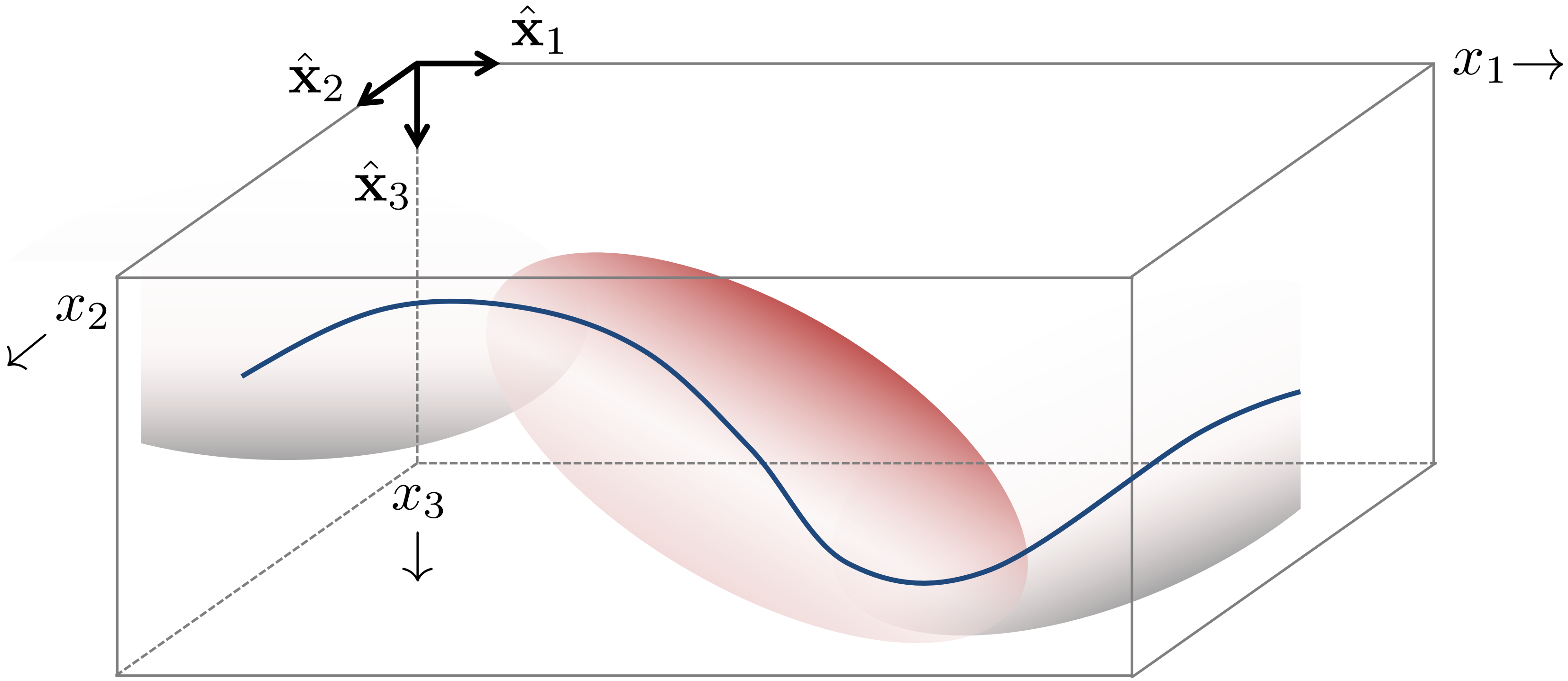
II. Estimating 3C of **u** from u_t



Input: $u_t(s) = \mathbf{u}^T \mathbf{t}(s)$

Applications

II. Estimating 3C of **u** from u_t



Problem: within some reconstruction volume, determine **u** from { $u_t(s_1)$, $u_t(s_2)$, ... $u_t(s_N)$ }

Applications

II. Estimating 3C of **u** from u_t

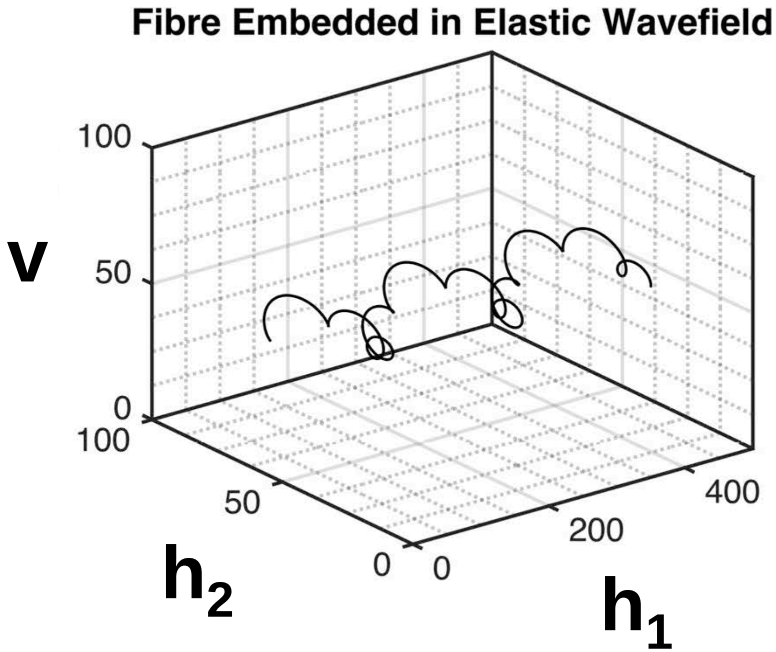
$$\begin{bmatrix} u_1' \\ \boxed{u_1''} \\ \vdots \\ u_1^{(N)} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{x}}_1' \cdot \hat{\mathbf{x}}_1 & \hat{\mathbf{x}}_1' \cdot \hat{\mathbf{x}}_2 & \hat{\mathbf{x}}_1' \cdot \hat{\mathbf{x}}_3 \\ \hat{\mathbf{x}}_1'' \cdot \hat{\mathbf{x}}_1 & \hat{\mathbf{x}}_1'' \cdot \hat{\mathbf{x}}_2 & \hat{\mathbf{x}}_1'' \cdot \hat{\mathbf{x}}_3 \\ & \vdots & \\ \hat{\mathbf{x}}_1^{(N)} \cdot \hat{\mathbf{x}}_1 & \hat{\mathbf{x}}_1^{(N)} \cdot \hat{\mathbf{x}}_2 & \hat{\mathbf{x}}_1^{(N)} \cdot \hat{\mathbf{x}}_3 \end{bmatrix} \boxed{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}$$

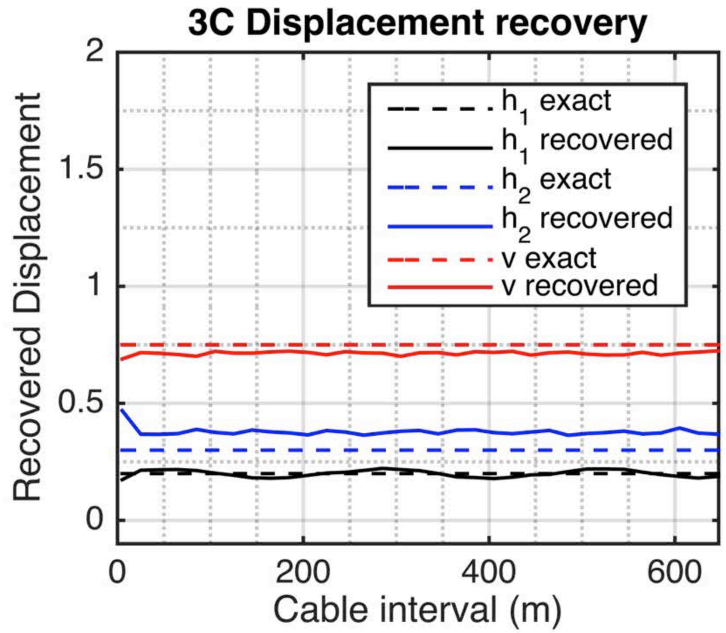
Single components $u_t(s_i)$ in reconstruction window

3C vector **u** in reconstruction window

Applications

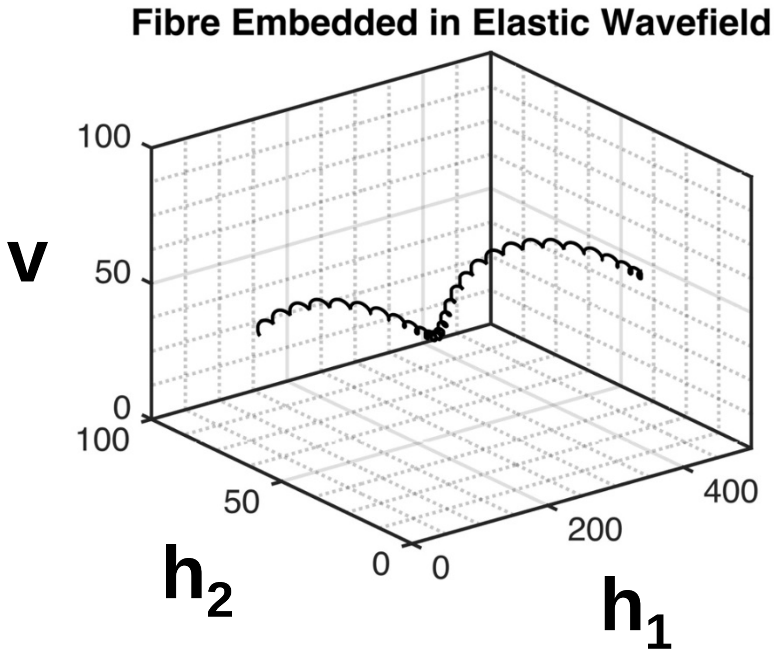
II. Estimating 3C of **u** from u_t

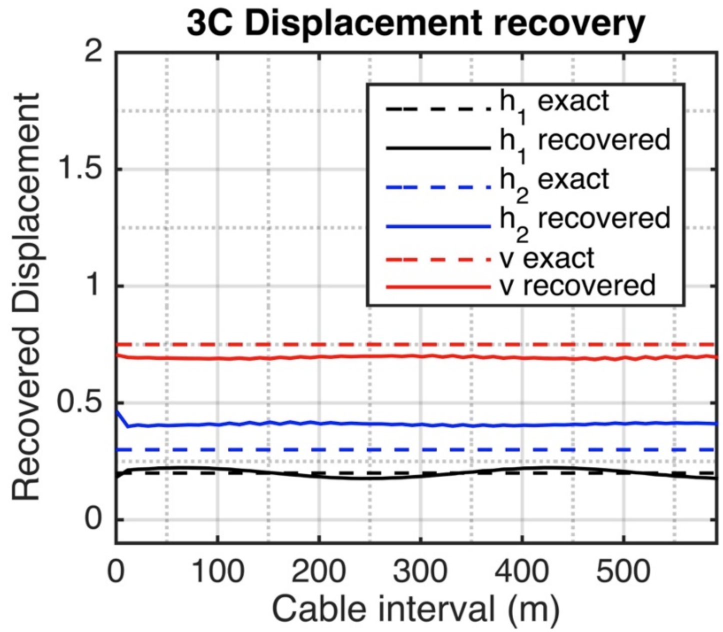




Applications

II. Estimating 3C of **u** from u_t



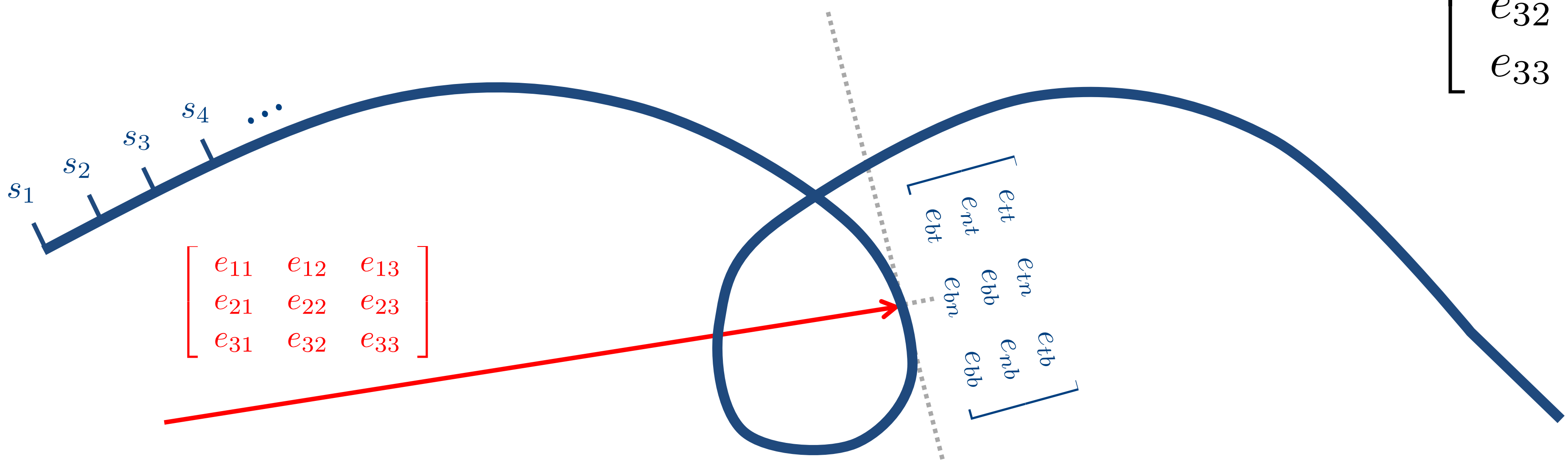


Applications

III. Estimating tensor strain from $e_{tt}(s)$

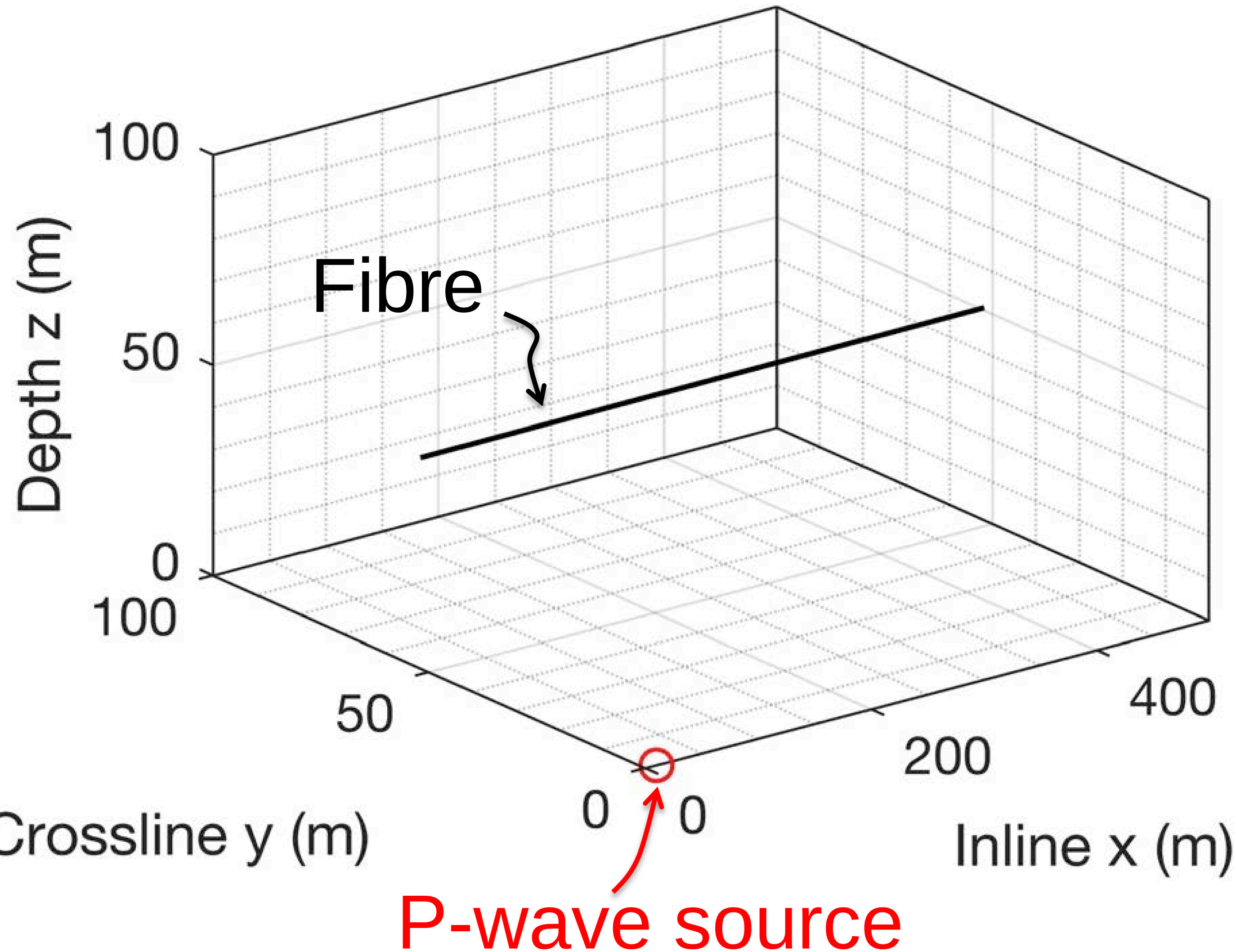
$$\lambda_{ij}^{k} = \left(\hat{\mathbf{t}}(s_k) \cdot \hat{\mathbf{i}} \right) \left(\hat{\mathbf{t}}(s_k) \cdot \hat{\mathbf{j}} \right)$$

$$\begin{bmatrix} e_{tt}(s_1) \\ e_{tt}(s_2) \\ \vdots \\ e_{tt}(s_N) \end{bmatrix} = \begin{bmatrix} \lambda_{11}^{1} & \lambda_{12}^{1} & \lambda_{13}^{1} & \lambda_{21}^{1} & \lambda_{22}^{1} & \lambda_{23}^{1} & \lambda_{31}^{1} & \lambda_{32}^{1} & \lambda_{33}^{1} \\ \lambda_{11}^{2} & \lambda_{12}^{2} & \lambda_{13}^{2} & \lambda_{21}^{2} & \lambda_{22}^{2} & \lambda_{23}^{2} & \lambda_{31}^{2} & \lambda_{32}^{2} & \lambda_{33}^{2} \\ & & & & \vdots & & & & \\ \lambda_{11}^{N} & \lambda_{12}^{N} & \lambda_{13}^{N} & \lambda_{21}^{N} & \lambda_{22}^{N} & \lambda_{23}^{N} & \lambda_{31}^{N} & \lambda_{32}^{N} & \lambda_{33}^{N} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \\ e_{21} \\ e_{22} \\ e_{23} \\ e_{31} \\ e_{32} \\ e_{33} \end{bmatrix}$$



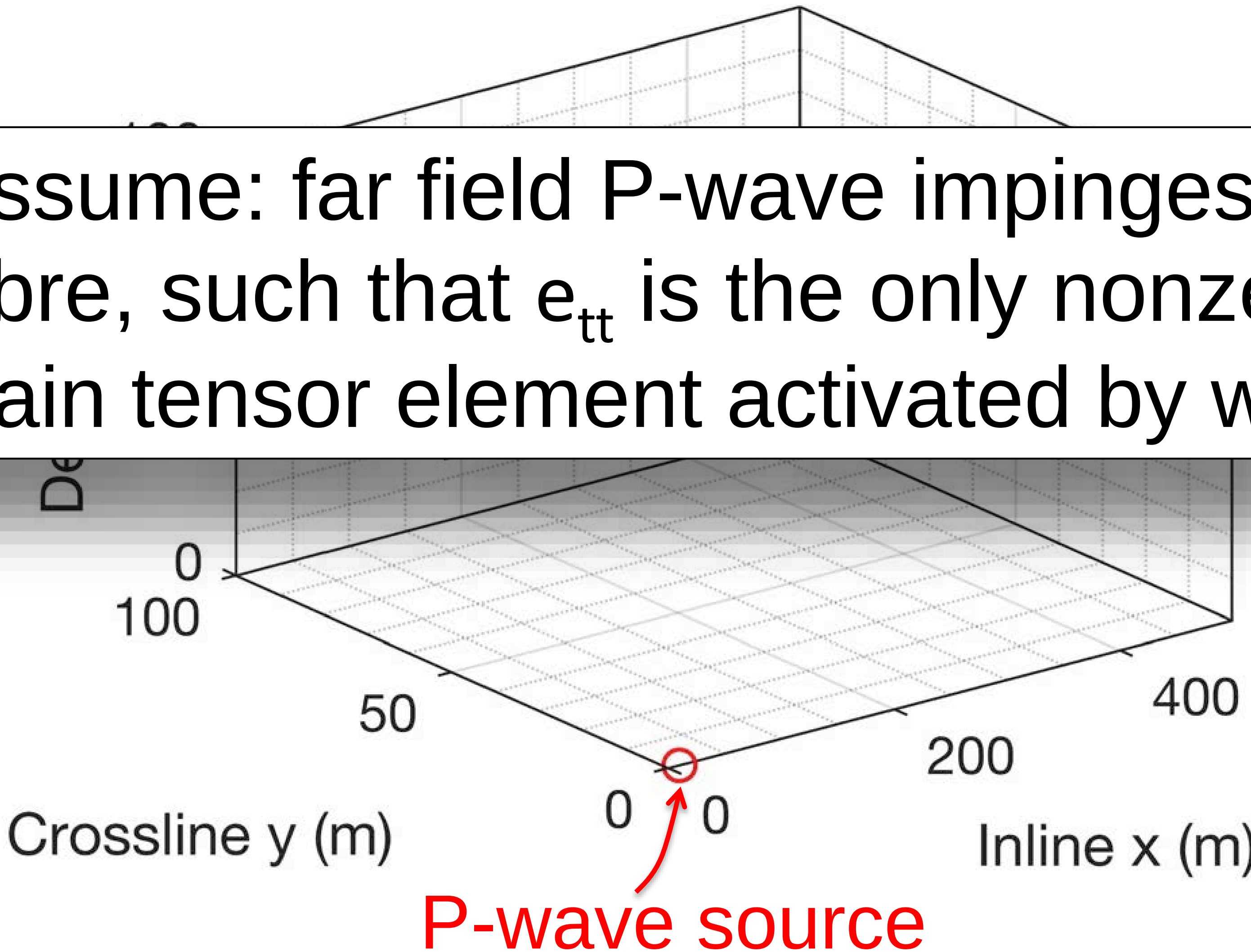
Applications

III. Generalizing $\cos^2\theta$ directionality rules



Applications

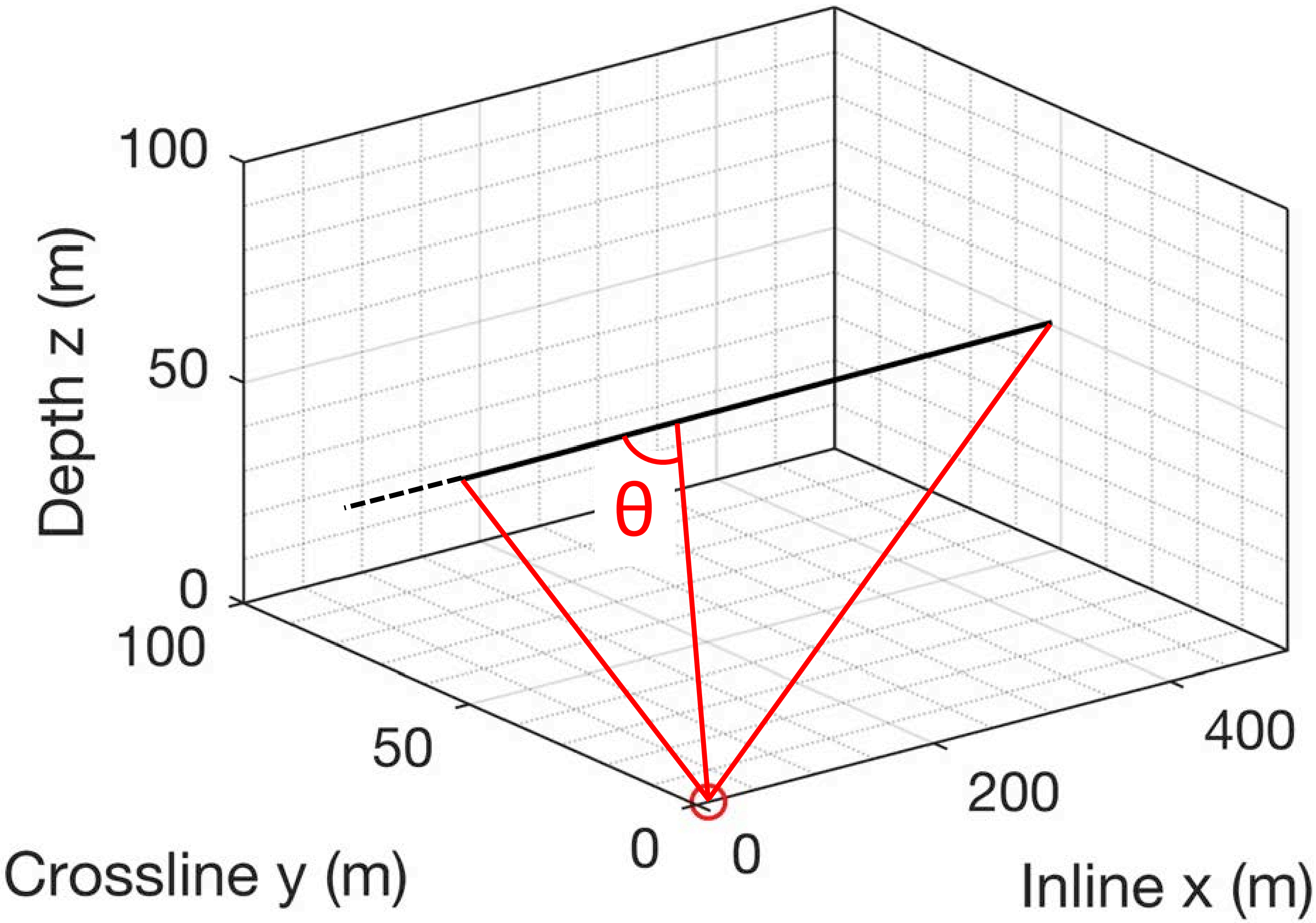
III. Generalizing $\cos^2\theta$ directionality rules



Assume: far field P-wave impinges on fibre, such that e_{tt} is the only nonzero strain tensor element activated by wave

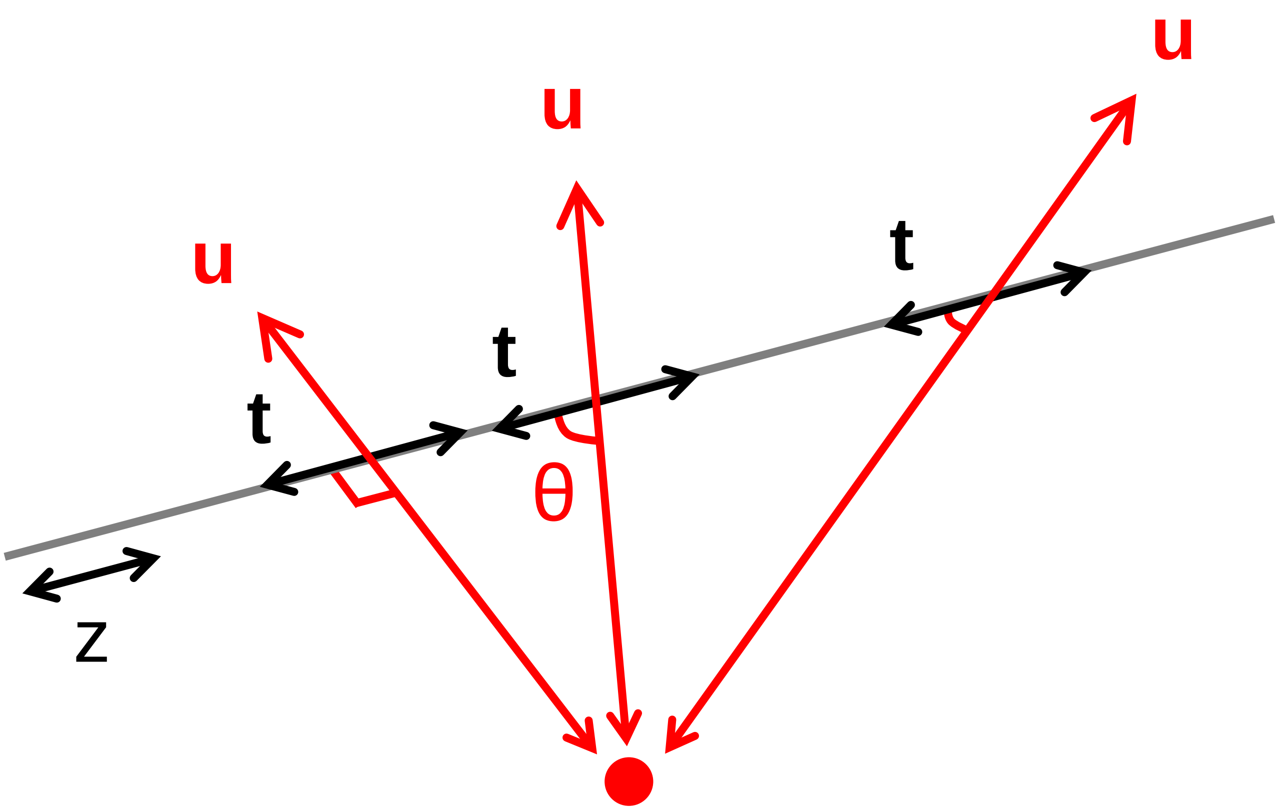
Applications

III. Generalizing $\cos^2\theta$ directionality rules



Applications

III. Generalizing $\cos^2\theta$ directionality rules

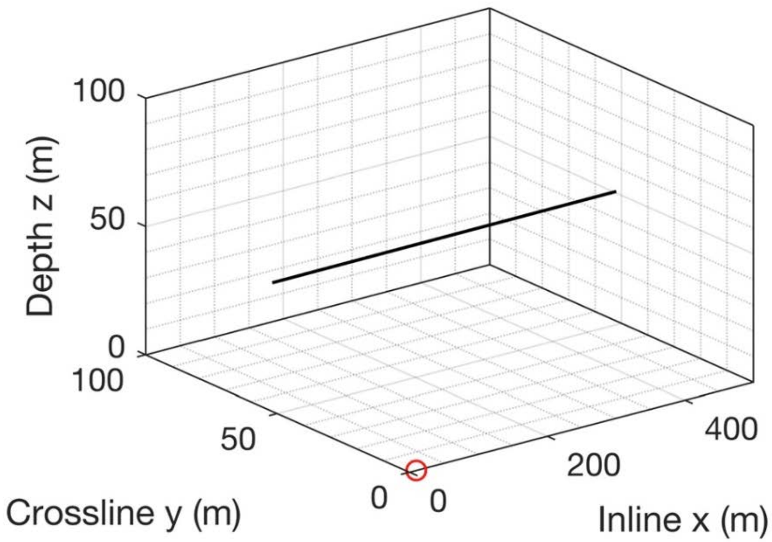


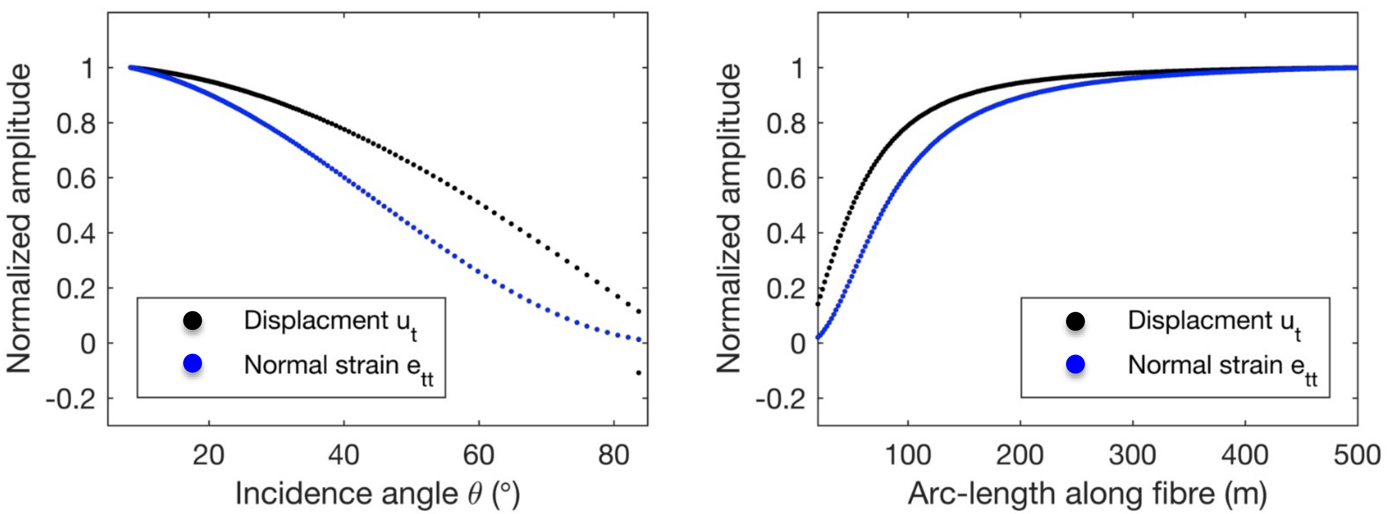
$u_t = |\mathbf{u}|\ \hat{\mathbf{u}} \bullet \mathbf{t} = |\mathbf{u}| \cos\theta$ Displacement response

$e_{tt} = (\hat{\mathbf{u}} \bullet \mathbf{t})\ |\mathbf{e}|\ (\hat{\mathbf{u}} \bullet \mathbf{t}) = |\mathbf{e}| \cos^2\theta$ Strain response

Applications

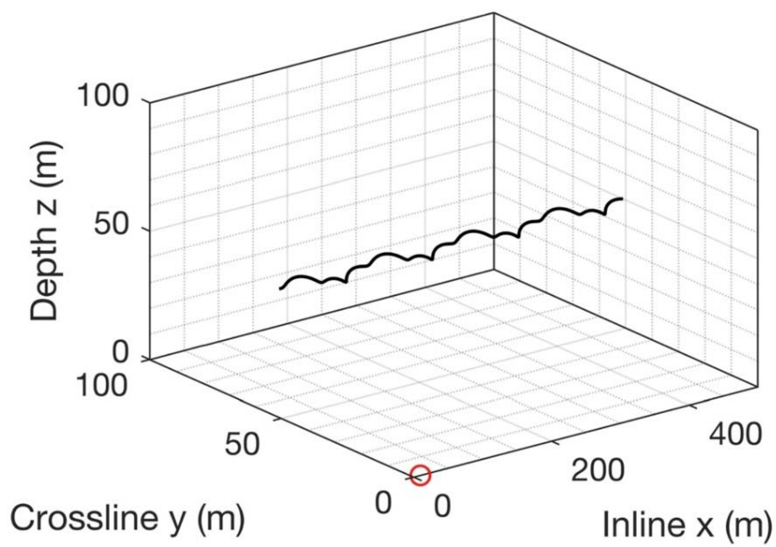
III. Generalizing $\cos^2\theta$ directionality rules

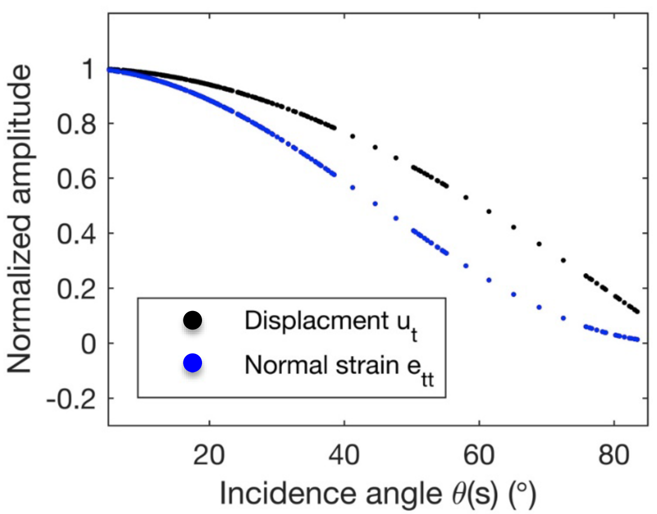


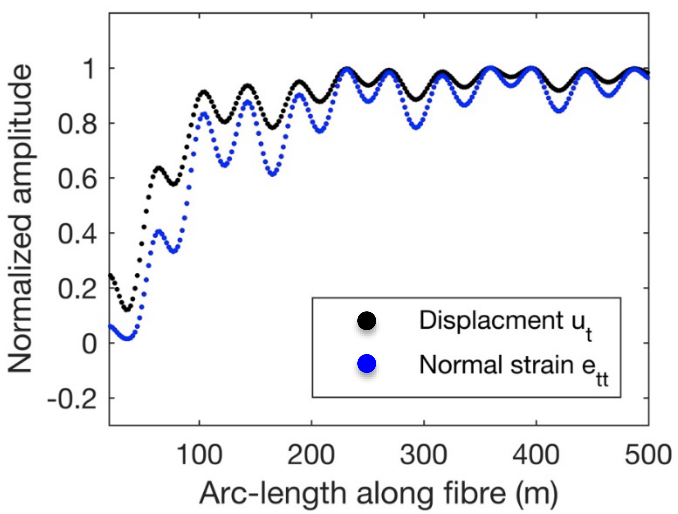


Applications

III. Generalizing $\cos^2\theta$ directionality rules







Conclusions

1. The next generation of geophysical methods for monitoring of production and exploration requires
   a) dense sampling
   b) repeatability
   c) cost-effectiveness
2. Fibre / DAS systems a strong contender. But:
   a) overcome / characterize broadside insensitivity?
   b) to what degree can we estimate 3C from fibre?
3. Modeling moves us towards answers. Going forward:
   a) design optimum practical geometries
   b) ...with outcomes (AVO/AVAz inversion, FWI) in mind
   c) Field tests

Background

Optical scattering

Mie

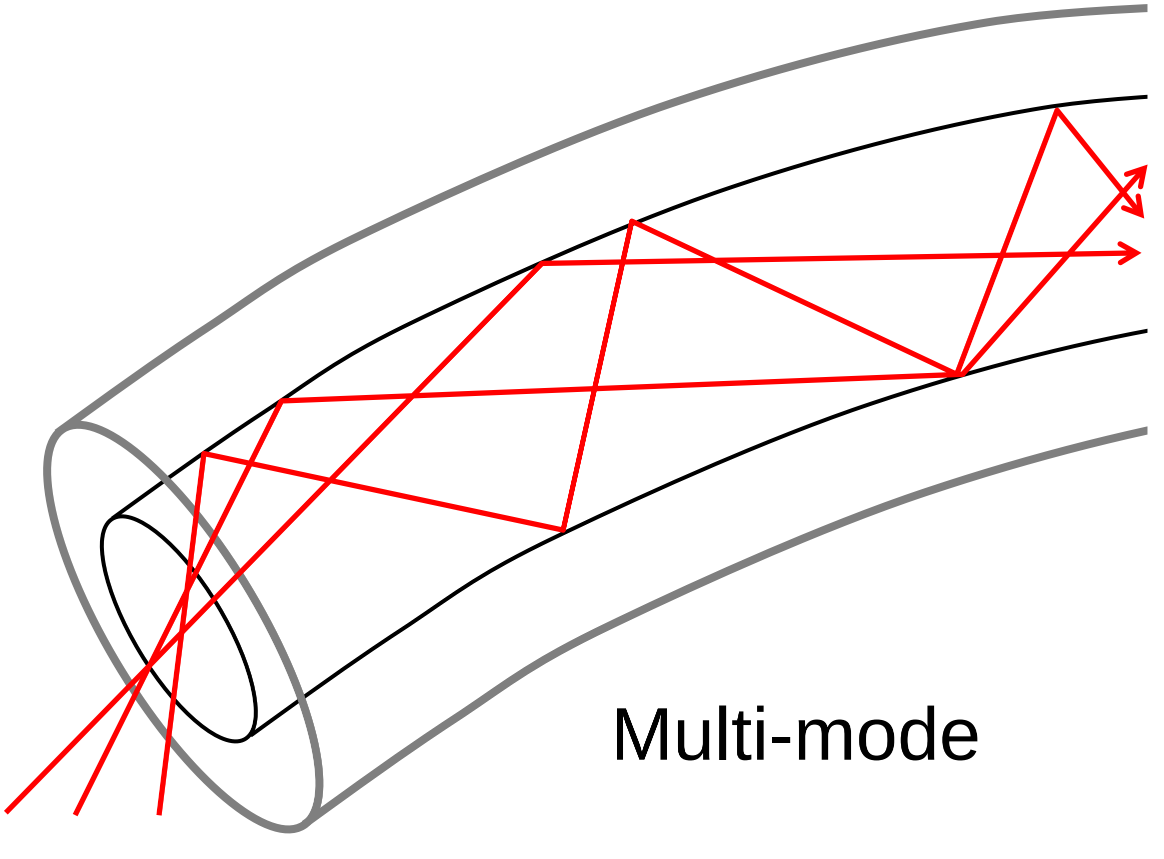
Raman

Bragg Rayleigh

Brillouin

Background

Fibre-optic acoustic sensing



Background

Fibre-optic acoustic sensing