# Sequential Gaussian Simulation —using multi-variable cokriging

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# Outline

- Motivation
- Methodology
- Case study ---  Blackfoot data
- Conclusion
- Acknowledgements

# Motivation

- A limitation of the deterministic method is the result generated has difficulty capturing the natural heterogeneity of reservoirs due to the smoothing effect.
- The sequential simulation technique provides a series of equal valid and possible realizations that reflect the distribution of reservoir properties.
- One of the key steps to generate a realization is to build the probability density distribution based on the conditional mean and covariance at each unsampled location.
- Heiskanen and Moritz (1967) and Dermanis (1984) showed that least square prediction is equivalent to a simple geostatistical system.
- Tarantola (2005) demonstrated that the least square problem can also be described as the posterior Gaussian probability density distribution.
- Hansen et al. (2006) expanded Tarantola's work to use two types datasets.

# Methodology

$$P(m|a,b) = const.\exp[-\frac{1}{2}(\boldsymbol{m} - \boldsymbol{\mu}_{\boldsymbol{m}|\boldsymbol{a},\boldsymbol{b}})^T \sum\nolimits_{m|a,b}^{-1} (\boldsymbol{m} - \boldsymbol{\mu}_{\boldsymbol{m}|\boldsymbol{a},\boldsymbol{b}})]$$

*where the conditional mean as*

$$\boldsymbol{\mu}_{\boldsymbol{m}|\boldsymbol{a},\boldsymbol{b}} = \boldsymbol{\mu}_{\boldsymbol{m}} + (\boldsymbol{G}\boldsymbol{C}_{\boldsymbol{m}})^{\boldsymbol{T}} \left(\boldsymbol{G}\boldsymbol{C}_{\boldsymbol{m}}\boldsymbol{C}^{\boldsymbol{T}} + \boldsymbol{C}_{\boldsymbol{d}}\right)^{-1} (\boldsymbol{d} - \boldsymbol{\mu}_{\boldsymbol{d}})$$

*where the conditional covariance matrix*

$$\Sigma_{\boldsymbol{m}|\boldsymbol{d}} = \boldsymbol{C}_{\boldsymbol{m}} - \boldsymbol{C}_{\boldsymbol{m}}\boldsymbol{C}^{\boldsymbol{T}}\left(\boldsymbol{G}\boldsymbol{C}_{\boldsymbol{m}}\boldsymbol{C}^{\boldsymbol{T}} + \boldsymbol{C}_{\boldsymbol{d}}\right)^{-1} (\boldsymbol{d} - \boldsymbol{\mu}_{\boldsymbol{d}})$$

$$\boldsymbol{d} = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}, \boldsymbol{C}_d = \begin{bmatrix} \boldsymbol{C}_{aa} & \boldsymbol{C}_{ab} \\ \boldsymbol{C}_{ab} & \boldsymbol{C}_{bb} \end{bmatrix}, \boldsymbol{\mu}_d = \begin{bmatrix} \boldsymbol{\mu}_a \\ \boldsymbol{\mu}_b \end{bmatrix},$$

$\boldsymbol{\mu}_m$ *and* $\boldsymbol{C}_m$ are the mean and covariance of target object.

- For multiple types of datasets $\boldsymbol{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}, \boldsymbol{C}_d = \begin{bmatrix} \boldsymbol{C}_{d_1 d_2} & \cdots & \boldsymbol{C}_{d_1 d_n} \\ \vdots & \ddots & \vdots \\ \boldsymbol{C}_{d_1 d_n} & \cdots & \boldsymbol{C}_{d_n d_n} \end{bmatrix}, \boldsymbol{\mu}_d = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \vdots \\ \boldsymbol{\mu}_n \end{bmatrix}$

# Methodology

- Rescaled Ordinary CoKriging (ROCK)

- We can extend rescaled ordinary cokriging from one to two secondary datasets :

$$\hat{u}_0 = \sum_{i=1}^{n} a_i u_i + \sum_{j=1}^{m} b_j (v_j - m_v + m_u) + \boxed{\sum_{k=1}^{p} c_k (x_k - m_x + m_u)}$$

- The weights are computed using the matrix equation:

$$\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} C_{uu} & C_{uv} & C_{ux} & \boldsymbol{1} \\ C_{vu} & C_{vv} & C_{vx} & \boldsymbol{1} \\ C_{xu} & C_{xv} & C_{xx} & \boldsymbol{1} \\ \boldsymbol{1}^T & \boldsymbol{1}^T & \boldsymbol{1}^T & \boldsymbol{0} \end{bmatrix}^{-1} \begin{bmatrix} C_{u_0 u} \\ C_{u_0 v} \\ C_{u_0 x} \\ 1 \end{bmatrix}$$

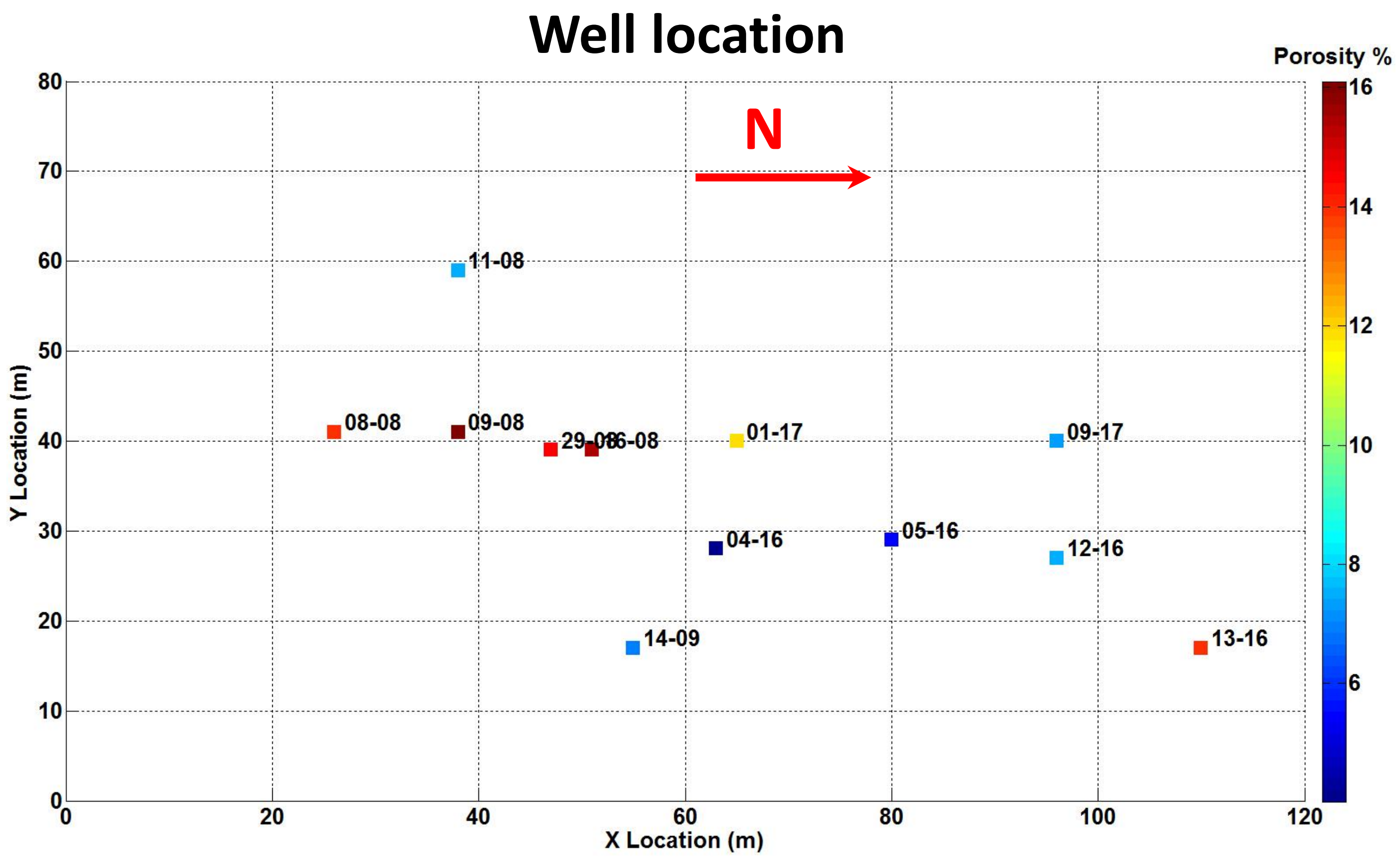
$C_{ux}$ = well to 2nd seismic covariance,
$C_{vx}$ = 1st seismic to 2nd seismic covariance,
$C_{xx}$ = 2nd seismic to 2nd seismic covariance,
$C_{u_0 x}$ = unknown well to 2nd seismic covariance.

# Methodology

- ROCK with n secondary datasets

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \\ \mu \end{bmatrix} = \begin{bmatrix} C_{uu} & C_{uv_1} & C_{uv_2} & \cdots & C_{uv_n} & 1 \\ C_{v_1u} & C_{v_1v_1} & C_{v_1v_2} & \cdots & C_{v_1v_n} & 1 \\ C_{v_2u} & C_{v_2v_1} & C_{v_2v_2} & \vdots & C_{v_2v_n} & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ C_{v_nu} & C_{v_nv_1} & C_{v_nv_2} & \cdots & C_{v_nv_n} & 1 \\ 1^T & 1^T & 1^T & \cdots & 1^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} C_{u_0u} \\ C_{u_0v_1} \\ C_{u_0v_2} \\ \vdots \\ C_{u_0v_n} \\ 1 \end{bmatrix}$$

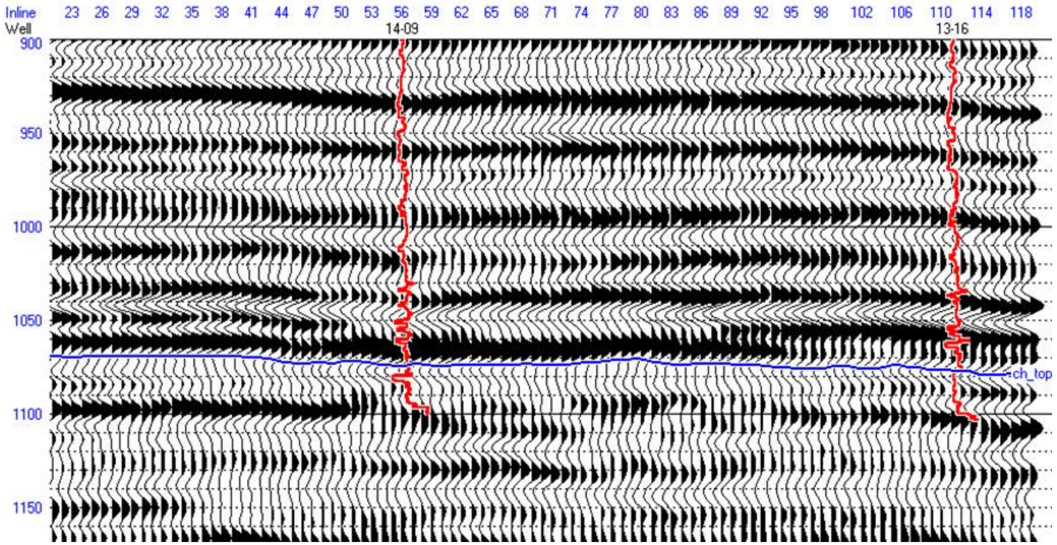
# Case study --- Blackfoot



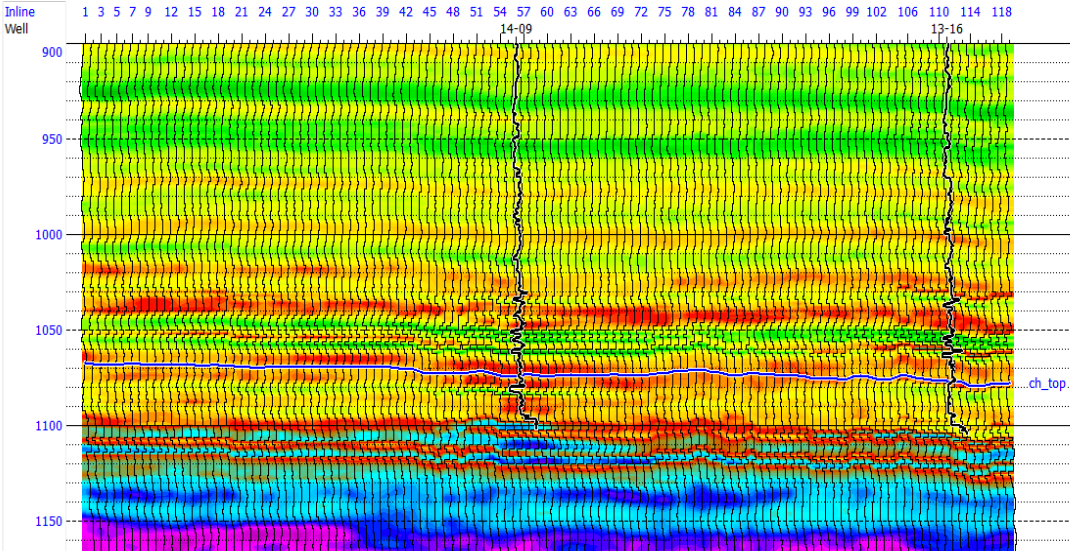

- 12 wells are located in the seismic survey area
- The color indicates the average porosity value of each well in the target zone

# Case study

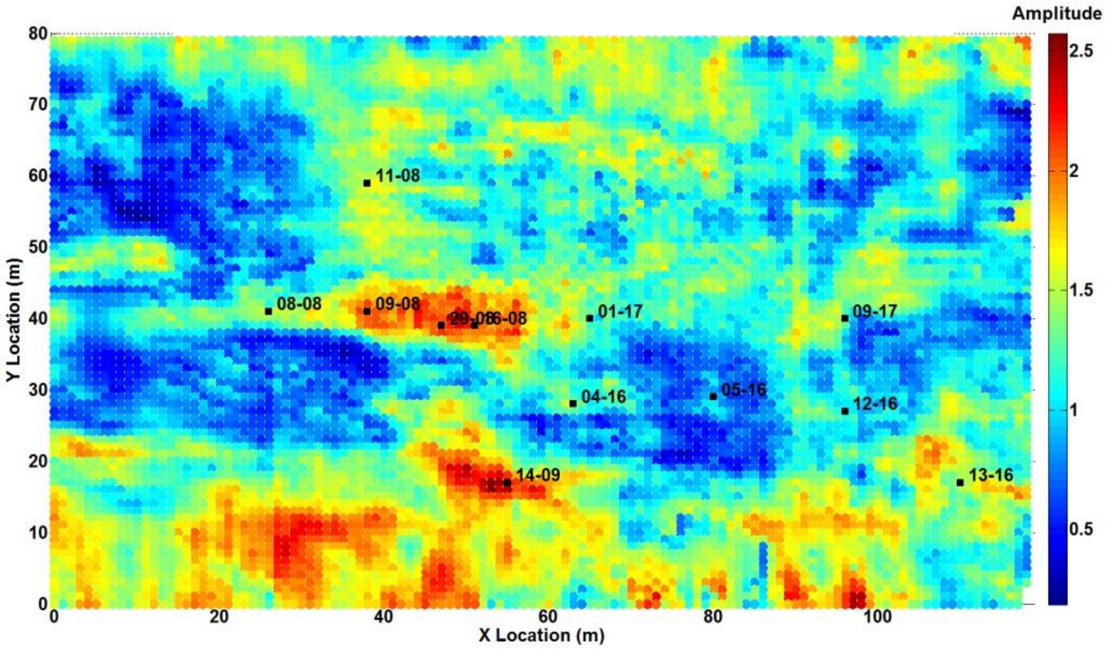
**crossline 18 from the seismic volume**



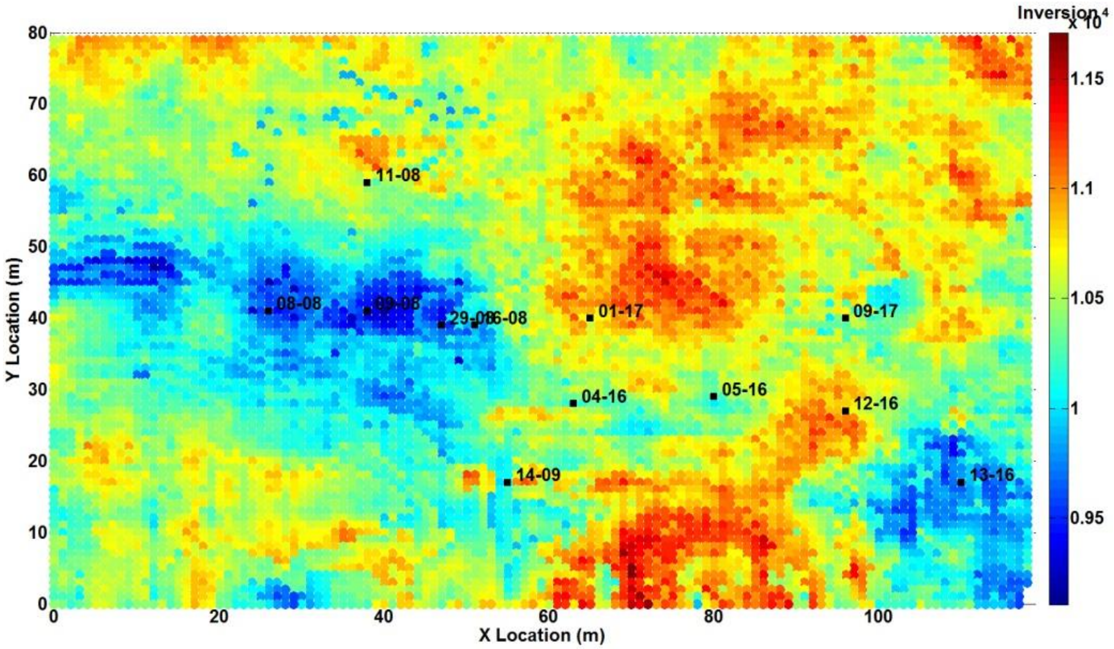

**crossline 18 from the inverted volume**



## Amplitude of near-angle stack

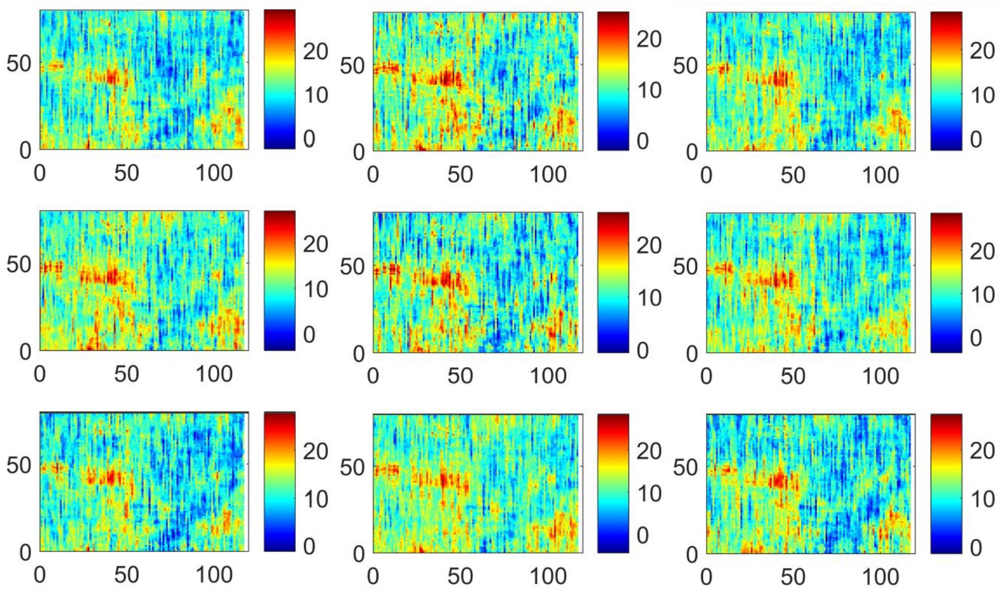



## P-impedance inversion

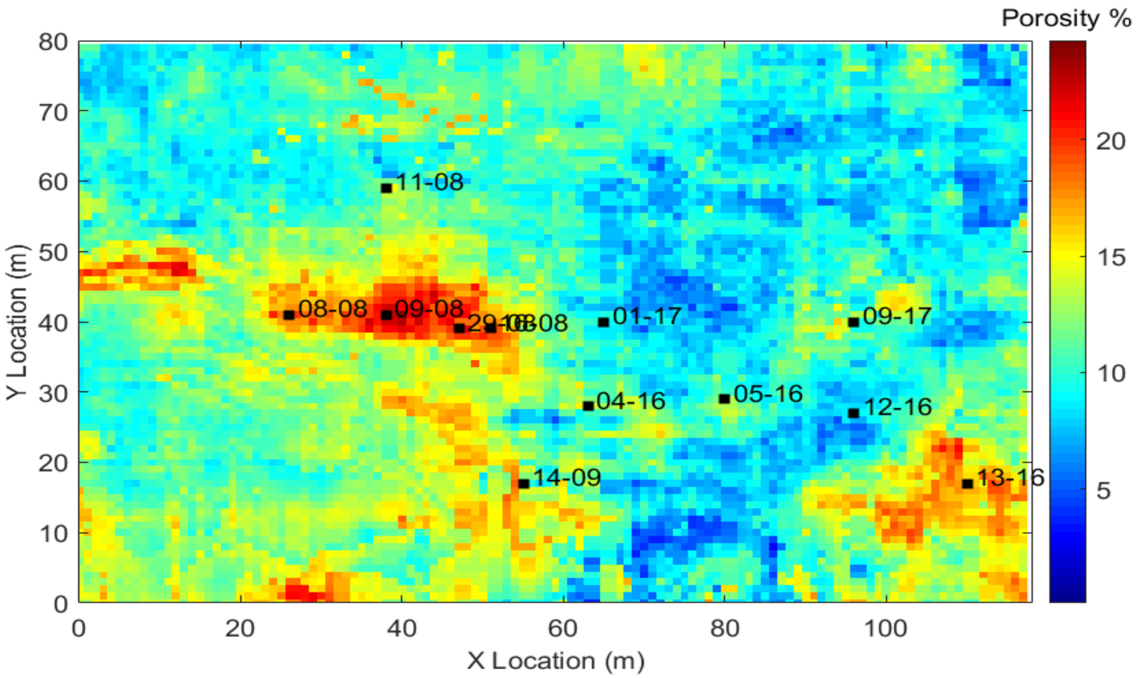


# Result

**9 realizations (SGS-extended ROCK)**

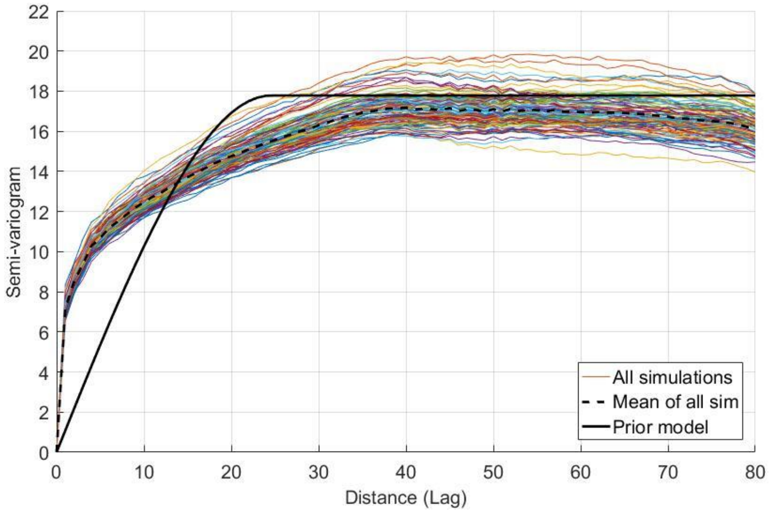



**Porosity mean value map of all 100 realizations**

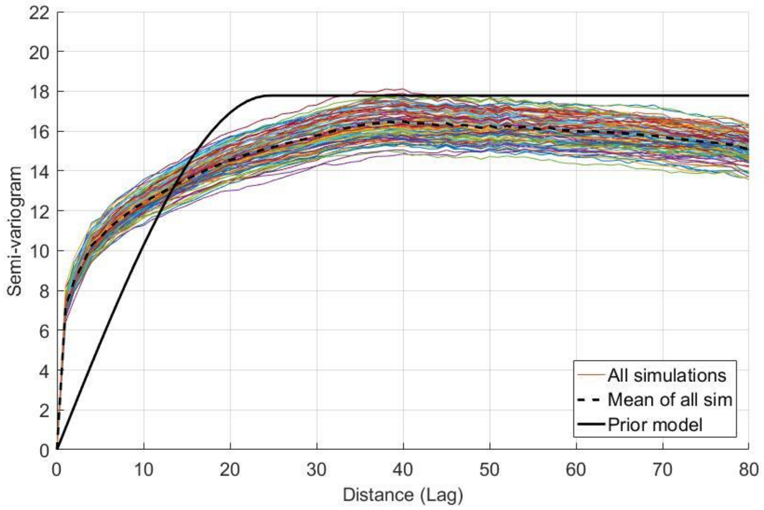


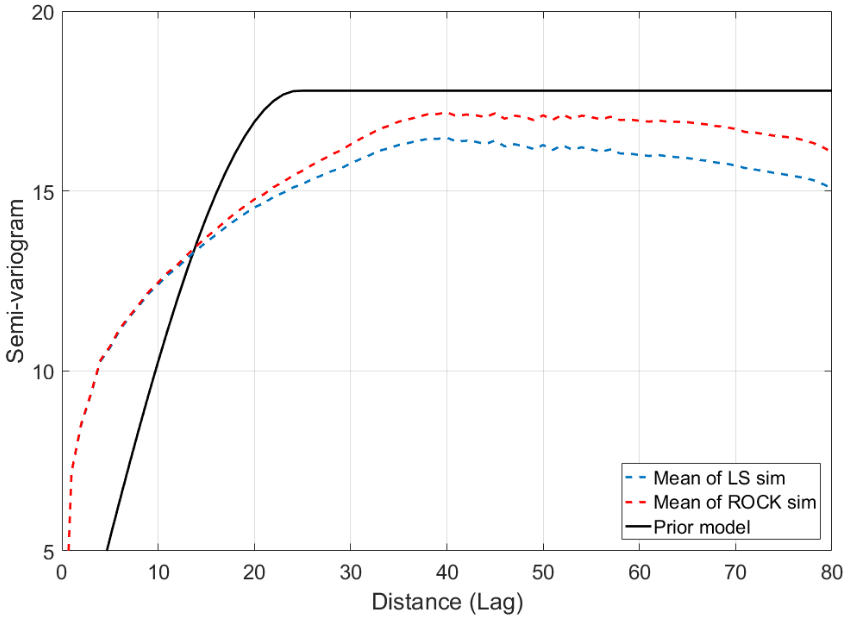
# Comparison

Semi-variograms (SGS-extended ROCK)



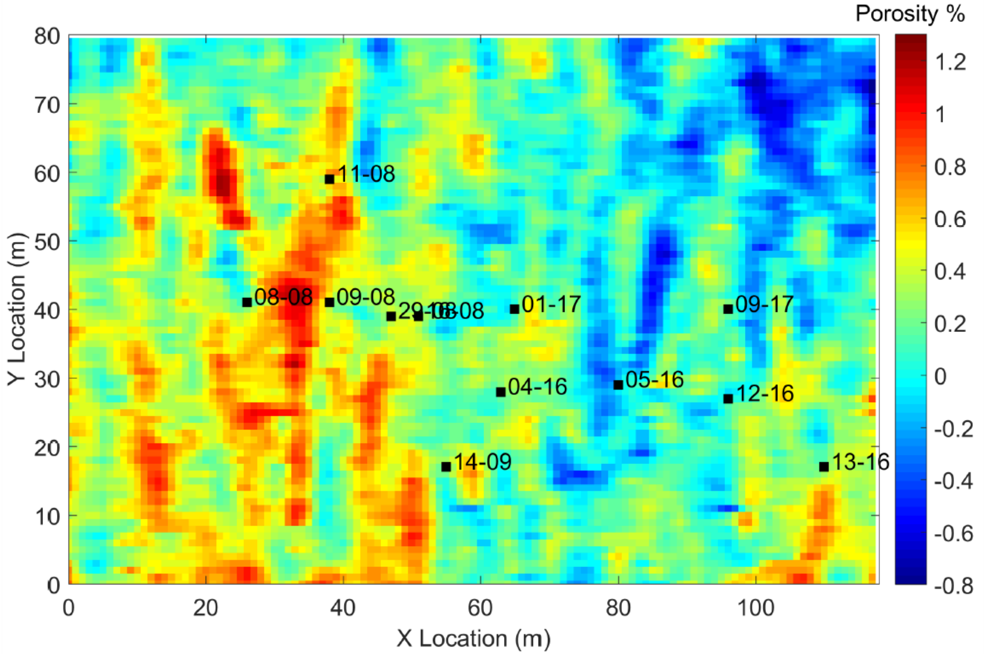

Semi-variograms (SGS-LS)



# Comparison

- The difference of mean maps from 100 realizations between SGS-ROCK and SGS-LS




- Goovaerts (1998)

$$z_{RCK}(X) - z_{SCK}(X) = \left[\boldsymbol{W}^{SCK}_{Primary} + \boldsymbol{W}^{SCK}_{Secondary}\right]\left[m_{RCK}(X) - m_{Stationary}\right]$$

- The difference between extended ROCK and Least Square (SCK)

$$z_{ROCK}(X) - z_{SCK}(X) = \left[\boldsymbol{W}^{SCK}_{Primary} + \boldsymbol{W}^{SCK}_{Secondary1} + \boldsymbol{W}^{SCK}_{Secondary2}\right]\left[m_{ROCK}(X) - m_{Stationary}\right]$$

# Conclusion

- We presented an approach to implement the sequential Gaussian simulation(SGS) by extended Rescaled Ordinary CoKriging, which allows more than one secondary variable to participate in the process.

- The comparison of semi-variograms demonstrates that the realizations created by SGS-extended ROCK honor better to the prior information.

- The difference of mean maps shows that the SGS-Least Square is limited due to the global mean.
  - If local mean is higher than global mean, values are underestimated  with SGS-LS.
  - If local mean is lower than global mean, values are overestimated  with SGS-LS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