SH Waves, Rays, and Full Waveform Inversion

- L.R. Lines, E.S. Krebes and P.F. Daley

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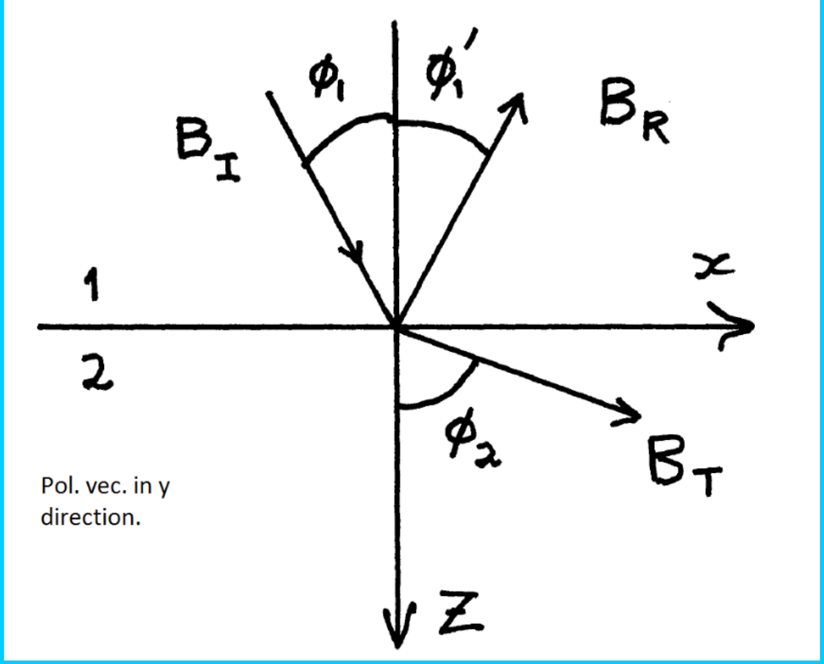
SH-Waves

SH-waves are shear waves with only a horizontal component normal to the propagation direction

Therefore, SH-waves are the least complicated elastic body waves in terms of mode conversions at boundaries.

What are the appropriate seismic modeling programs for utilizing SH-waves in full waveform seismic inversions?

Rays and Waves for incident, reflected and transmitted SH-waves from Krebes 2016 lecture "Computing Reflection Coefficients for Viscoelastic Media"



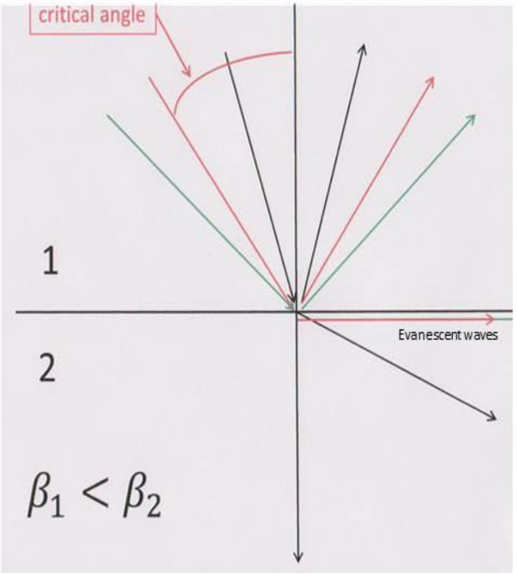
$$\frac{\sin\phi_1}{\beta_1} = \frac{\sin\phi_1'}{\beta_1} = \frac{\sin\phi_2}{\beta_2}$$

$$R_{SH} = \frac{A_r}{A_0} = \frac{\rho_1\beta_1\cos\phi_1 - \rho_2\beta_2\cos\phi_2}{\rho_1\beta_1\cos\phi_1 + \rho_2\beta_2\cos\phi_2}$$

$$\rho\frac{\partial^2 u}{\partial t^2} = \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

SH Reflection coefficients reparameterized

(Krebes, 2016)



$$R_{SH} = \frac{\rho_1 \beta_1 \cos\phi_1 - \rho_2 \beta_2 \cos\phi_2}{\rho_1 \beta_1 \cos\phi_1 + \rho_2 \beta_2 \cos\phi_2}$$

$$p = \frac{\sin\phi_n}{\beta_n}\,, \quad \eta_n = \frac{\cos\phi_n}{\beta_n}\,, \quad n = 1, 2$$

$$\frac{1}{{\beta_n}^2} = p^2 + {\eta_n}^2\,, \quad {\eta_n}^2 = \frac{1}{{\beta_n}^2} - p^2$$

$$\mu_n = \rho_n {\beta_n}^2\,, \quad \rho_n \beta_n \cos\phi_n = \mu_n \eta_n$$

$$R_{SH} = \frac{\mu_1 \eta_1 - \mu_2 \eta_2}{\mu_1 \eta_1 + \mu_2 \eta_2}$$

$$\eta_n = \sqrt{\frac{1}{{\beta_n}^2} - p^2}$$

Viscoelastic Media

$$\beta^2 = {\beta_0}^2\left(1 - \frac{i}{Q}\right) = \frac{\mu}{\rho}$$

$$\frac{1}{\beta^2} \approx \frac{1}{{\beta_0}^2}\left(1 + \frac{i}{Q}\right), \quad Q \gg 1$$

$$\beta \approx \beta_0\left(1 - \frac{i}{2Q}\right) = \sqrt{\frac{\mu}{\rho}}$$

$$\frac{1}{\beta} \approx \frac{1}{\beta_0}\left(1 + \frac{i}{2Q}\right)$$

$$p = \frac{\sin\phi}{\beta} \text{ is also complex}$$

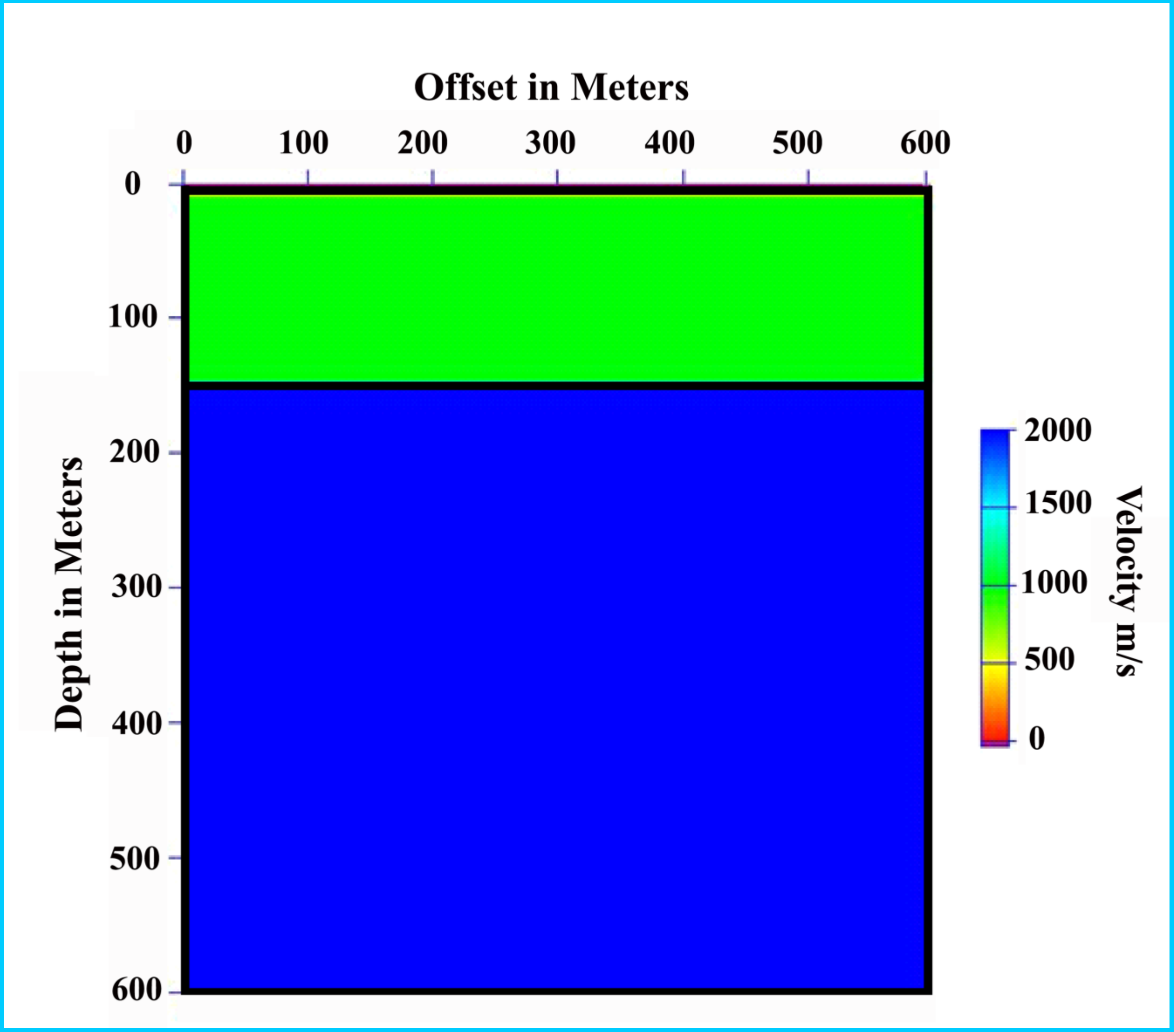
Reflection Coefficients from Geometrical Ray Theory

Geometrical ray theory is not at all accurate in the critical zone. There is a singularity at the critical angle and there are jump-discontinuities in η (Krebes and Daley, Geophys. J. Int., 2007)

In the near critical zone and post-critical zone we shall use finite-difference solutions to the wave equation as described by Boore (1970) for the elastic case and by Carcione (2007) for the viscoelastic case.

$$\rho \frac{\partial^2 v}{\partial t^2} = \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right]$$

SH-Wave Model

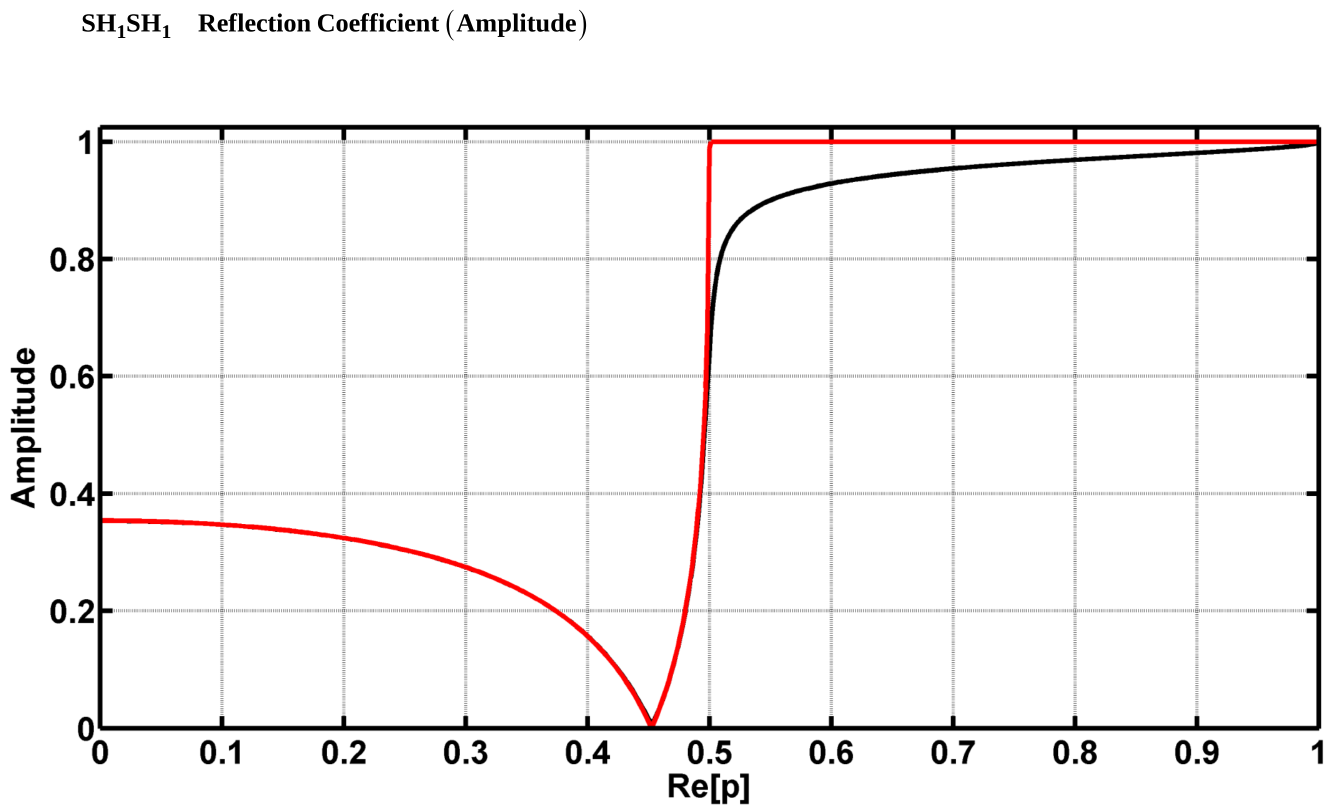


Layer number	Density (gm/cc)	S-wave velocity (m/s)	Q
1	2.1	1000	15
2	2.2	2000	20

Source-receiver geometry for modeling

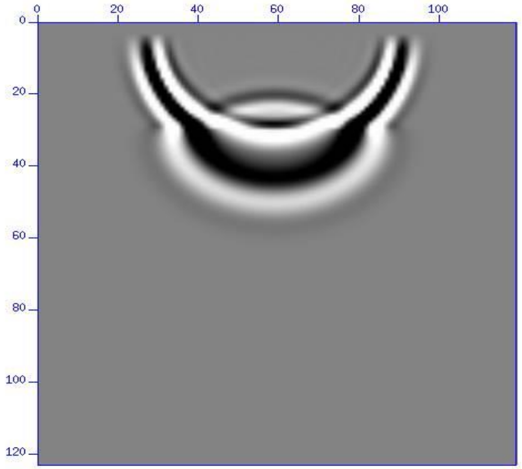
Source number	Source offset	Receiver locations	Trace number
1	95	0-595m	1-120
2	145	0-595m	121-240
3	195	0-595m	241-360
4	245	0-595m	361-480
5	295	0-595m	481-600
6	345	0-595m	601-720
7	395	0-595m	721--840
8	445	0-595m	841-960
9	495	0-595m	961-1080

SH-wave reflection coefficient as a function of ray parameter . Red curve = elastic wave reflection coefficient; black curve = viscoelastic reflection coefficient.



2-D finite-difference code for viscoelastic SH-wave propagation from Carcione (2007)

$$\rho \frac{\partial v_2}{\partial t} = \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_3} + f_2$$

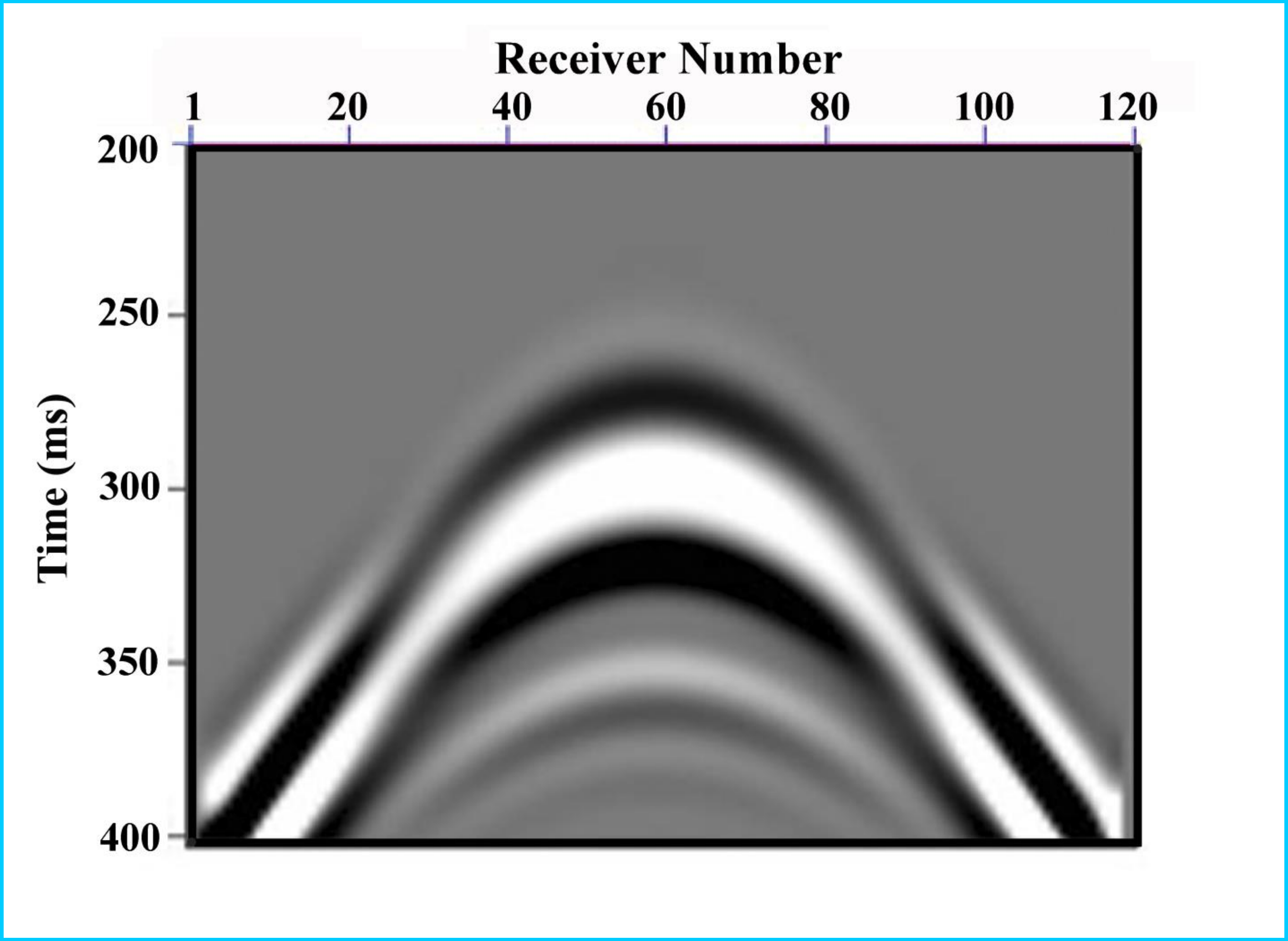


$$v_2 = \frac{\partial u_2}{\partial t}$$

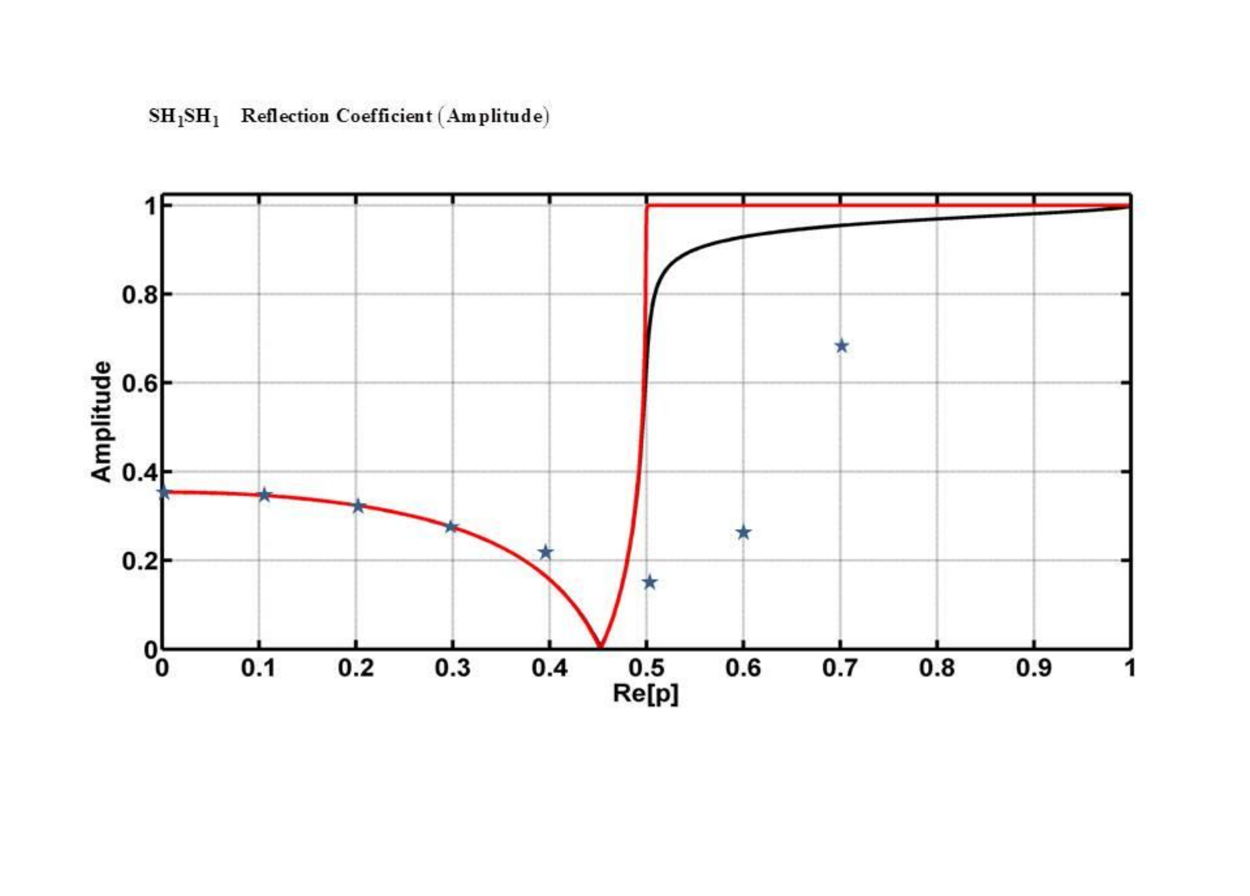
$$\frac{\partial \sigma_{23}}{\partial t} = \mu \frac{\partial v_2}{\partial x_3}$$

$$\frac{\partial \sigma_{12}}{\partial t} = \mu \frac{\partial v_2}{\partial x_1}$$

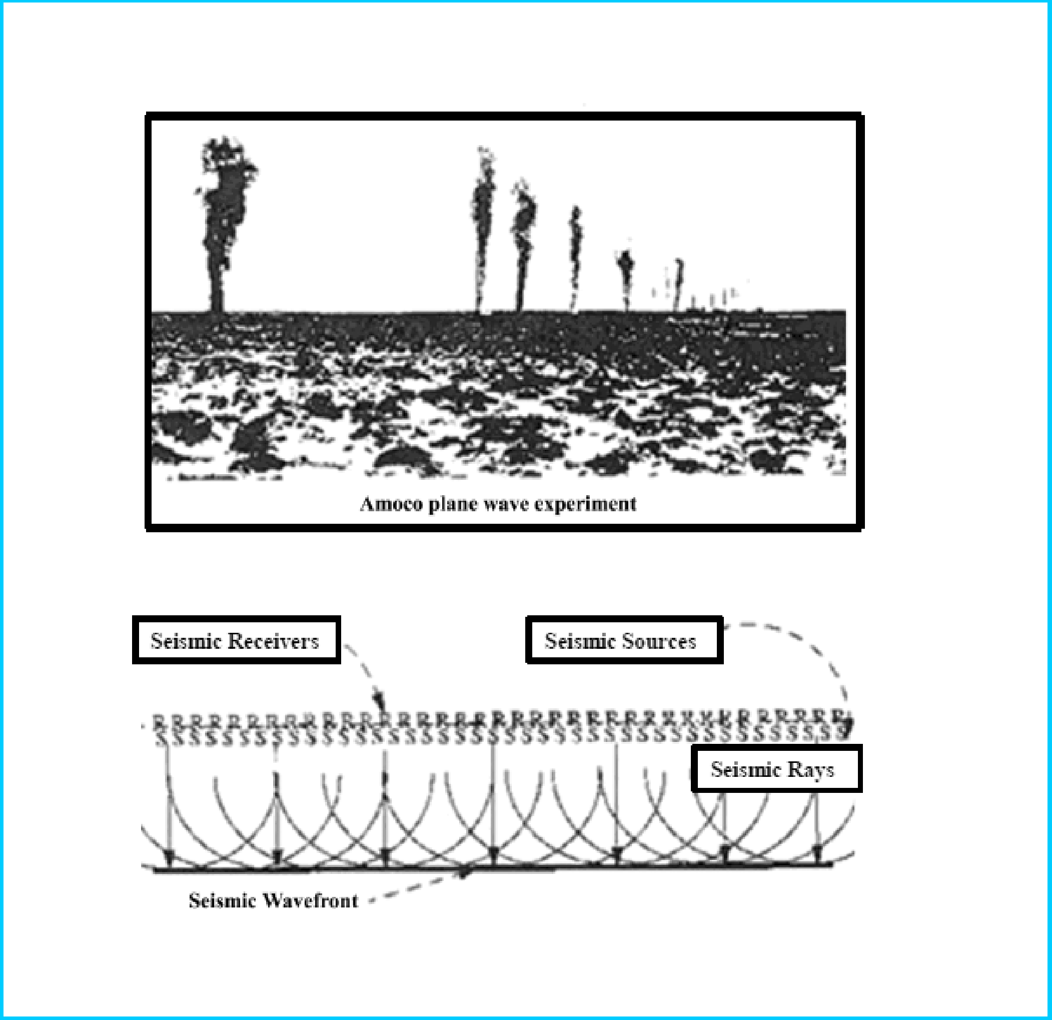
SH-Wave Reflection Response for source near centre of model



Comparison SH-wave reflection coefficients from ray-reflectivity with amplitudes from FD synthetic seismogram as a function of ray parameter . Red curve = elastic wave reflection coefficient; black curve = viscoelastic reflection coefficient. Blue stars were amplitudes from FD synthetic seismograms -obtained by taking maxima of trace envelopes for the wave equation seismograms. Excellent agreement for precritical reflections. (Critical angle at p=0.5)

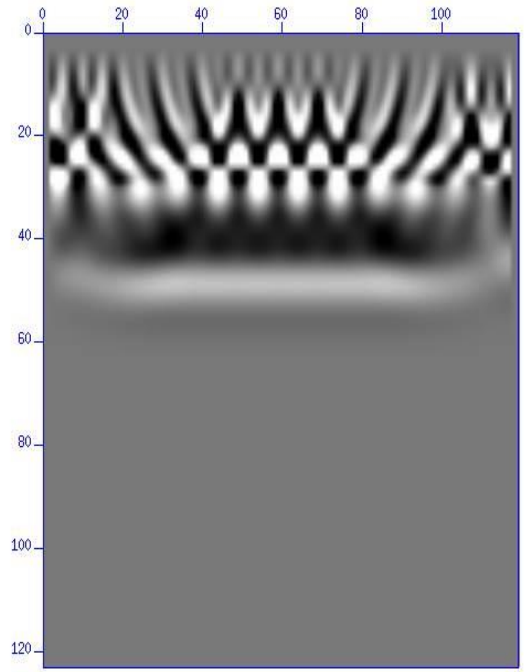


Relation of Plane Waves to Cylindrical Waves; figure from Whitmore (1995 PhD thesis). Plane wave constructed from superposition of wavefronts. Plane wave function in terms of Hankel/Bessel functions (Sommerfeld, 1964, Pilante, 1979).



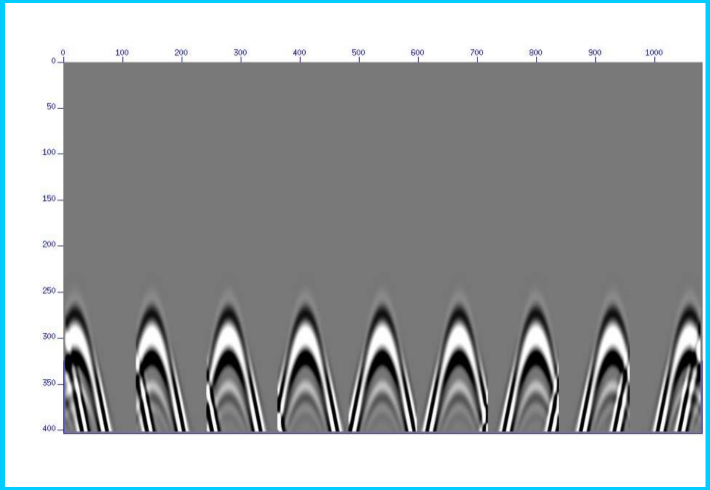
$$e^{i\mathbf{k}\bullet\mathbf{r}} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\theta}$$

Numerical Calculations from FD seismograms – summing to approximate plane waves

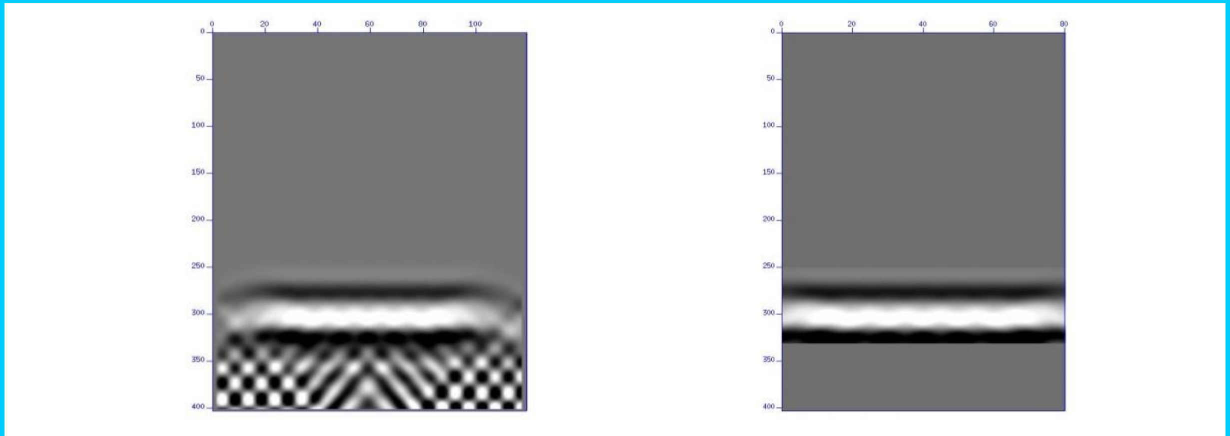


Wavefronts snapshots generated by initiating 9 shots for our SH-wave model

Numerical Calculations from FD seismograms – summing to approximate plane waves

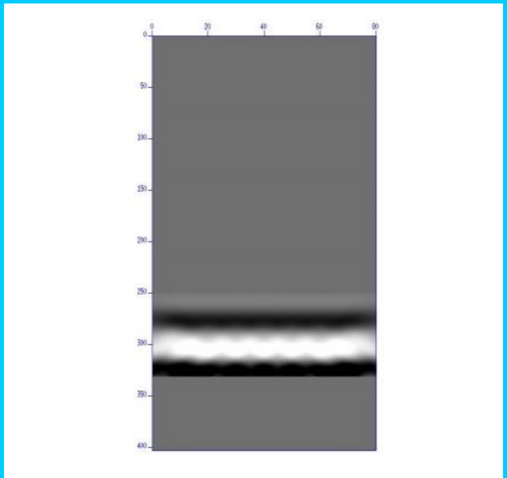


Shot records obtained after filtering off direct arrivals

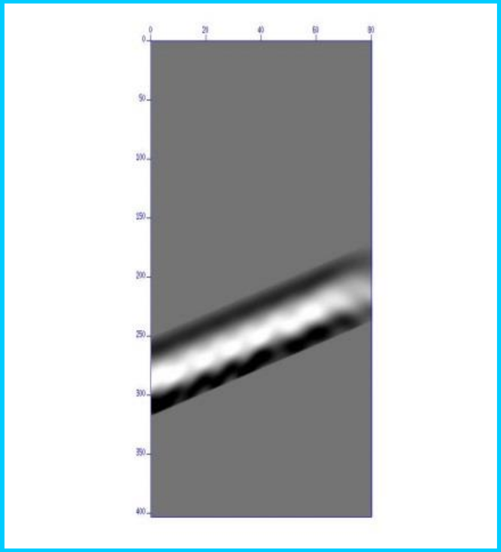


Left: Summation of shot records. Right: Windowed version

Plane waves at different angles through sum and delay

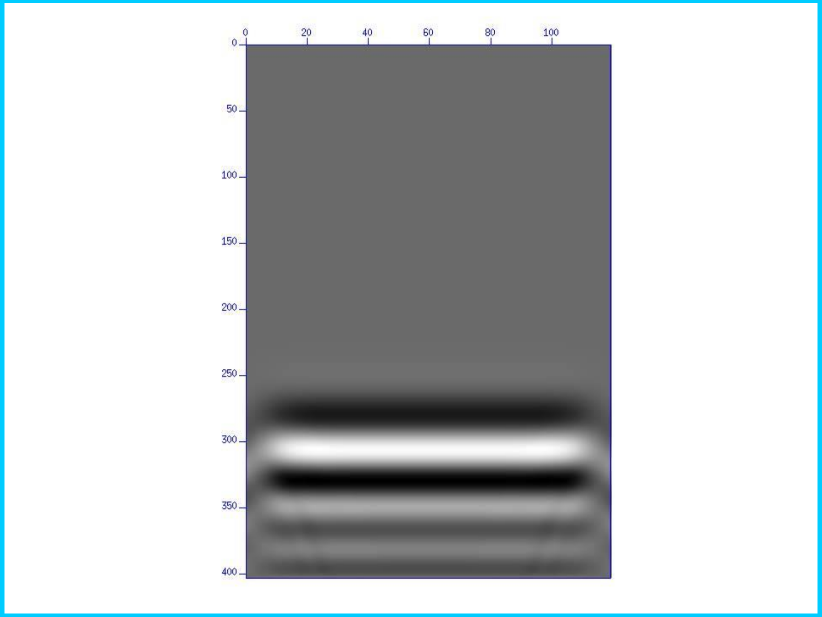


Summation to simulate p=0



Sum and delay to simulate p=0.1

Quick and better simulation of normal incidence plane wave through use of exploding reflector model



Simulation of p=0 with source at every node point

Question: Which model is best for full waveform inversion?

- For small offsets (ray parameters) in the precritical range, ray reflectivity approximated by FD seismograms. In far field at small offset, a cylindrical wavefront is closely approximated by plane waves
- In critical and postcritical region, use FD modeling
- For 3-D models with relatively flat reflectors, 2.5 hybrid modeling may be appropriate.

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Acknowledgements

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