A coupled DAS shaped-fibre and 3D elastic finite difference wave model

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Talk Outline

1. Motivation
2. Theoretical Basis for DAS
3. Geometrical model of DAS fibres
4. 3D velocity-stress method for elastic wave propagation
5. Bridging the gap
6. Examples
7. Future Work

Motivation

- All inversion algorithms require forward model.
- Distributed acoustic sensing could help remedy some of the current issues facing FWI.
  - Low cost monitoring
  - Has potential to supply the lower frequencies crucial to FWI.
  - It is hoped that improved spatial sampling of DAS fibres will also aid in FWI frameworks.
- A forward model will be required to optimize DAS fibre shapes for elastic wave mode discrimination.
- Potential for synthetic modeling of time lapse seismic with DAS fibres

Theoretical Basis for DAS

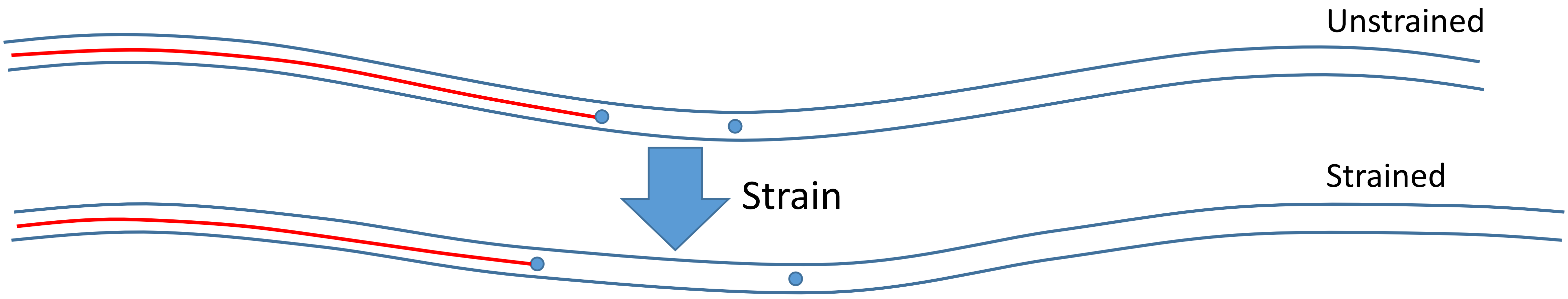
- When light interacts with matter, it scatters in a predictable manner dependent on particle size, based on the scale factor:

$$x = \frac{2\pi r}{\lambda}$$

- In the case $x \ll 1$, light scatters according to Rayleigh scattering, and light is backscattered with the same phase.

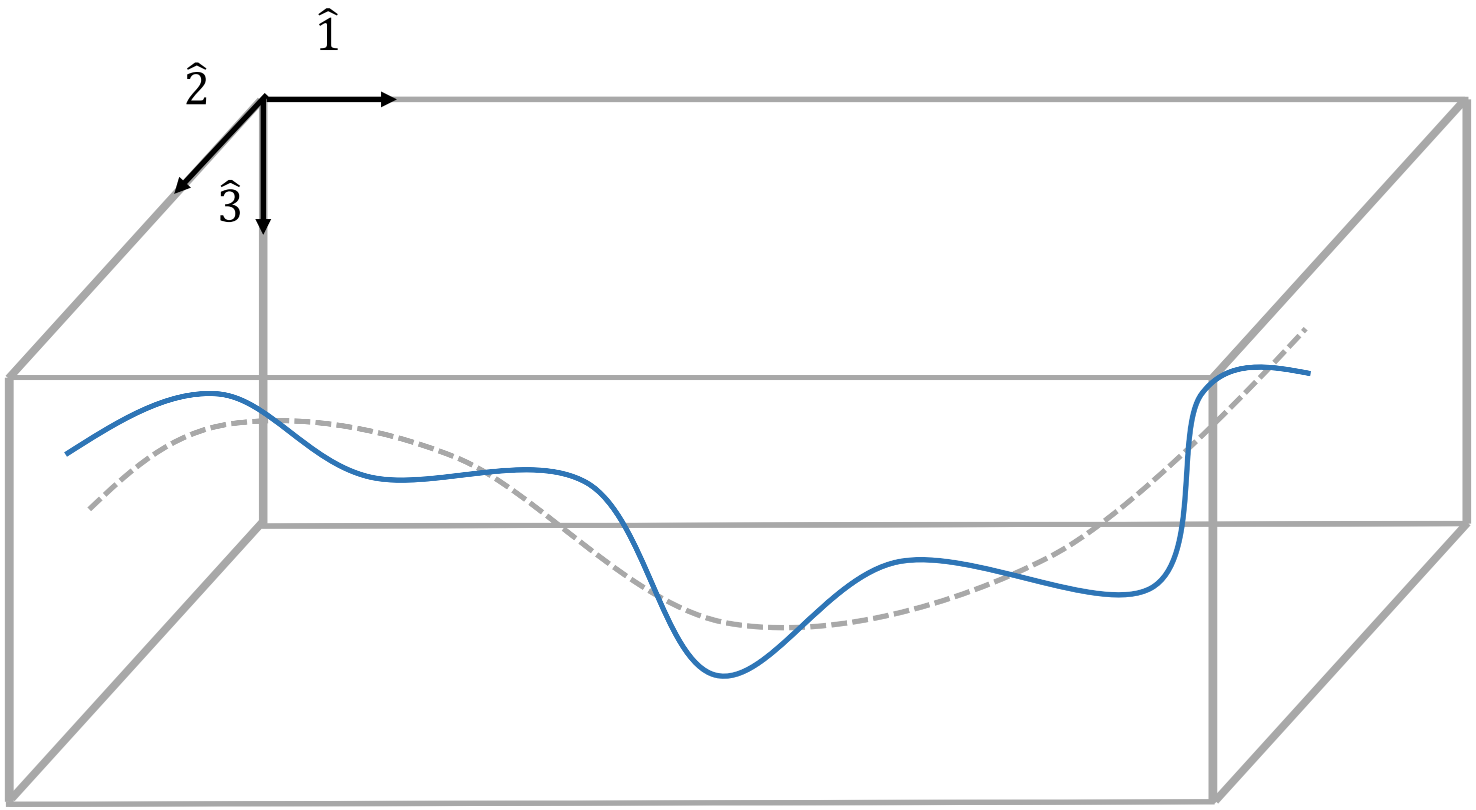
- DAS systems operate based on Rayleigh backscattered light.

Theoretical Basis for DAS

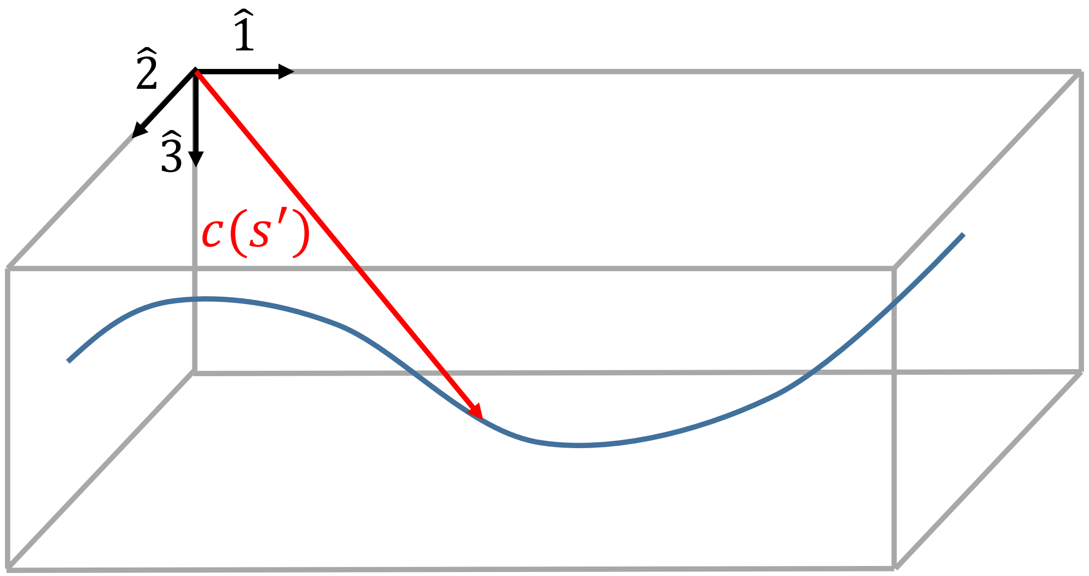


- Seismic strain causes stretching an squeezing of fibre, along its tangent, changing the distance between Rayleigh scattering centers.
- This changes the optical path length of the light pulse, altering the light intensity originating from a portion of the fibre.

Cable-Fibre System



Cable geometry – Cable Axis



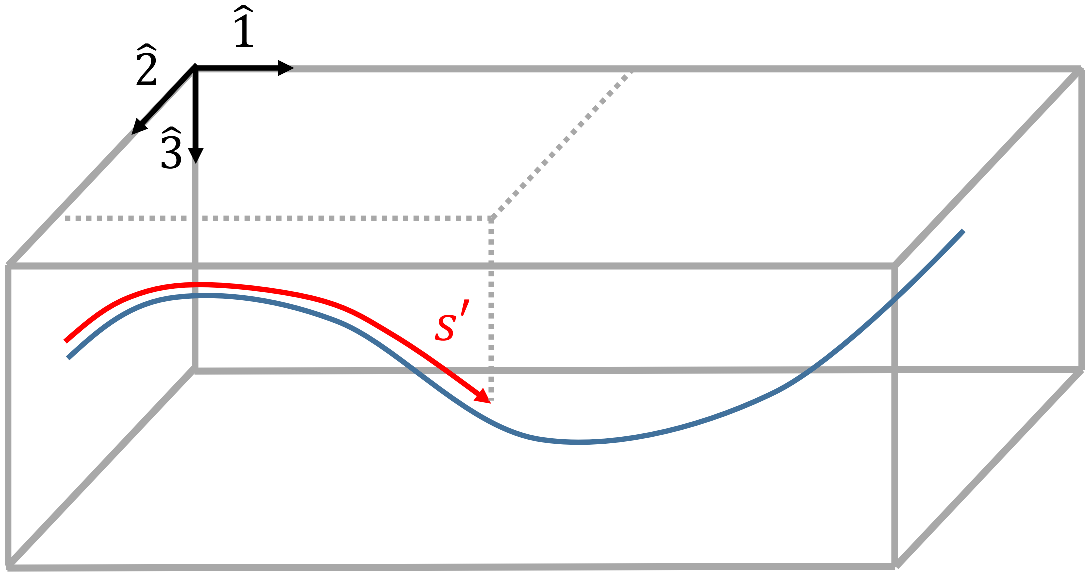
Let **c** be a vector pointing from the origin to a point at a distance s' along the cable, such that,

$$\boldsymbol{c}(\boldsymbol{s}') = [\boldsymbol{c}_1(\boldsymbol{s}'), \boldsymbol{c}_2(\boldsymbol{s}'), \boldsymbol{c}_3(\boldsymbol{s}')]^{\boldsymbol{T}}$$

If we choose to instead parameterize the cable in Cartesian coordinates then **c** may take the form

$$\boldsymbol{c}(\boldsymbol{x}) = [\boldsymbol{x}, \boldsymbol{c}_2(\boldsymbol{x}), \boldsymbol{c}_3(\boldsymbol{x})]^{\boldsymbol{T}}$$

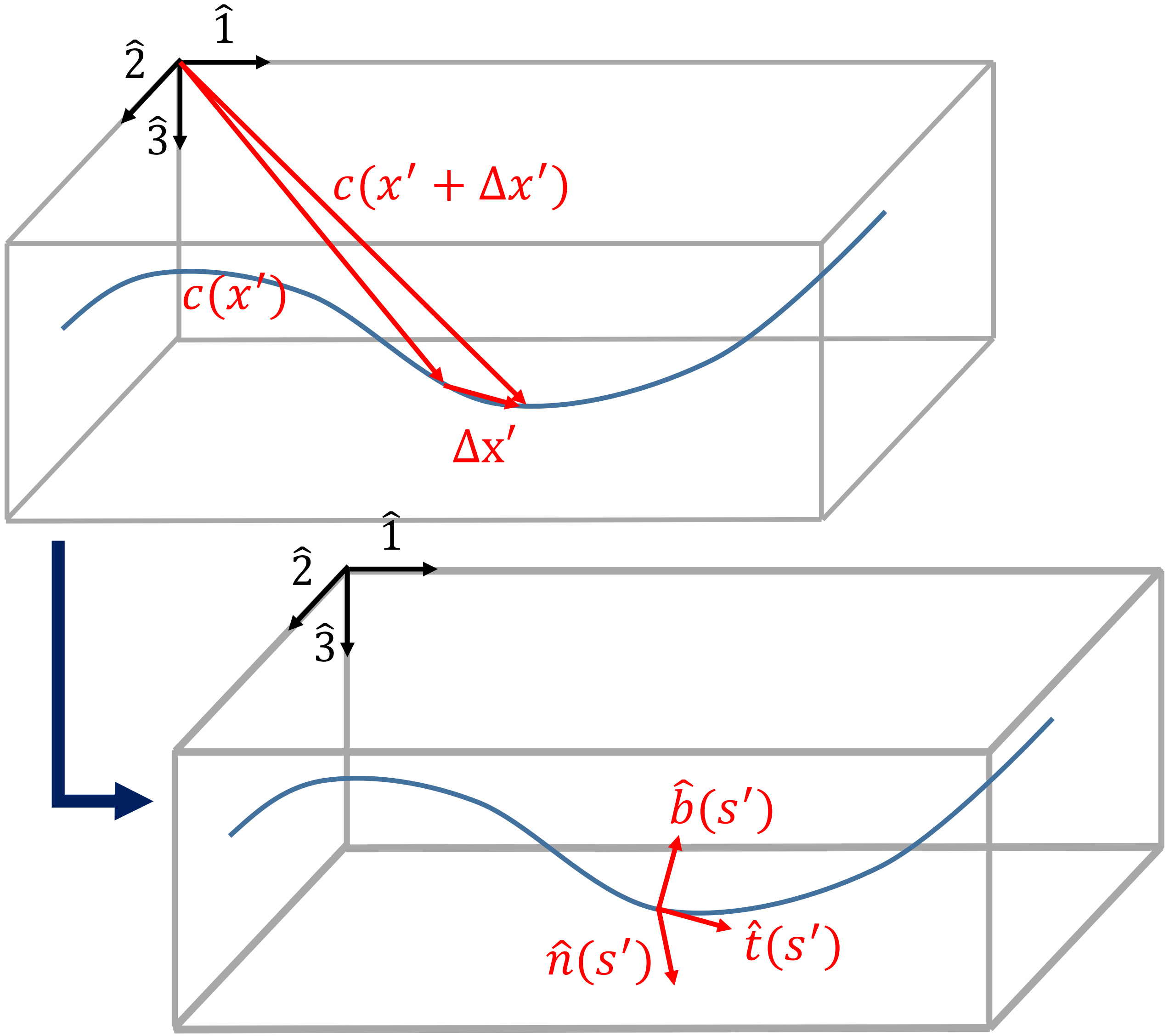
Cable geometry – Arc length



The arc length is the sum of the length of each derivative component along the cable,

$$s'(x) = \int_0^x dx' \left[\frac{d\mathbf{c}(x')}{dx'} \cdot \frac{d\mathbf{c}(x')}{dx'}\right]^{1/2}$$

Cable geometry – Tangential Coordinate System

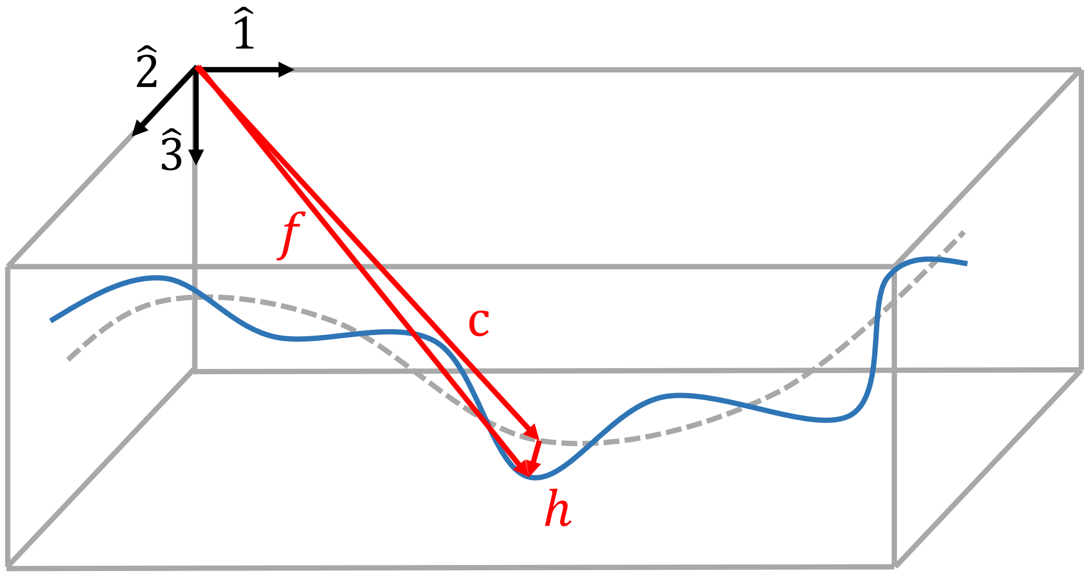


We may now develop a coordinate system that varies along the cable, where

$$\hat{\boldsymbol{t}}(s') = \frac{d\boldsymbol{c}}{ds'}$$

describes the tangent of the cable at every point.

Fibre Geometry – Fibre position



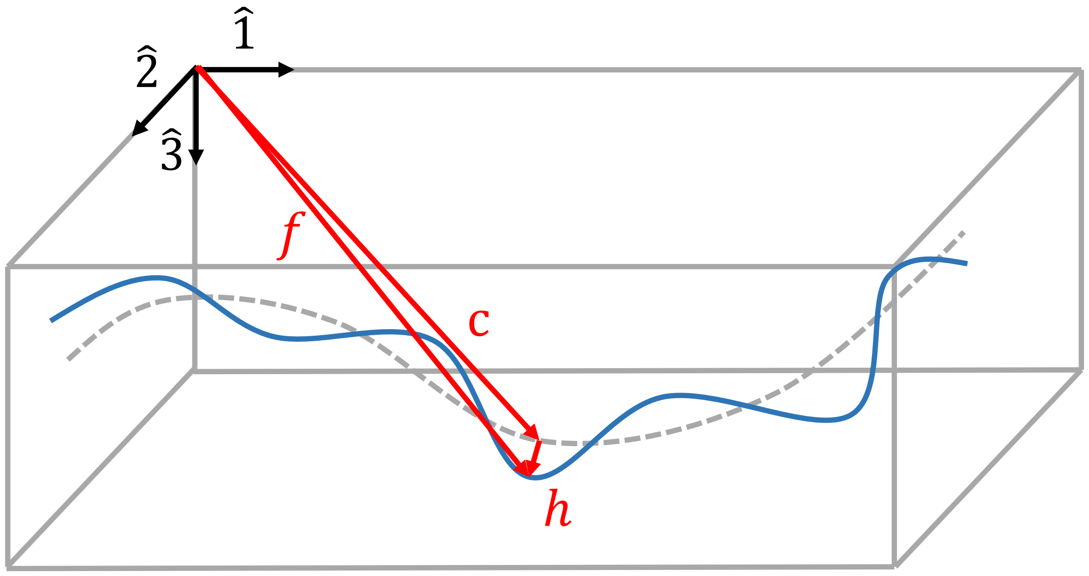
It is important to keep track of the fibre positions, in order to know where a particular measurement is being made:

$$\boldsymbol{f} = \boldsymbol{c} + \boldsymbol{h} = \begin{bmatrix} c_1(s') \\ c_2(s') \\ c_3(s') \end{bmatrix} + \begin{bmatrix} h_1(s') \\ h_2(s') \\ h_3(s') \end{bmatrix}$$

For the special case of a helically wound fibre, with radius r and velocity v, then in the $\{\hat{\boldsymbol{t}}, \hat{\boldsymbol{n}}, \hat{\boldsymbol{b}}\}$ frame, we have

$$\boldsymbol{h} = \begin{bmatrix} 0 \\ r\cos(s'/v) \\ r\sin(s'/v) \end{bmatrix}$$

Fibre Geometry – Fibre position

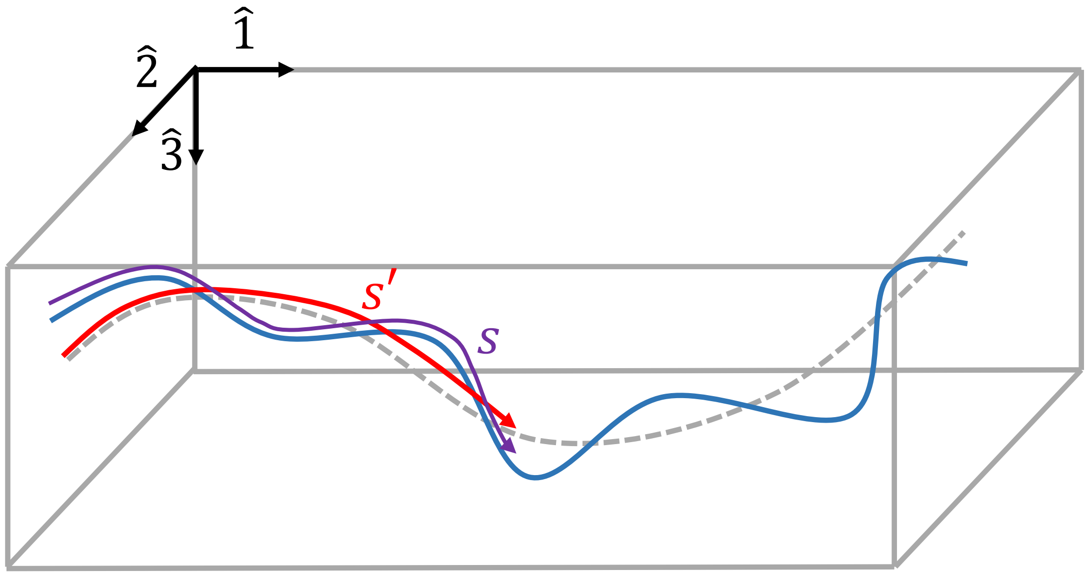


Rotating the helix back into the Cartesian frame,

$$\boldsymbol{f} = \boldsymbol{c} + \boldsymbol{R}\boldsymbol{h}$$

where R is rotation matrix taking the helix from the frame $\{\hat{\boldsymbol{t}}, \hat{\boldsymbol{b}}, \hat{\boldsymbol{n}}\} \rightarrow \{\hat{\boldsymbol{1}}, \hat{\boldsymbol{2}}, \hat{\boldsymbol{3}}\}$

Fibre Geometry – Arc length



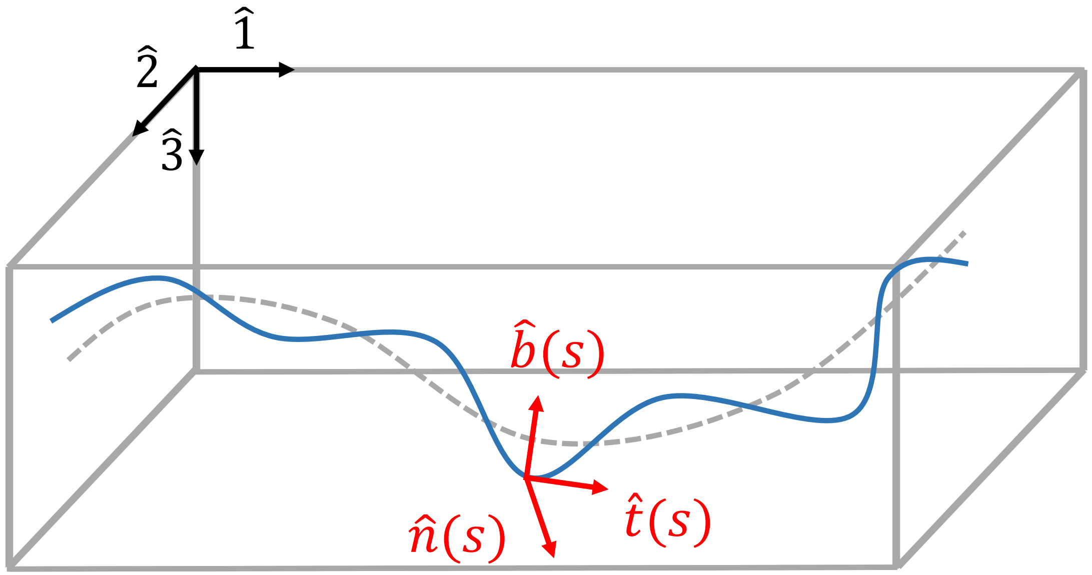
Knowing the arc length of the total cable-fibre system $\boldsymbol{s}$ is crucial to the model. Following the previous formulaic development for the cable arc length.

$$s = \left[1 + \frac{r^2}{v^2}\right]^{1/2} s'$$

The following re-parameterization allows knowledge of the fibre position as a function of total arc length,

$$s' = \left[1 + \frac{r^2}{v^2}\right]^{-1/2} s$$

Fibre Geometry – Tangents



Finally, the most important quantity in any DAS geometric model, the fibre tangent is,

$$\hat{\boldsymbol{t}}(s) = \frac{d\boldsymbol{f}}{ds}$$

It is along these tangents that the cable-fibre system senses strain.

Velocity-stress finite difference

Velocity-stress finite difference simulations rely on computations of the particle velocity, and stress to propagate the wavefield.

1. **Elastodynamic equation of motion (EOM)**

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \nabla \cdot \sigma + f_i = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} + f_i$$

2. **Hooke's Law (stress-strain relation)**

$$\sigma_{ij} = C_{ijkl} e_{kl}$$

3. **Strain tensor**

$$e_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)$$

Velocity-stress finite difference in one dimension

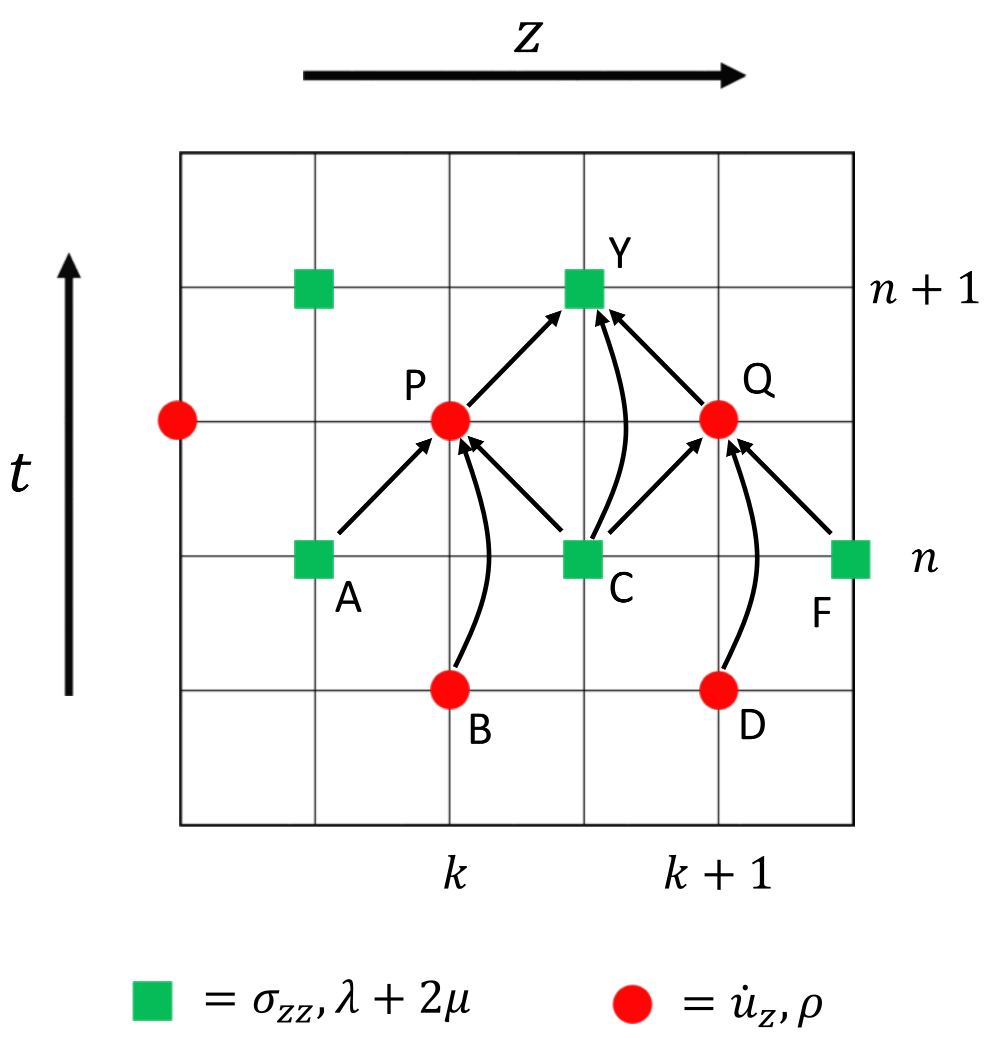
Consider a p-wave propagating in the vertical direction, then only the vertical component of displacement u_z, and the normal stress σ_{zz} are nonzero.

$$\rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \sigma_{zz}}{\partial z} \qquad \sigma_{zz} = (\lambda + 2\mu) e_{zz}$$

Invoking the formula for the strain tensor, taking the derivative of the stress-strain relation with respect to time, and letting the particle velocity be $\dot{u} = \frac{\partial u}{\partial t}$.

$$\rho \frac{\partial \dot{u}_z}{\partial t} = \frac{\partial \sigma_{zz}}{\partial z} \qquad \frac{\partial \sigma_{zz}}{\partial t} = (\lambda + 2\mu) \frac{\partial \dot{u}_z}{\partial z}$$

Velocity-stress finite difference in one dimension



The velocity-stress finite difference formulation is unique because it solves the system of equations on a staggered grid, improving accuracy and stability,

$$\rho \frac{\partial \dot{u}_z}{\partial t} = \frac{\partial \sigma_{zz}}{\partial z}$$

$$\frac{\partial \sigma_{zz}}{\partial t} = (\lambda + 2\mu) \frac{\partial \dot{u}_z}{\partial z}$$

Velocity-stress finite difference in three dimensions

In the more general, two and three dimensional cases, the number of nonzero components grows,

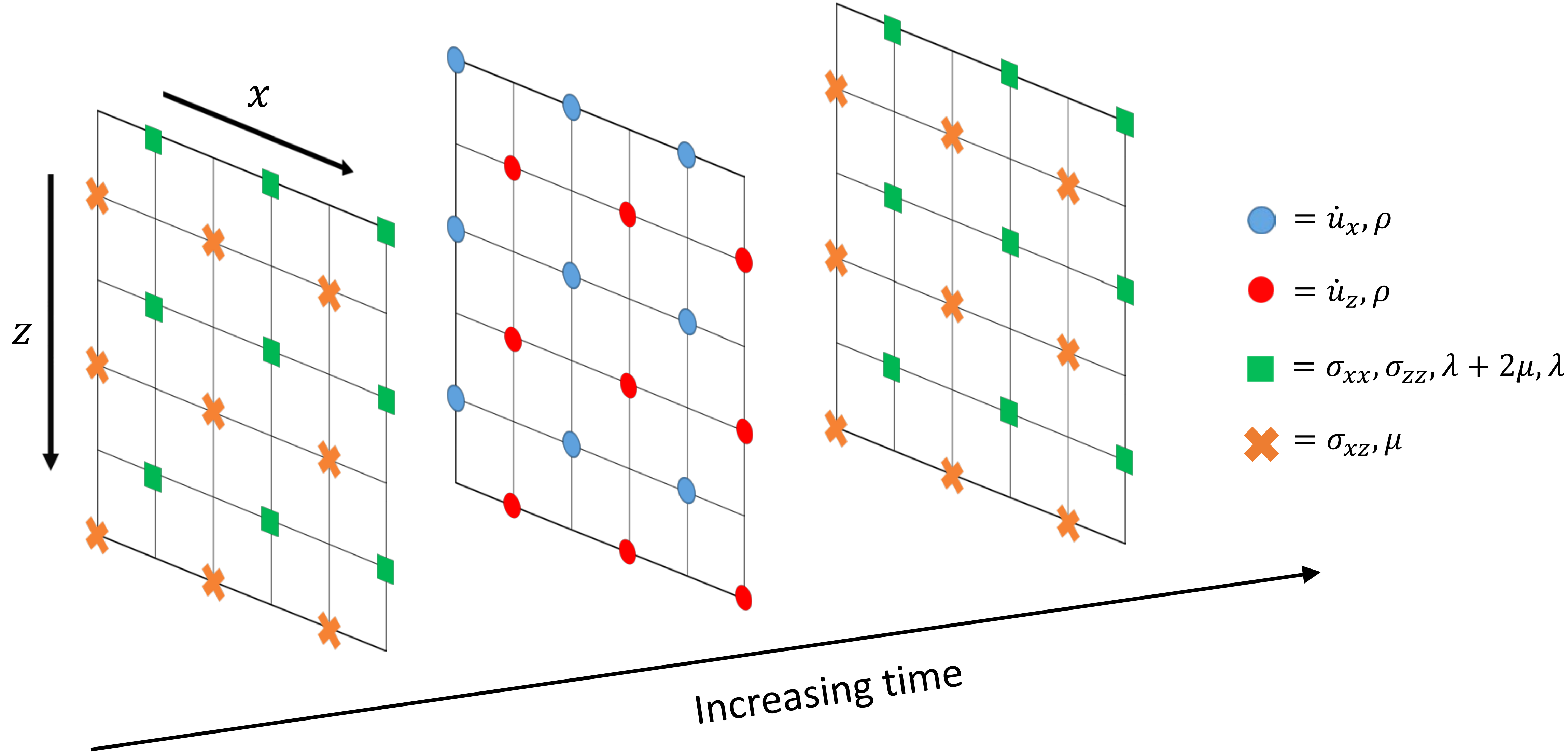
Velocity

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_j \frac{\partial \sigma_{ij}}{\partial x_j}$$

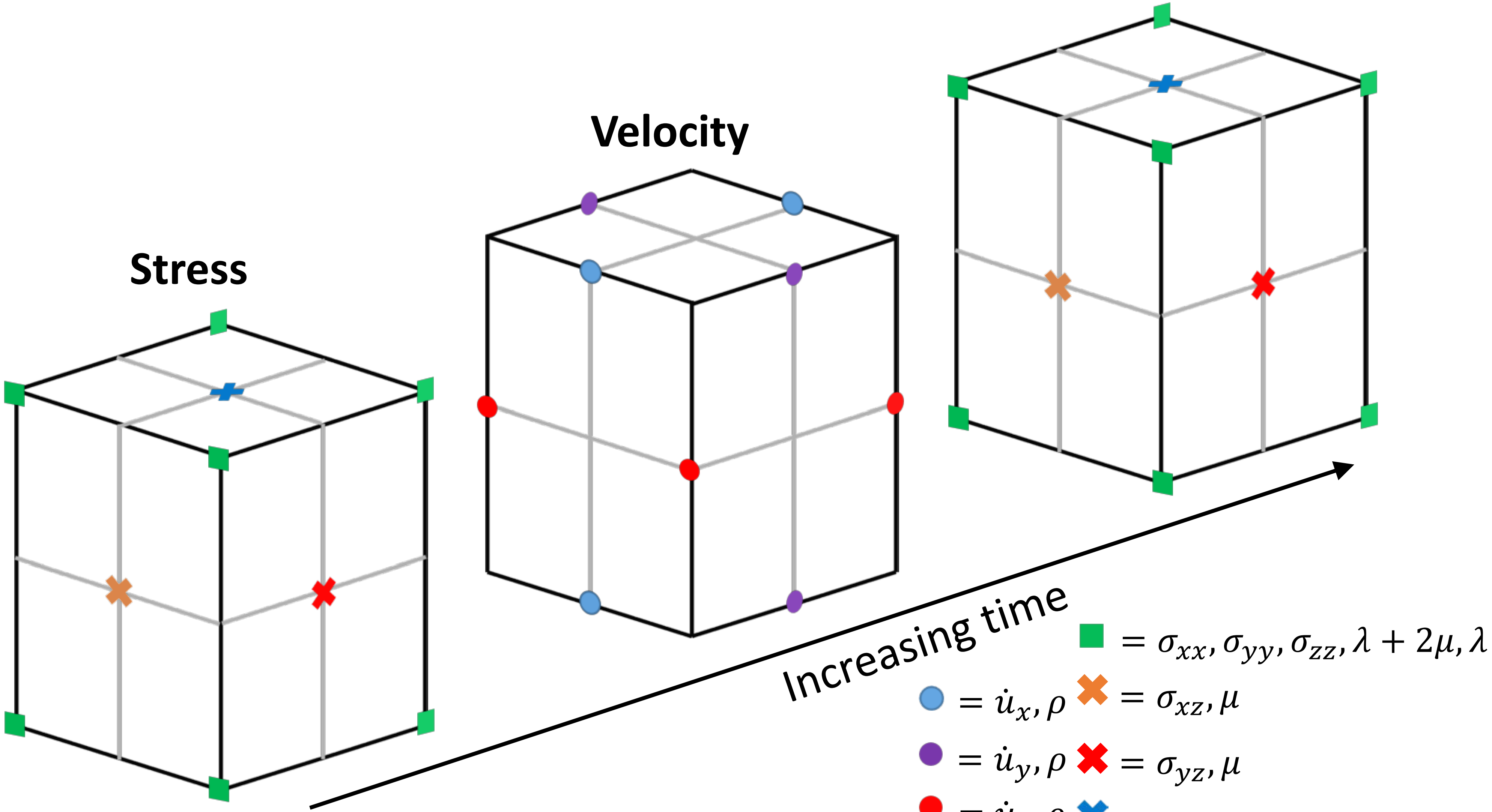
Stress

$$\frac{\partial \sigma_{ij}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t}\left[C_{ijkl}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)\right]$$

Velocity-stress finite difference in two dimensions



Velocity-stress finite difference in three dimensions



The strain rate tensor

DAS fibres measure strain along the axial tangent. Our first goal is then to quantify the strain at every point, from values computed during the velocity-stress propagation.

$$\frac{\partial e_{kl}}{\partial t} = \frac{1}{2}\left(\frac{\partial^2 u_k}{\partial x_l \partial t} + \frac{\partial^2 u_l}{\partial x_k \partial t}\right) = \frac{1}{2}\left(\frac{\partial \dot{u}_k}{\partial x_l} + \frac{\partial \dot{u}_l}{\partial x_k}\right)$$

$$\dot{e} = \frac{1}{2}\begin{bmatrix} 2\dot{u}_{x,x} & \dot{u}_{x,y} + \dot{u}_{y,x} & \dot{u}_{x,z} + \dot{u}_{z,x} \\ \dot{u}_{x,y} + \dot{u}_{y,x} & 2\dot{u}_{y,y} & \dot{u}_{y,z} + \dot{u}_{z,y} \\ \dot{u}_{x,z} + \dot{u}_{z,x} & \dot{u}_{y,z} + \dot{u}_{z,y} & 2\dot{u}_{z,z} \end{bmatrix}$$

Projection onto the fibre

We must now rotate the strain rate in Cartesian coordinates, onto the fibre tangent,

$$\dot{e}_{tnb} = \begin{bmatrix} \dot{e}_{tt} & \dot{e}_{tn} & \dot{e}_{tb} \\ \dot{e}_{nt} & \dot{e}_{nn} & \dot{e}_{nb} \\ \dot{e}_{bt} & \dot{e}_{bn} & \dot{e}_{bb} \end{bmatrix} = R\dot{e}_{xyz}R^T$$

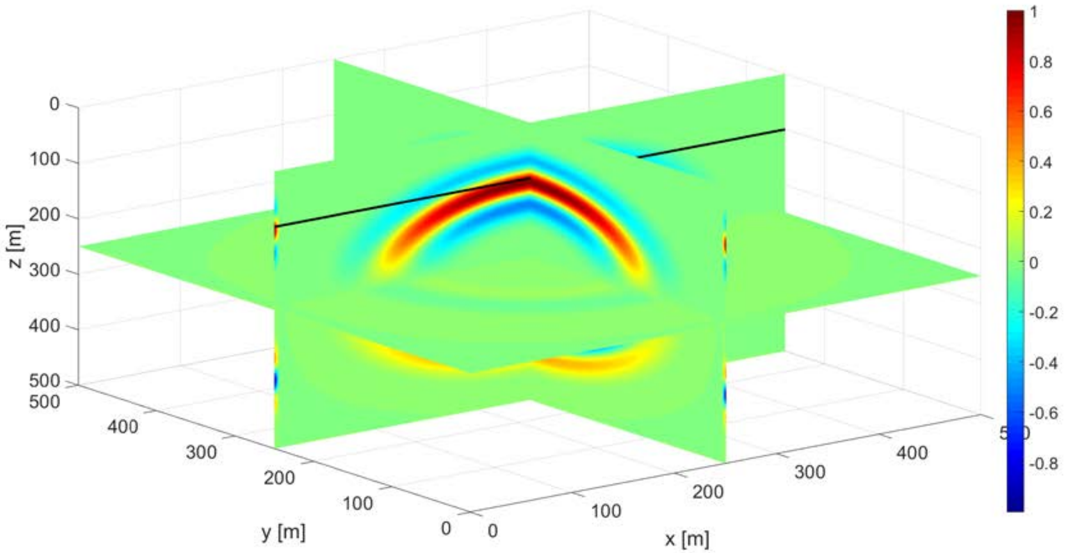
with ***R*** being the rotation matrix

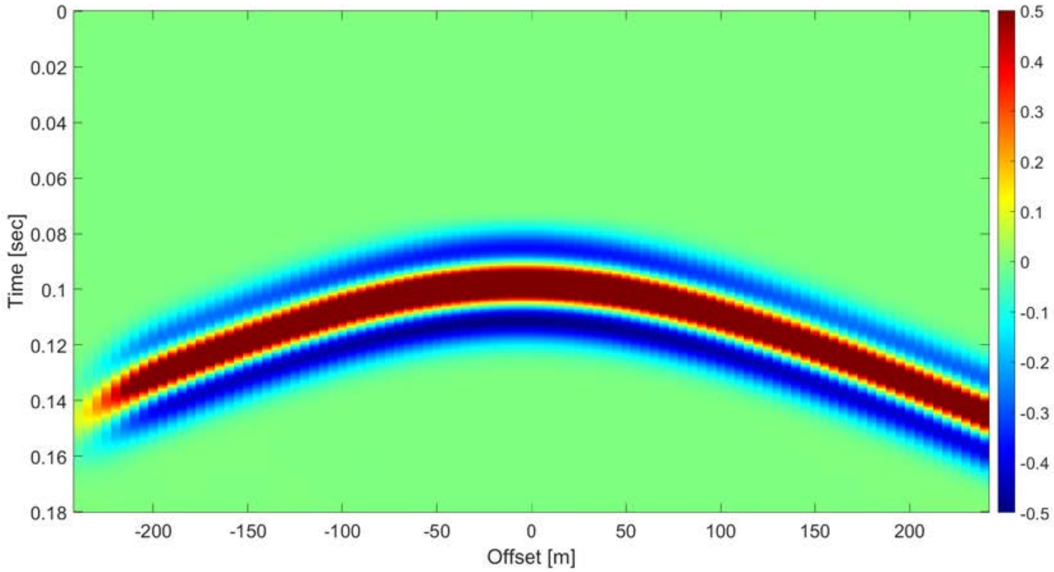
$$R = \begin{bmatrix} \hat{\boldsymbol{t}} \cdot \hat{1} & \hat{\boldsymbol{t}} \cdot \hat{2} & \hat{\boldsymbol{t}} \cdot \hat{3} \\ \hat{\boldsymbol{n}} \cdot \hat{1} & \hat{\boldsymbol{n}} \cdot \hat{2} & \hat{\boldsymbol{n}} \cdot \hat{3} \\ \hat{\boldsymbol{b}} \cdot \hat{1} & \hat{\boldsymbol{b}} \cdot \hat{2} & \hat{\boldsymbol{b}} \cdot \hat{3} \end{bmatrix}$$

The only measured component of fibre strain that is nonzero is $\dot{e}_{tt}$ resulting in,

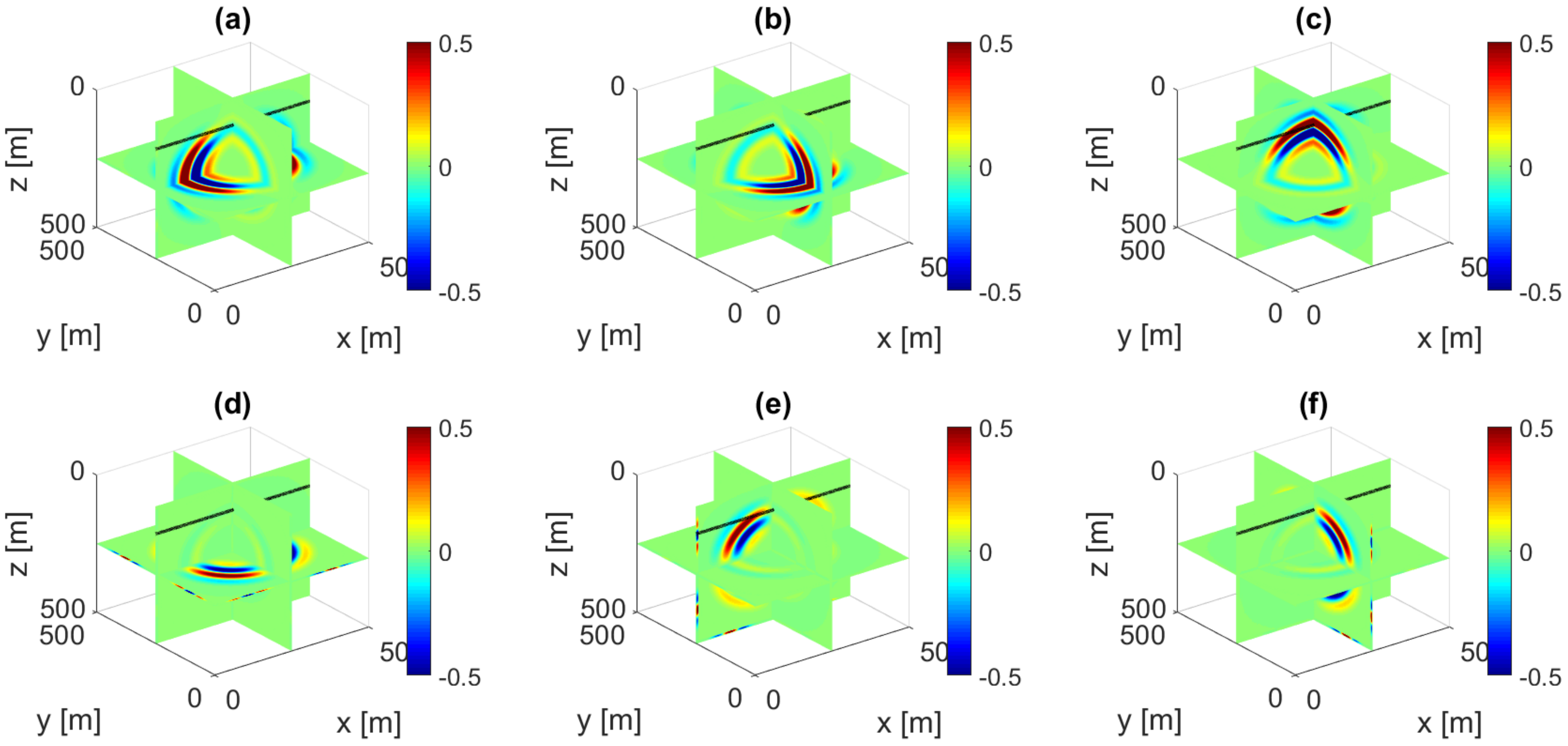
$$\dot{e}_{tt} = \left[R\dot{e}_{xyz}R^T\right]_{11}$$

Vertical velocity on 1-C geophones



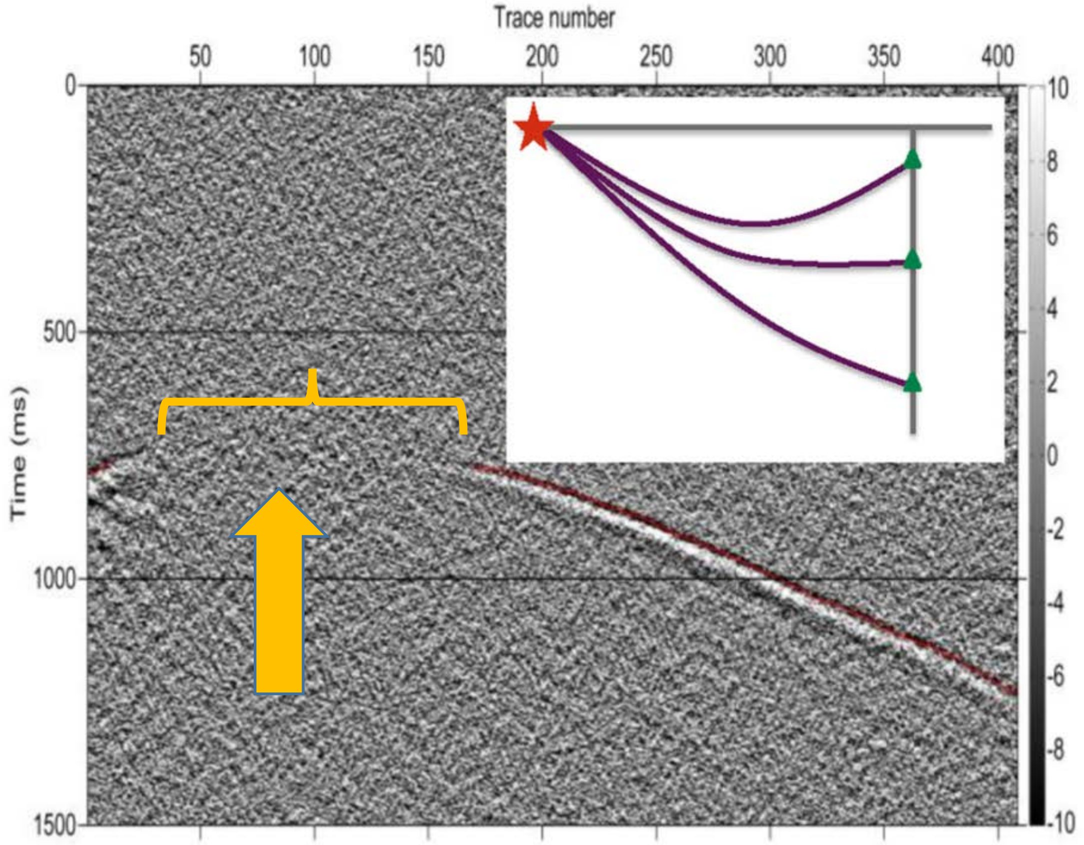


Strain tensor components required for DAS

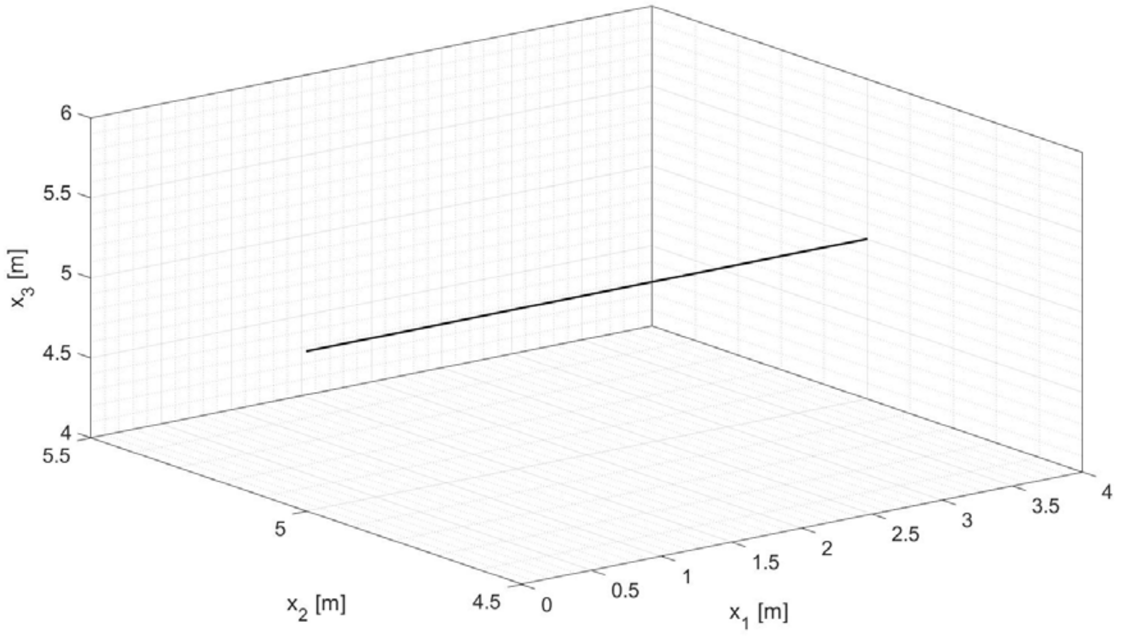


(a) e_{xx}, (b) e_{yy}, (c) e_{zz}
(d) e_{xy}, (d) e_{xz}, (e) e_{yz}

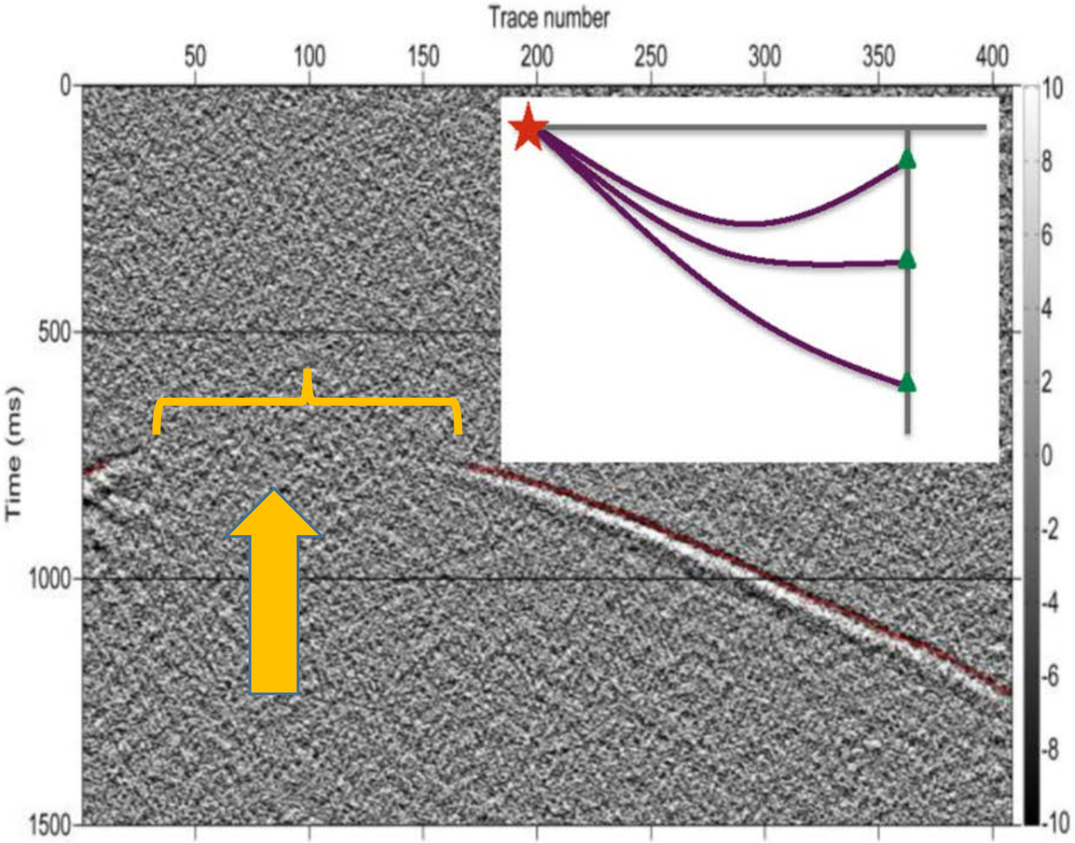
Response of a straight fibre



Modified from Mateeva et al., 2014



Response of a straight fibre



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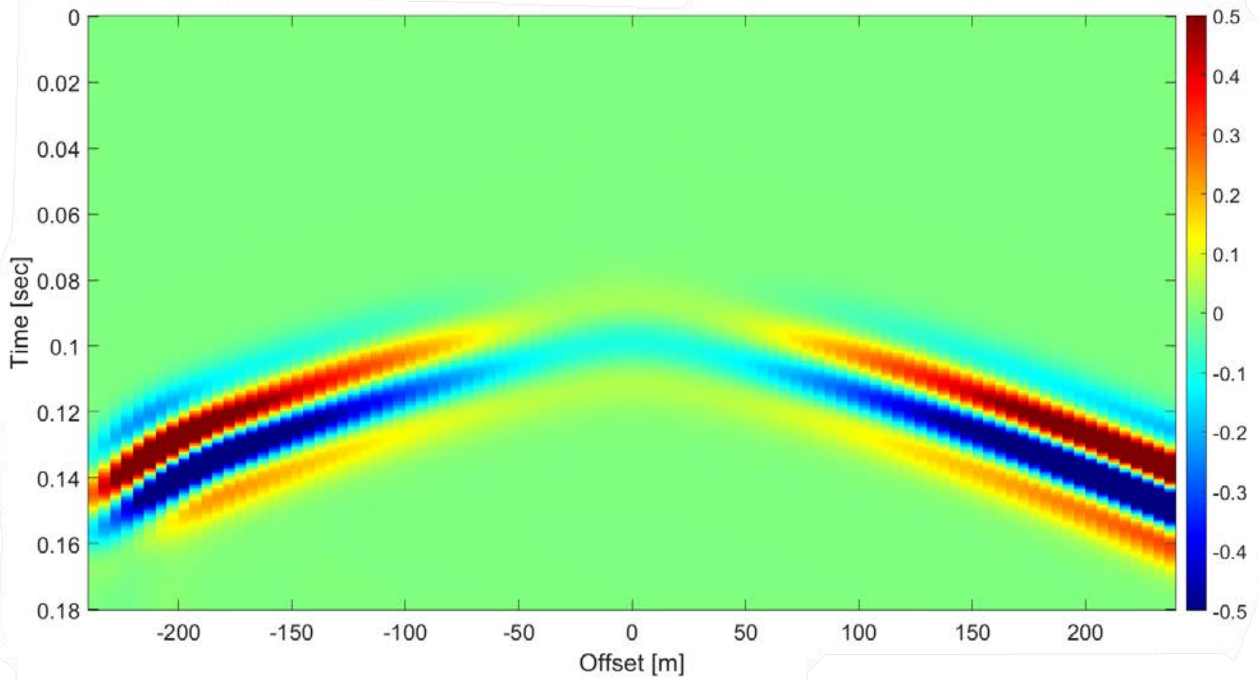
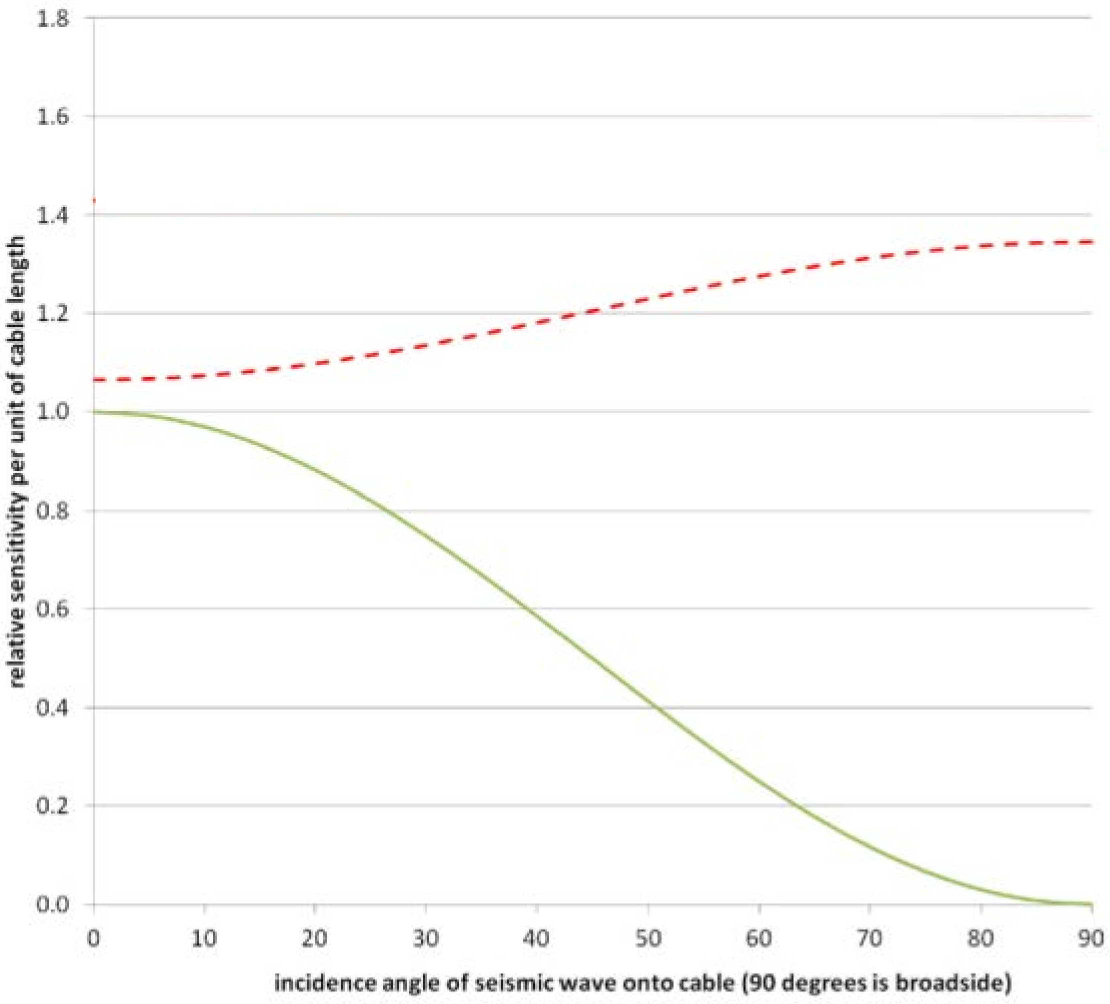
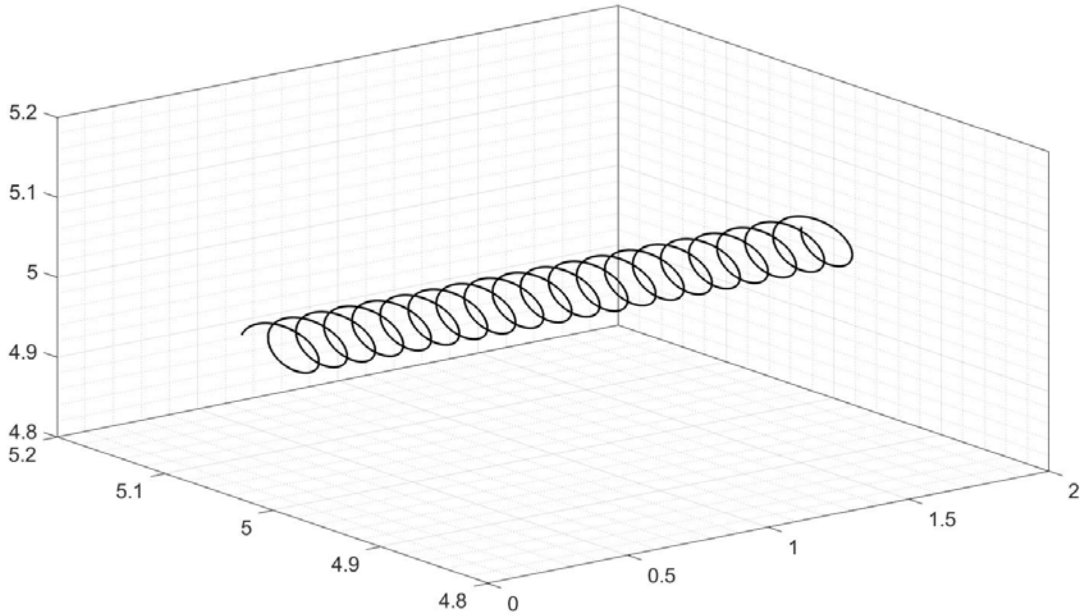


Response of a helical fibre

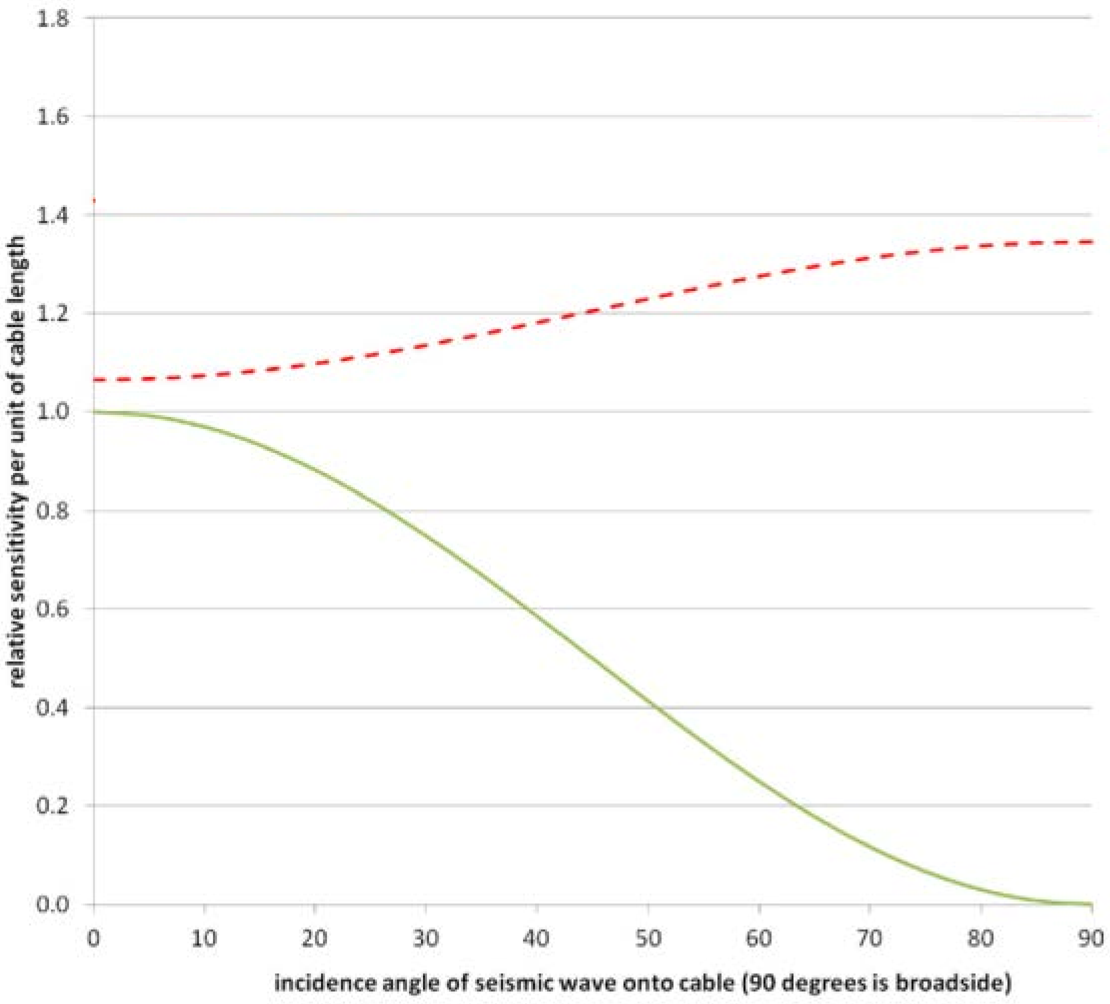




Modified from Mateeva et al., 2014

Response of a helical fibre

Motivation – Theoretical background – Geometric models – Elastic wave models – Bridging the gap – **Examples**



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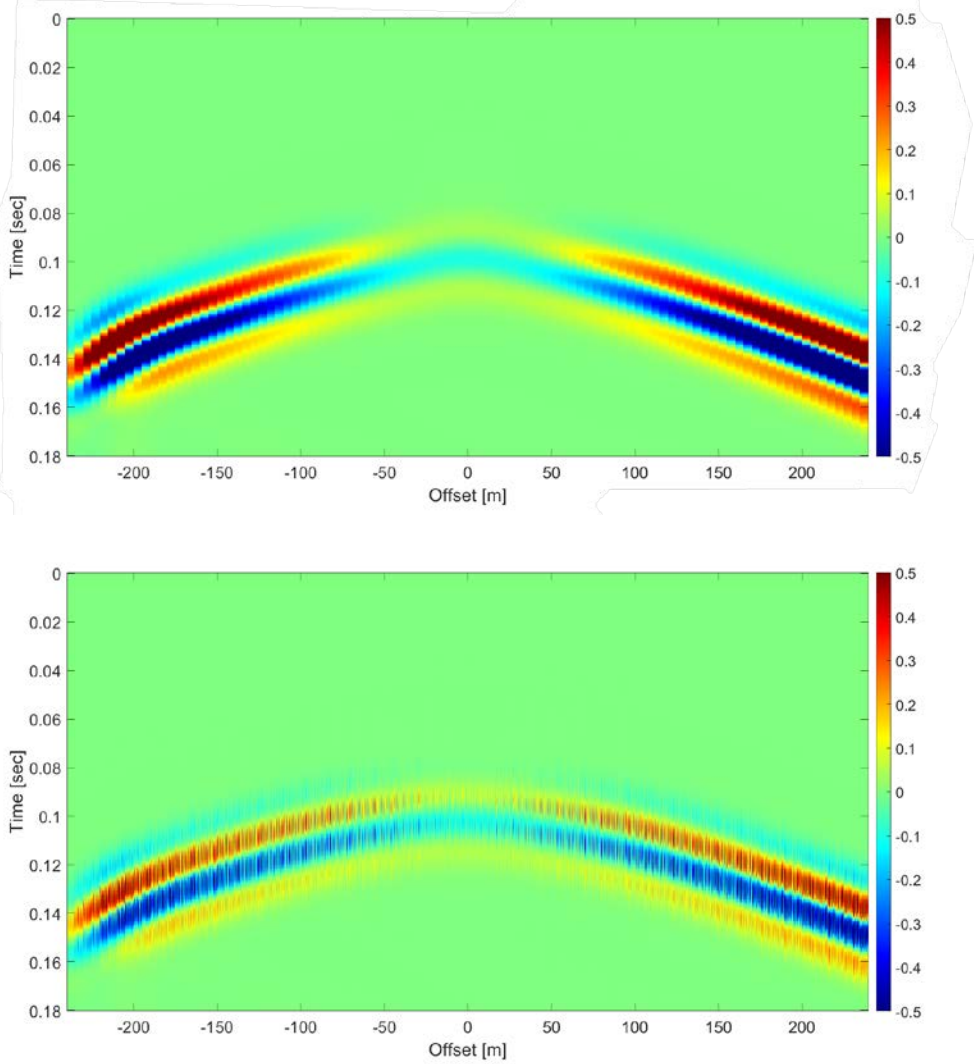
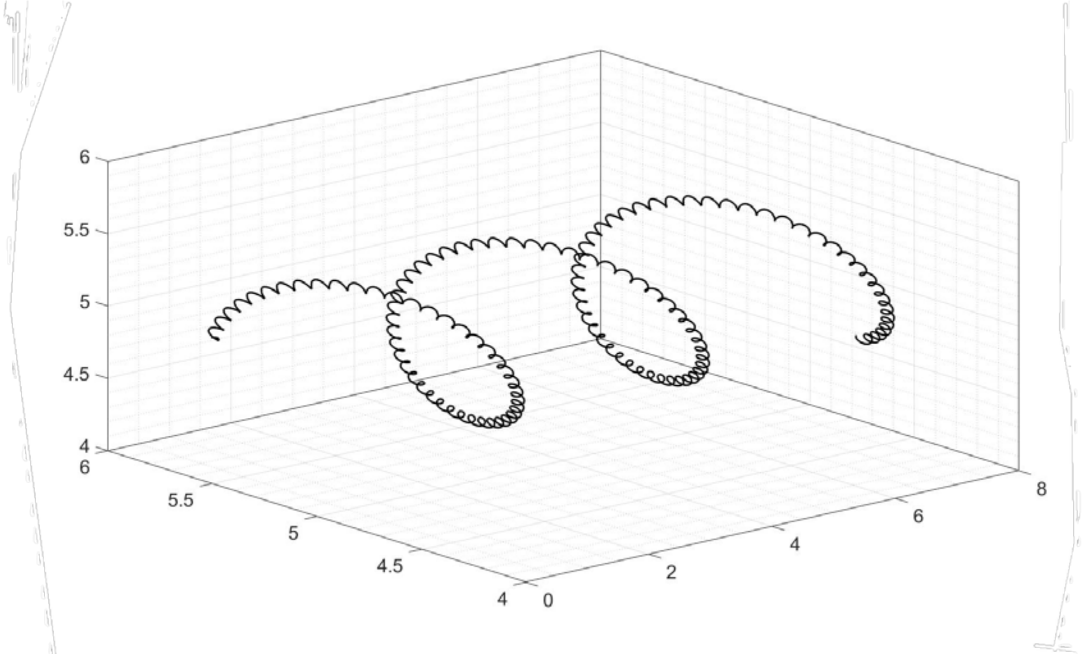
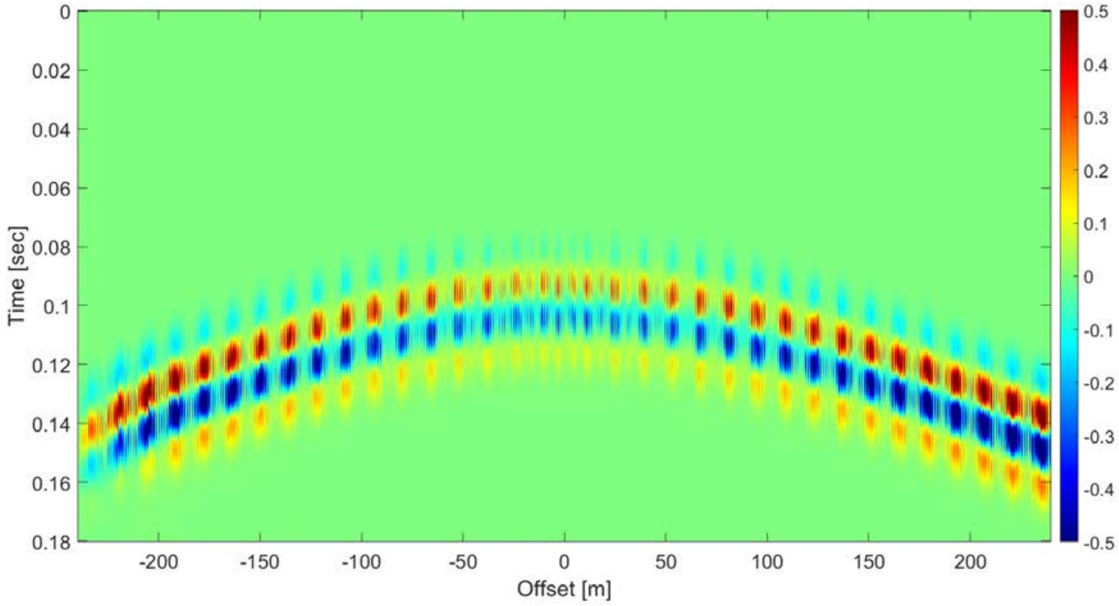


Response of a two-helix fibre





Future work

With a successful coupling of the DAS geometric model, and a full 3D elastic wave model, we are in a good position to further investigate the applications of DAS which may include but are not limited to:

1. Discrimination of elastic wave modes & identification of microseismic events in a propagating wavefield
2. Wave-based processing and imaging of DAS data
3. Full Waveform Inversion in a Distributed Acoustic Sensing framework
4. Acquisition design and appraisal: shape parameters, gauge length, channel separation and their influence on RTM, FWI, impedance inversion, etc .

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Questions?