Comparison of refraction inversion methods

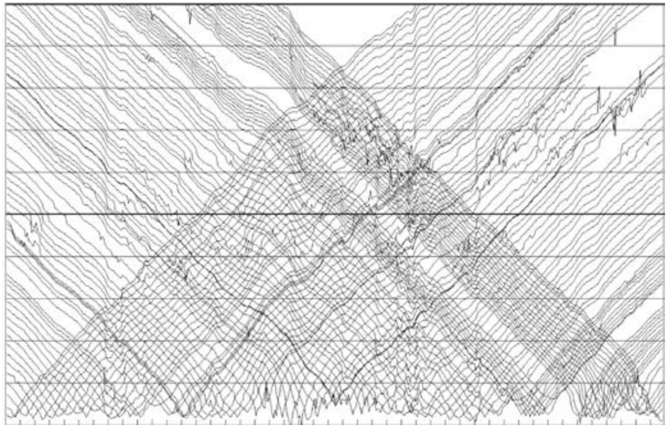
Bernard Law and Daniel Trad

Outline

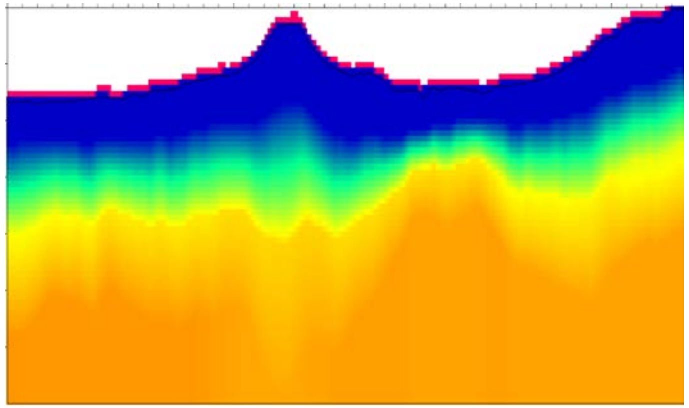
- Delay time method and refraction tomography
- Use reflection data in refraction inversion and near surface velocity model building
    - Long wavelength component of the reflection residual statics
    - FWI
- Field data example (Hussar 2D)
- FWI numerical example
- Conclusions

Refraction inversion

Refraction Data (d)



Model parameter (m)



Minimize $J = || d - Lm ||^2$

d = first arrival picks
L = forward modeling operator
m = model parameters
Lm = modeled data
J = cost function to minimize

Refraction inversion

Refraction Inversion

Refraction Data (d)

↓

Minimize $J = || d - Lm ||^2$

d = first arrival picks

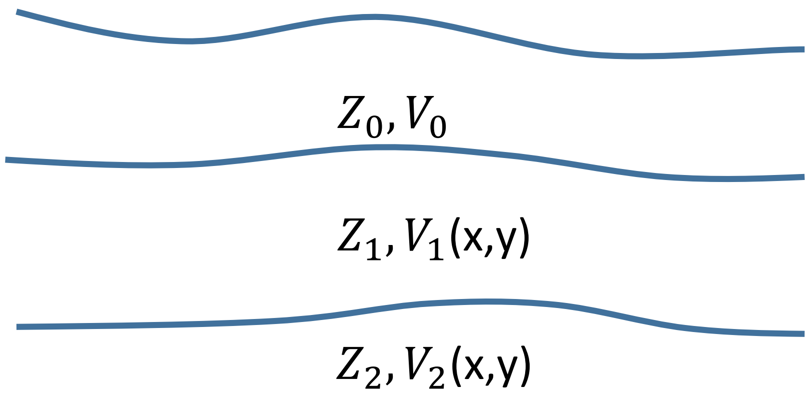
L = forward modeling operator

m = model parameters

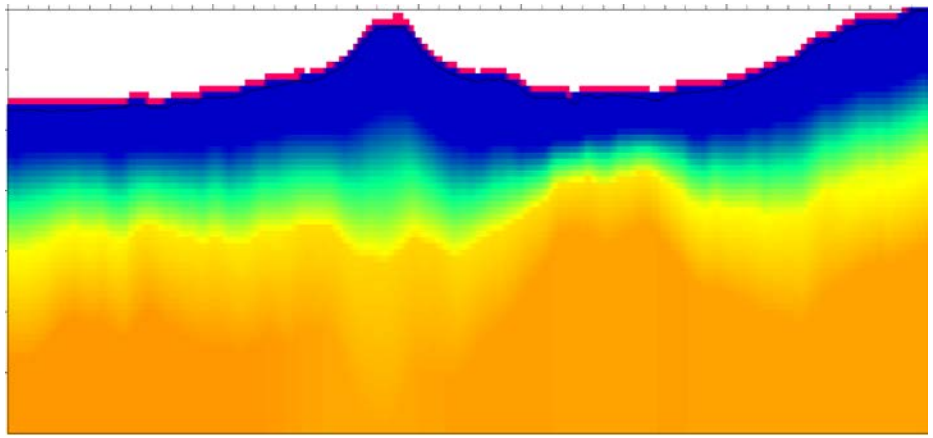
Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)



Grid model (Refraction Tomography)



Refraction inversion

Refraction Inversion

Refraction Data (d)

↓

Minimize $J = || d - Lm ||^2$

d = first arrival picks

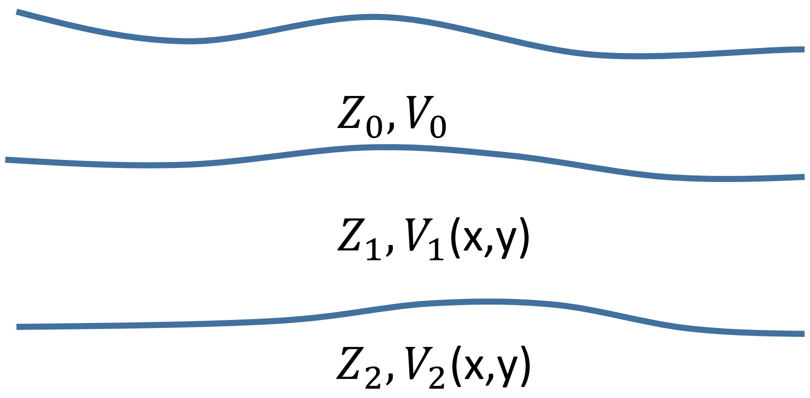
L = forward modeling operator

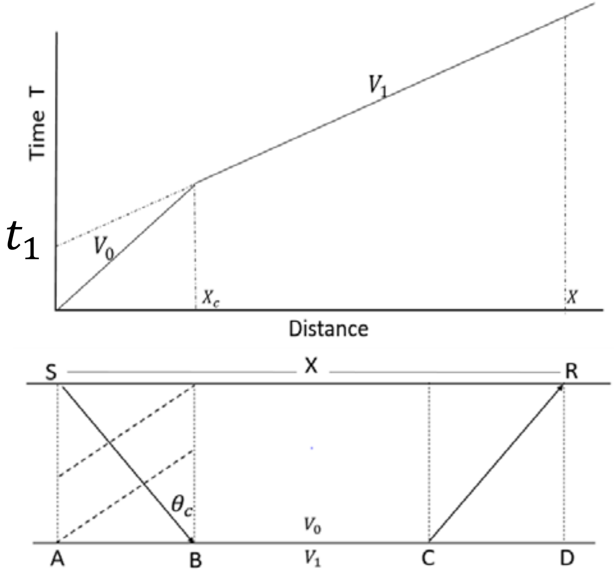
m = model parameters

Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)





Delay time equation

$$T_{SBCR} = \delta SB + \delta CR + ABCD/V_k$$

$$T_{SBCR} = t_k + X/V_k$$

$$t_k = intercept\ time$$

Layer thickness

$$Z_{k-1} = 0.5 * (t_k - t_{k-1}) \frac{V_k}{cos(\theta_{c_1})}$$

Refraction inversion

Refraction Inversion

Refraction Data (d)

↓

Minimize $J = ||\,\boldsymbol{d} - \boldsymbol{Lm}\,||^2$

d = first arrival picks

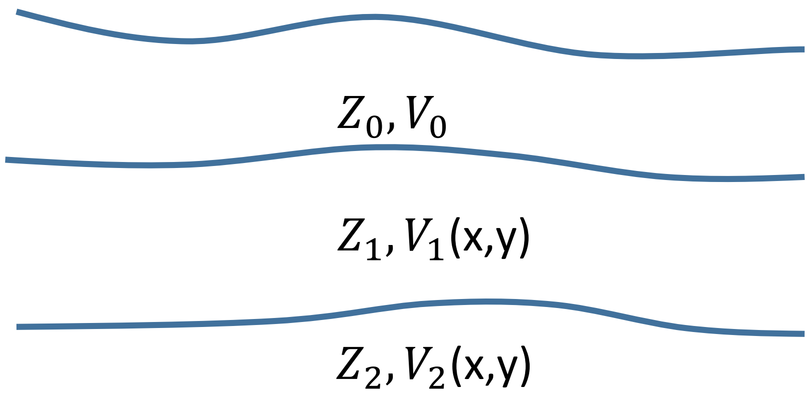
L = forward modeling operator

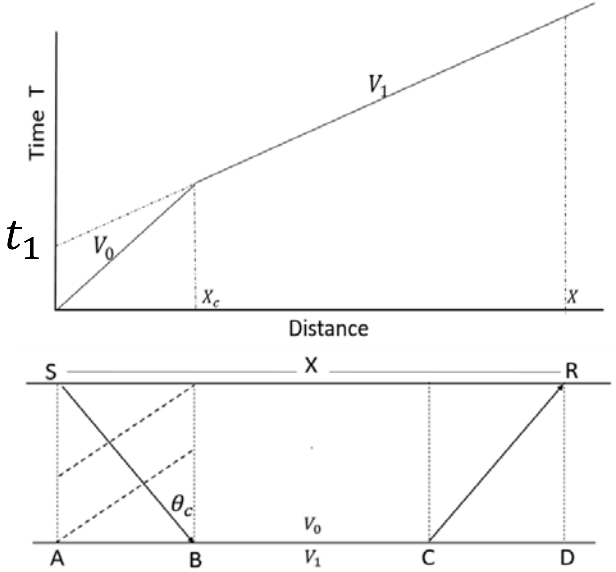
m = model parameters

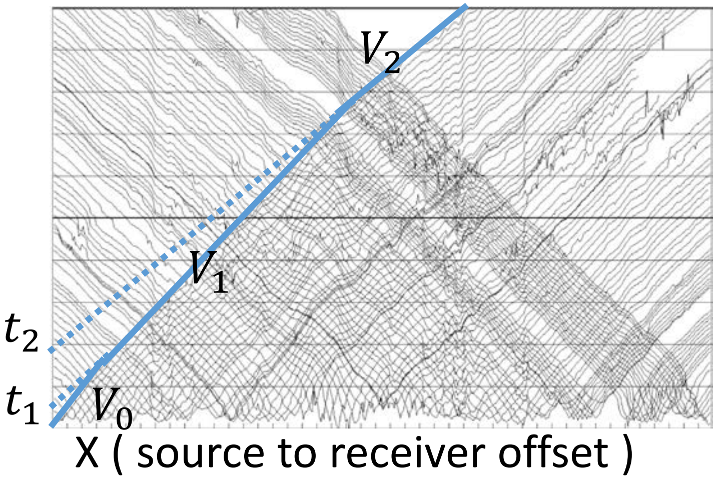
Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)







Delay time equation

$$T_{SBCR} = \delta SB + \delta CR + ABCD/V_k$$

$$T_{SBCR} = t_k + X/V_k$$

Layer thickness

$$Z_{k-1} = 0.5 * (t_k - t_{k-1}) \frac{V_k}{cos(\theta_{c_1})}$$

Refraction Inversion

Refraction Data (d)

↓

Minimize $J = || d - Lm ||^2$

d = first arrival picks

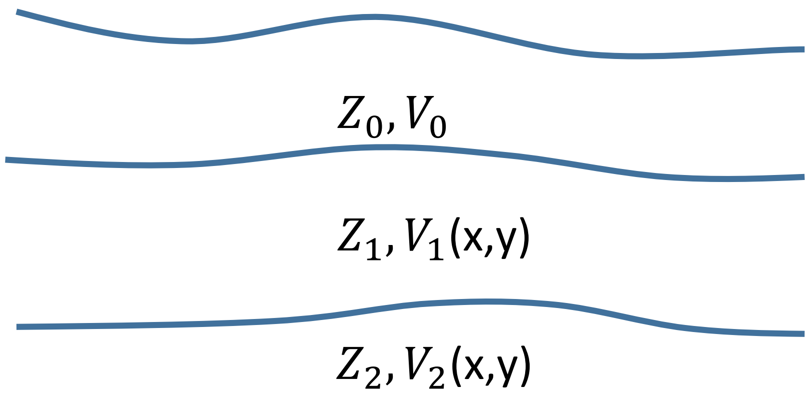
L = forward modeling operator

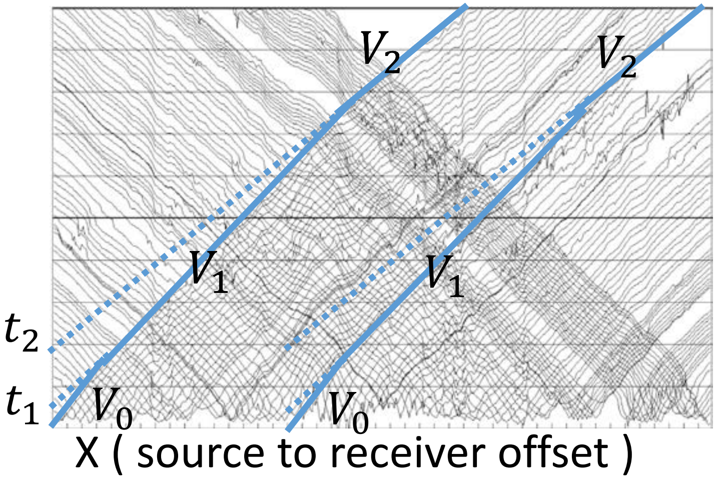
m = model parameters

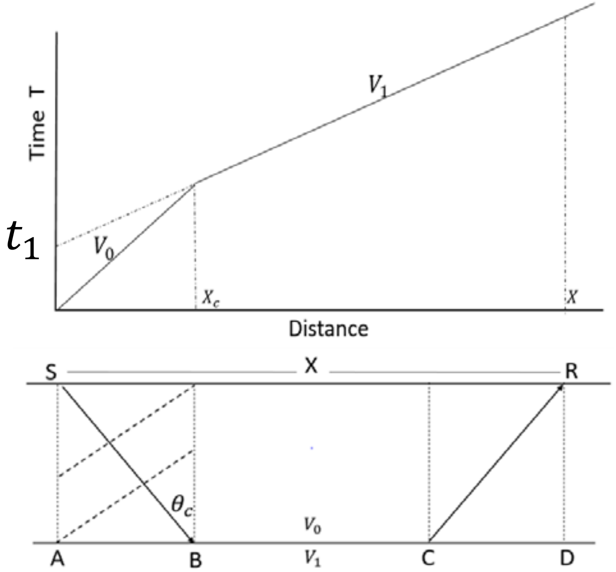
Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)







Delay time equation

$$T_{SBCR} = \delta SB + \delta CR + ABCD/V_k$$

$$T_{SBCR} = t_k + X/V_k$$

Layer thickness

$$Z_{k-1} = 0.5 * (t_k - t_{k-1}) \frac{V_k}{cos(\theta_{c_1})}$$

Refraction inversion

Refraction Inversion

Refraction Data (d)

↓

Minimize $J = || \boldsymbol{d} - \boldsymbol{Lm} ||^2$

d = first arrival picks

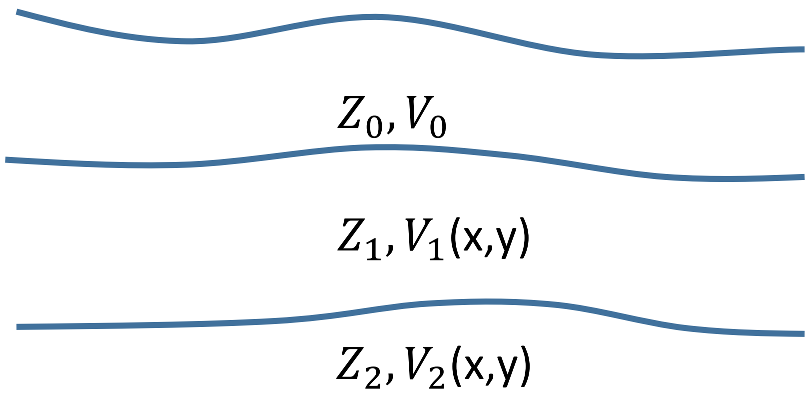
L = forward modeling operator

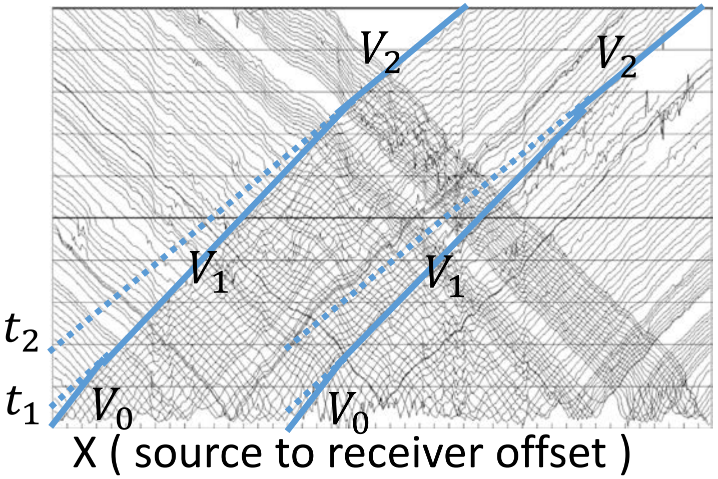
m = model parameters

Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)





Generalized linear inversion

$$\Delta T = B \Delta M$$

$$\Delta T = d - Lm$$

$$B = \partial T / \partial m$$

$$\Delta M = M^{l+1} - M^{l}$$

Least square solution:

$$M = (B^T B)^{-1} B^T T$$

Hampson and Russell 1984

Refraction inversion

Refraction Inversion

Refraction Data (d)

↓

Minimize $J = || d - Lm ||^2$

d = first arrival picks

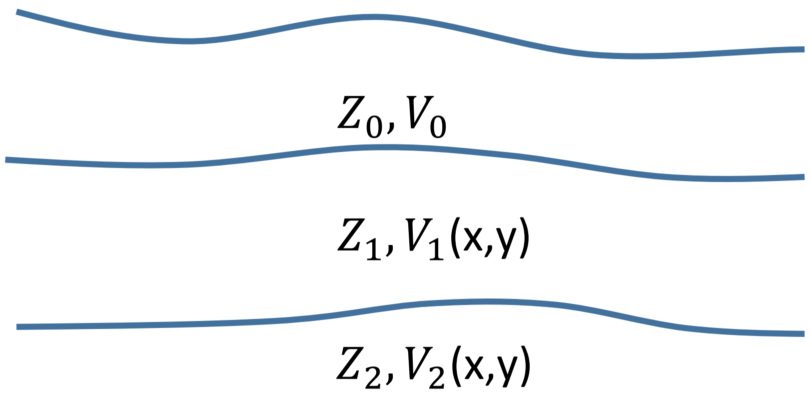
L = forward modeling operator

m = model parameters

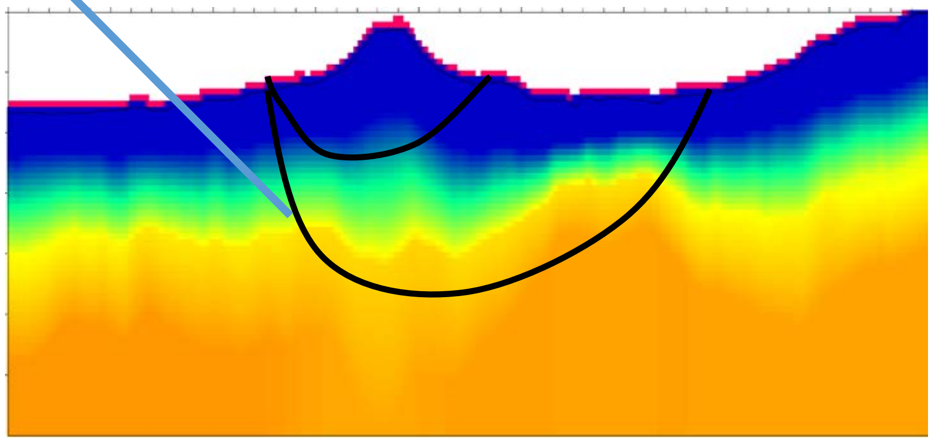
Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)



Grid model (Refraction Tomography)



Generalized linear inversion

$$\Delta T = B \Delta M$$

$$\Delta T = d - Lm$$

$$B = \partial T / \partial m$$

$$\Delta M = M^{l+1} - M^{l}$$

Least square solution:

$$M = (B^T B)^{-1} B^T T$$

Hampson and Russell 1984

Refraction inversion

Refraction Inversion

Refraction Data (d)

↓

Minimize $J = || d - Lm ||^2$

d = first arrival picks

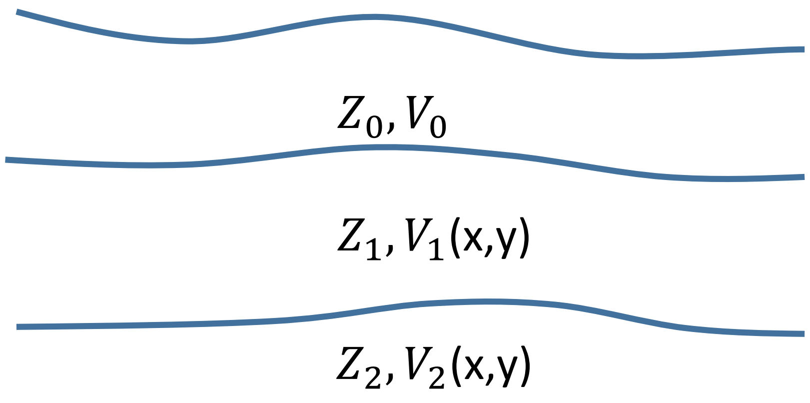
L = forward modeling operator

m = model parameters

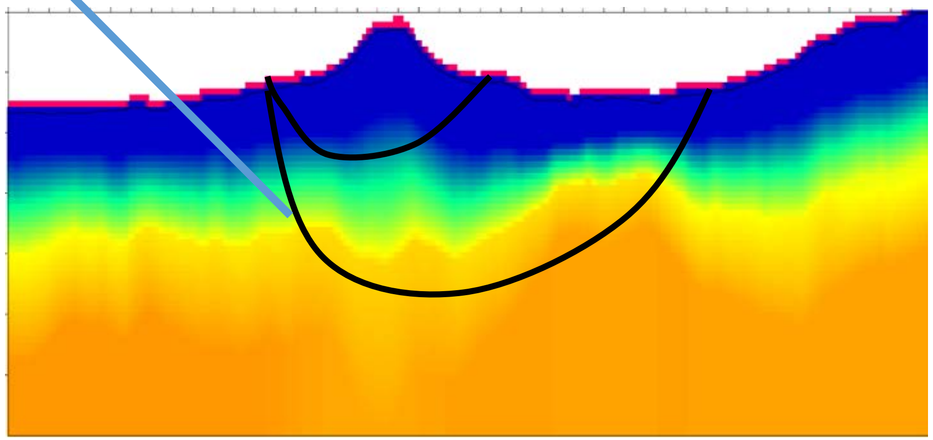
Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)



Grid model (Refraction Tomography)



Generalized linear inversion

$$\Delta T = B \Delta M$$

$$\Delta T = d - Lm$$

$$B = \partial T / \partial m$$

$$\Delta M = M^{l+1} - M^{l}$$

Least square solution:

$$M = (B^T B)^{-1} B^T T$$

Hampson and Russell 1984

Refraction tomography

$$\underline{G}\,\underline{m} = \underline{d}$$

$$\underline{G} = \begin{bmatrix} L1,1 & L1,2 & L1,3 & L1,4 & L1,5 & .. & L1,m \\ L2,1 & L2,2 & L2,3 & L2,4 & L2,5 & .. & L2,m \\ L3,1 & L3,2 & L3,3 & L3,4 & L3,5 & .. & L3,m \\ .. & .. & .. & .. & .. & .. & .. \\ Ln,1 & Ln,2 & Ln,3 & Ln,4 & Ln,5 & .. & Ln,m \end{bmatrix}$$

$$\underline{m} = \begin{bmatrix} \Delta M1 \\ \Delta M2 \\ \Delta M3 \\ .. \\ \Delta Mm \end{bmatrix} \qquad \underline{d} = \begin{bmatrix} \Delta T1 \\ \Delta T2 \\ \Delta T3 \\ .. \\ \Delta Tn \end{bmatrix}$$

$$\underline{m} = (\underline{G}^T \underline{G})^{-1} \underline{G}^T \underline{d}$$

Refraction inversion

Refraction Inversion

Refraction Data (d)

Minimize $J = \| d - Lm \|^2$

d = first arrival picks

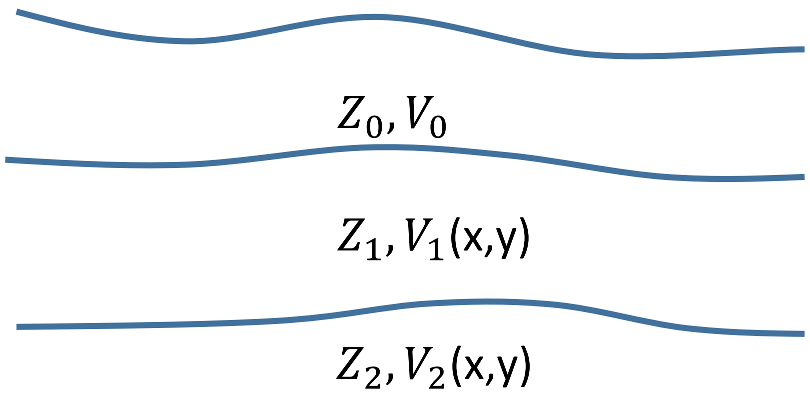
L = forward modeling operator

m = model parameters

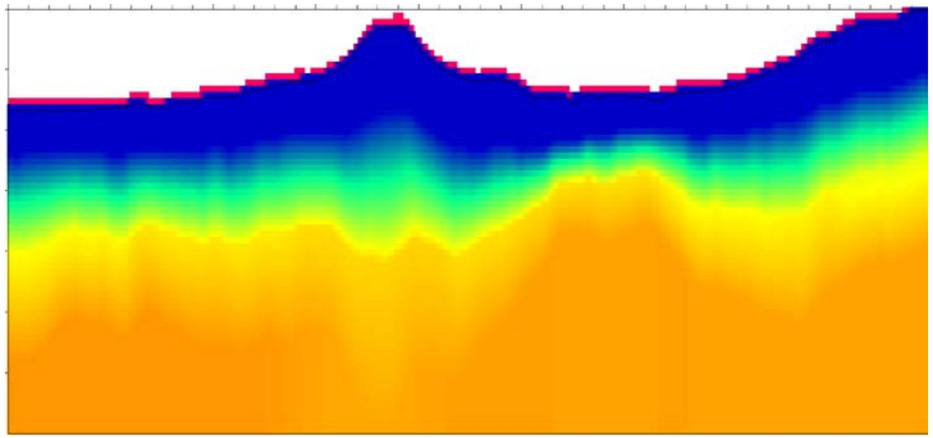
Lm = modeled data

J = cost function to minimize

Layer model (Delay time method)



Grid model (Refraction Tomography)



Weathering correction for near-surface effects

$$C_{wx} = \sum_{i=0}^{n-1} \left(\frac{Z_i}{V_r} - \frac{Z_i}{V_i} \right)$$

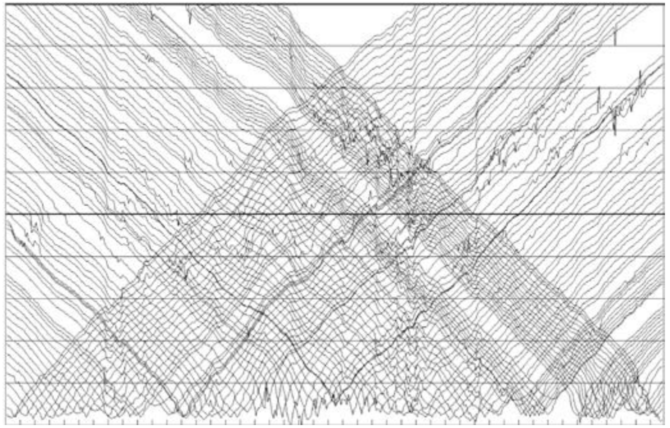
C_{wx} = Weathering correction

V_r = Replacement velocity

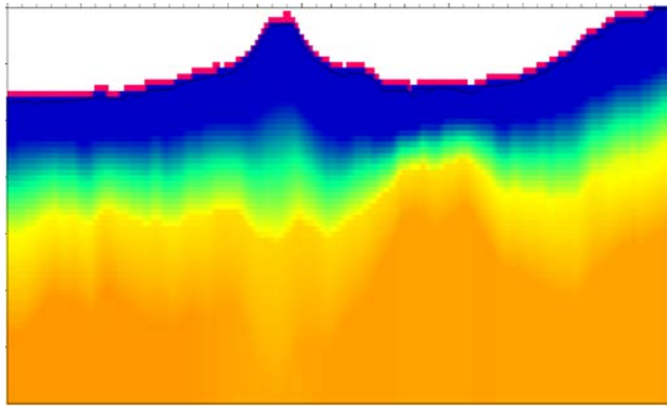
n = Number of layers or depth levels

Refraction inversion

Refraction Data

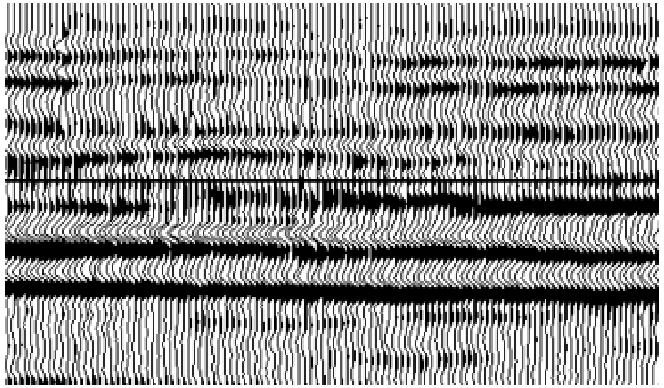


Refraction Inversion

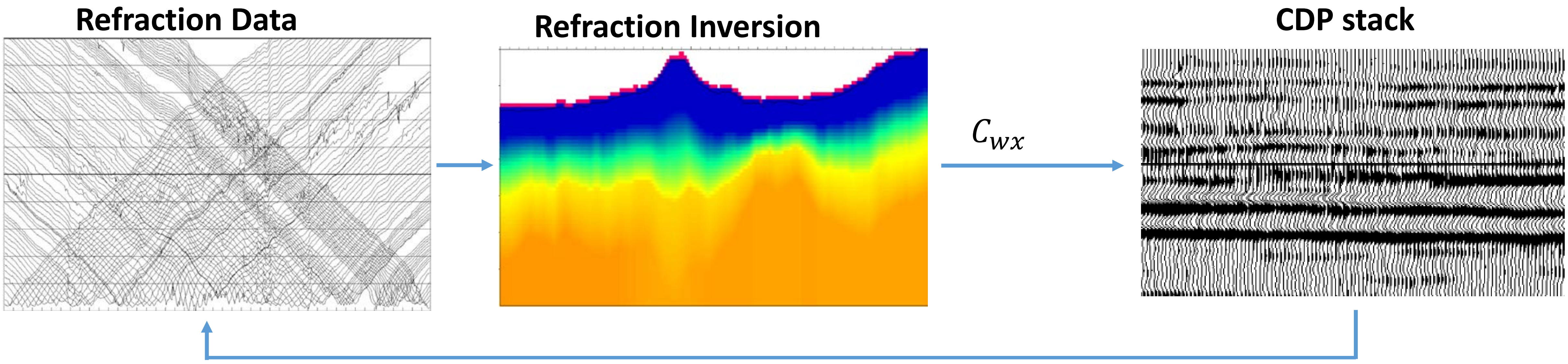


C_{wx}

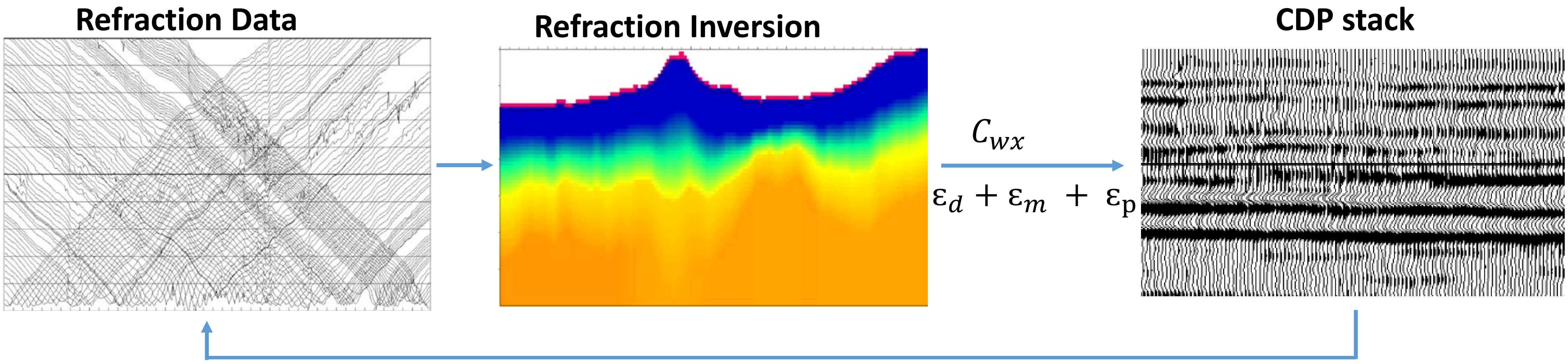
CDP stack



Refraction inversion using reflection data



Refraction inversion using reflection data

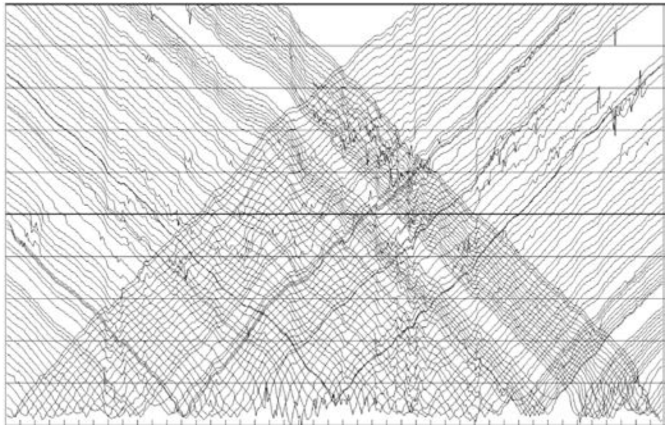


Some of the problems in the refraction solution can be caused by:

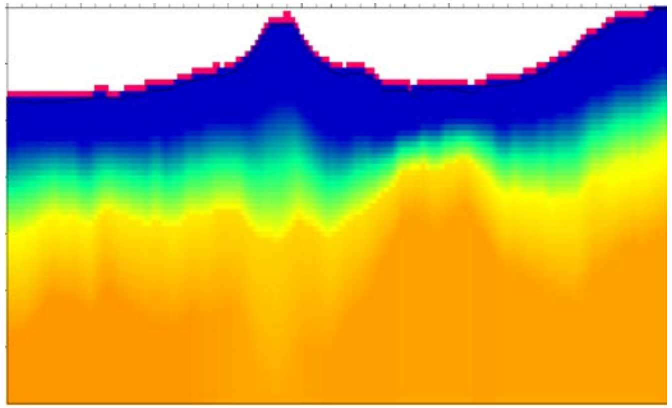
- First break picking errors (ε_d)
- Suboptimal inversion parameters and starting model (ε_m)
- Insufficient sampling (ε_p)
- Over-simplified assumptions used in refraction inversion algorithm (ε_p)

Refraction inversion using reflection data

Refraction Data



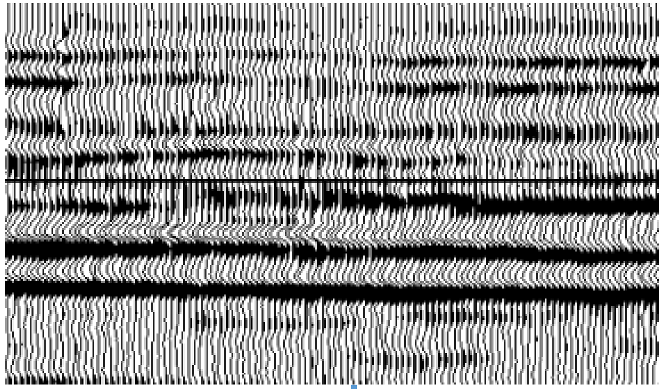
Refraction Inversion



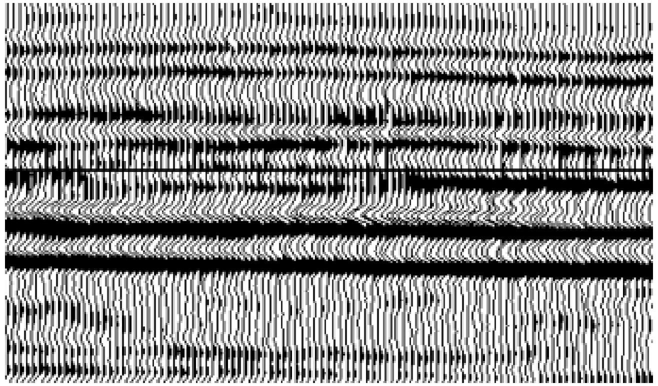
C_{wx}

$\varepsilon_d + \varepsilon_m + \varepsilon_p$

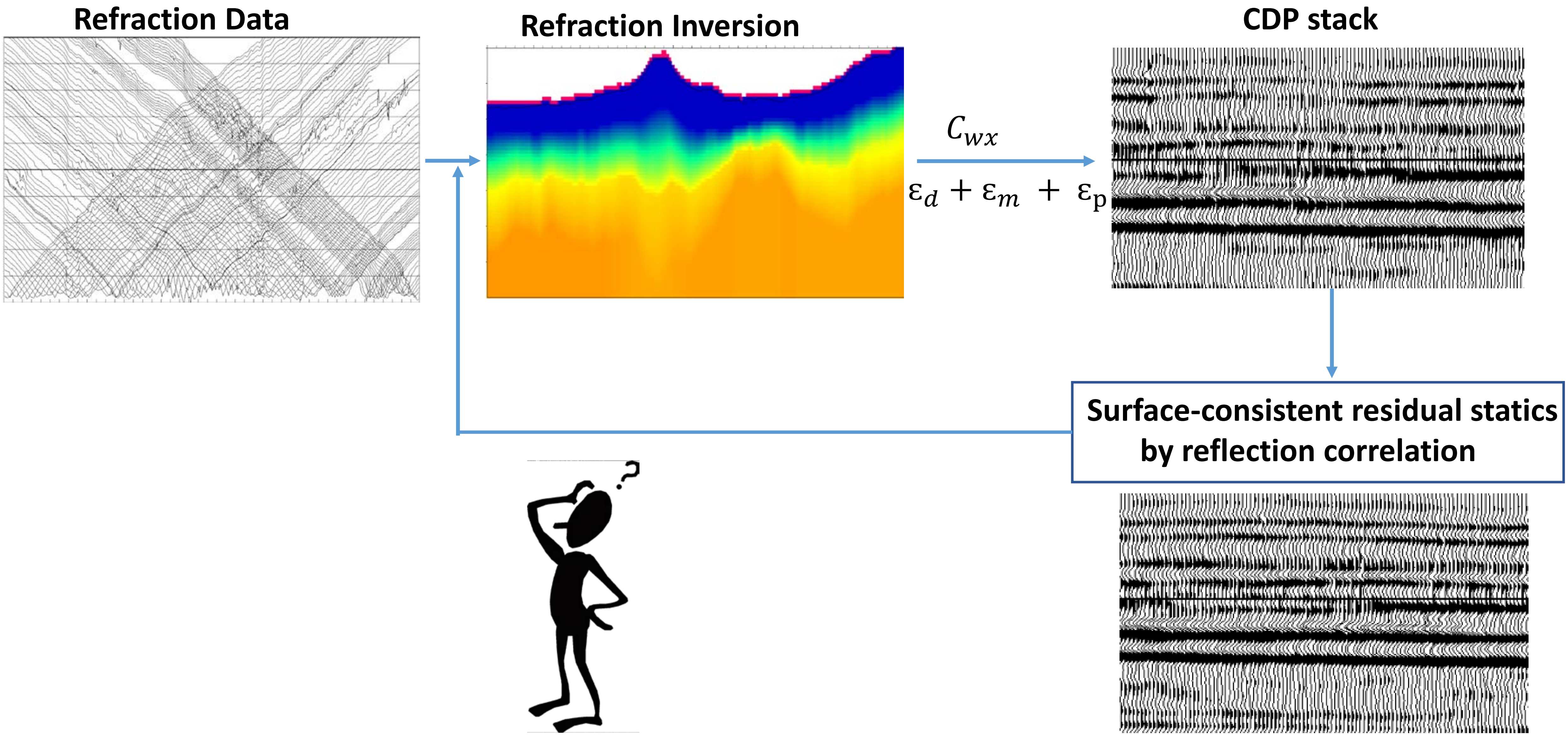
CDP stack



Surface-consistent residual statics by reflection correlation



Refraction inversion using reflection data



Non-linear optimization of near surface velocity model using reflection data

Refraction Data (d)

Reflection data U(t,x)

Minimize $\boldsymbol{J = || d - Lm||^2}$

$C_{wx} + \varepsilon_d + \varepsilon_m + \varepsilon_p$

Pre-stack Processes

$\boldsymbol{m}$

Maximize stack-power by Reflection Correlation

$-\varepsilon_d - \varepsilon_m - \varepsilon_p$

Minimize $\boldsymbol{J = ||W_d d - W_d L W_m m||^2}$

$\boldsymbol{W_m m}$

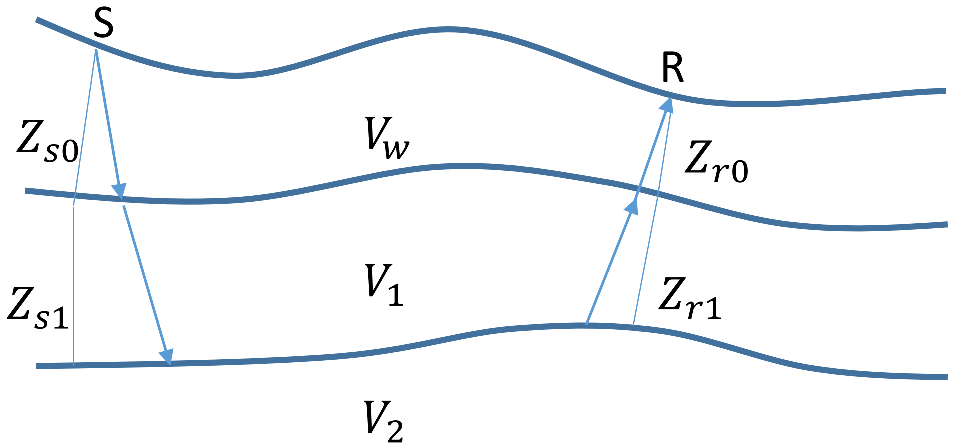
$C_{wx} + \varepsilon_p$

$\boldsymbol{W_m = model\ weight}$

$\boldsymbol{W_d = data\ weight}$

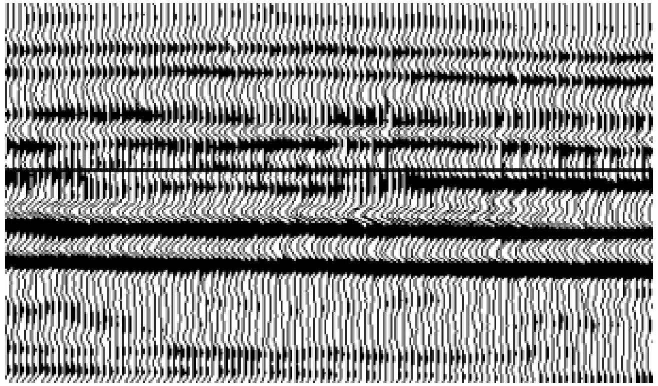
Computing model weight W_m

Refraction Inversion

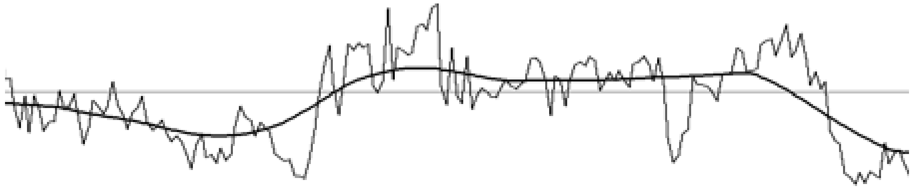


Weathering statics correction for layer i :

$$\boldsymbol{T_i = \frac{Z_i}{V_r} - \frac{Z_i}{V_i}}$$

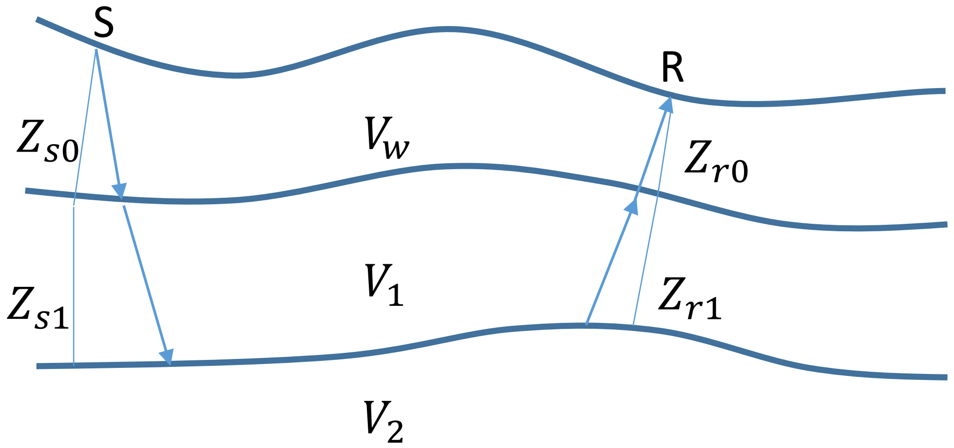


surface-consistent reflection residual statics



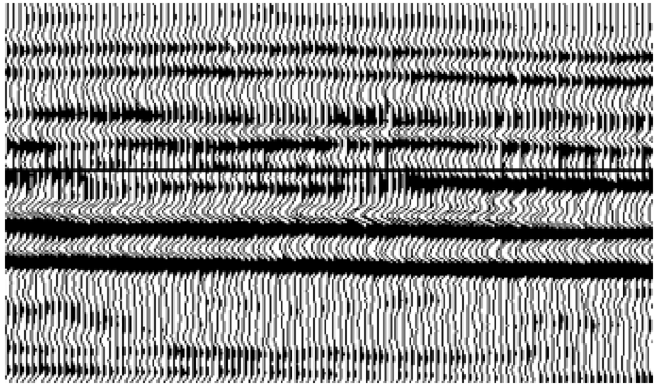
Computing model weight W_m

Refraction Inversion

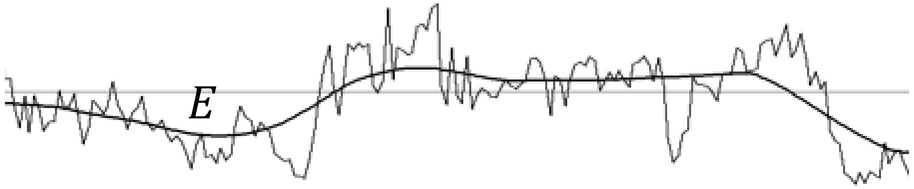


Weathering statics correction for layer i :

$$\boldsymbol{T_i = \frac{Z_i}{V_r} - \frac{Z_i}{V_i}}$$



surface-consistent reflection residual statics

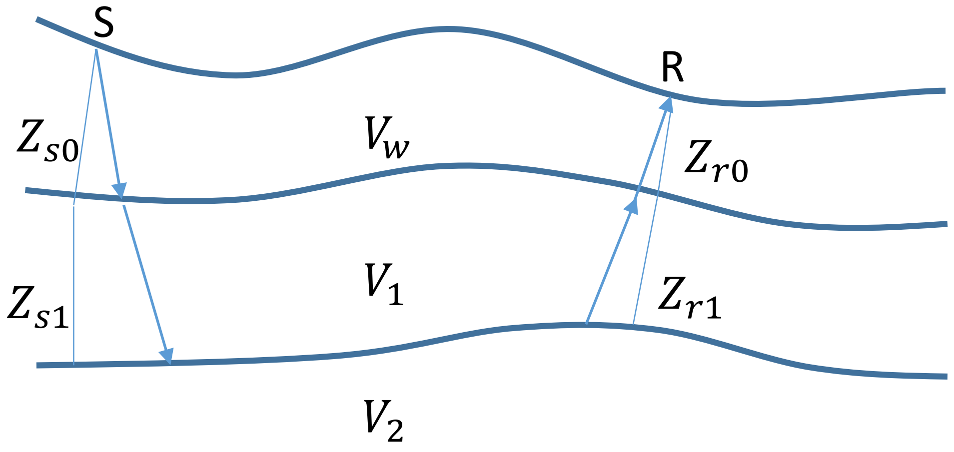


$$E = smoothed\ (long\ wavelength) residual\ statics$$

$$\boldsymbol{E_i} = E \frac{Z_i}{Total\ thickness}$$

Computing model weight W_m

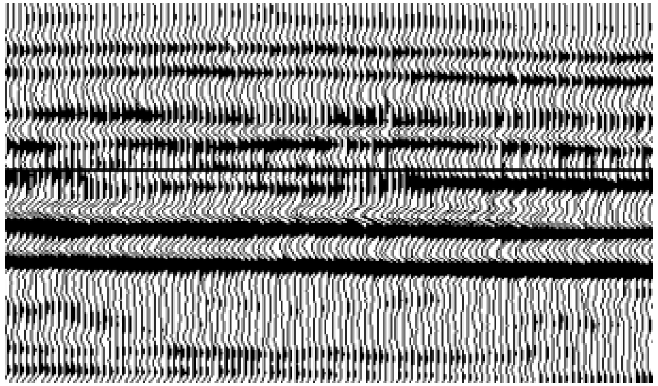
Refraction Inversion



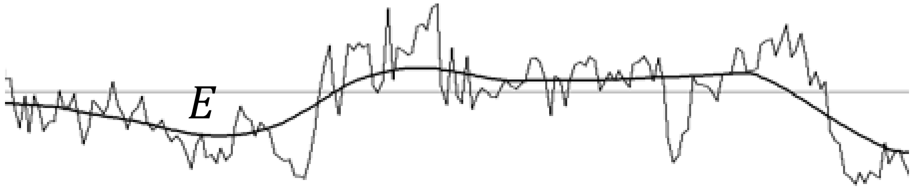
Weathering statics correction for layer i :

$$T_i = \frac{Z_i}{V_r} - \frac{Z_i}{V_i}$$

$$T_i + E_i = \frac{Z_i}{V_r} - Z_i P_i W_{mi}$$



surface-consistent reflection residual statics



$E \;= smoothed\ (long\ wavelength) residual\ statics$

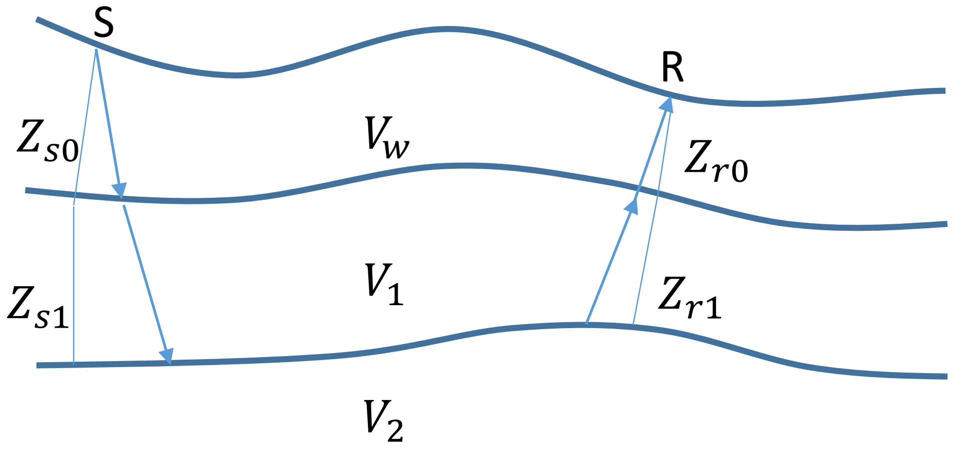
$E_i = E \dfrac{Z_i}{Total\ thickness}$

Define $P_i = 1 / V_i$

$W_{mi} = slowness\ model\ weight\ for\ layer\ i$

Computing model weight W_m

Refraction Inversion



Weathering statics correction for layer i :

$$\boldsymbol{T_i = \frac{Z_i}{V_r} - \frac{Z_i}{V_i}}$$

$$\boldsymbol{T_i + E_i = \frac{Z_i}{V_r} - Z_i P_i W_{mi}}$$

$$\boldsymbol{W_{mi} = 1 - E_i/(Z_i P_i)}$$

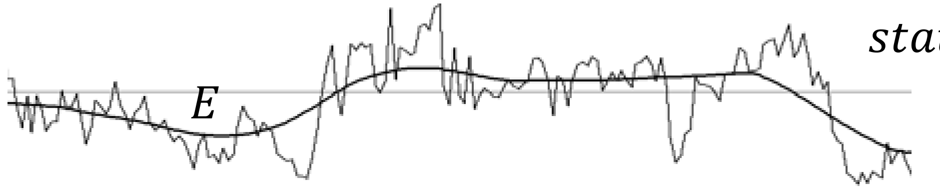
$$E \;= smoothed\ (long\ wavelength) residual\ statics$$

$$\boldsymbol{E_i} = E \; \frac{Z_i}{Total\ thickness}$$

Define $\boldsymbol{P_i} = 1\,/\,V_i$

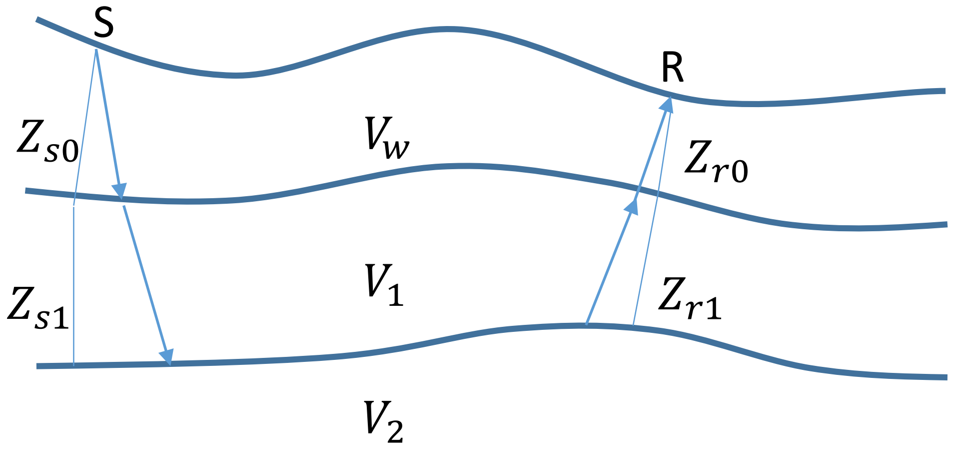
$$\boldsymbol{W_{mi}} = slowness\ model\ weight\ for\ layer\ i$$

surface-consistent reflection residual statics



Computing model weight W_m

Refraction Inversion



Weathering statics correction for layer i :

$$T_i = \frac{Z_i}{V_r} - \frac{Z_i}{V_i}$$

$$T_i + E_i = \frac{Z_i}{V_r} - Z_i P_i W_{mi}$$

$$W_{mi} = 1 - E_i/(Z_i P_i)$$

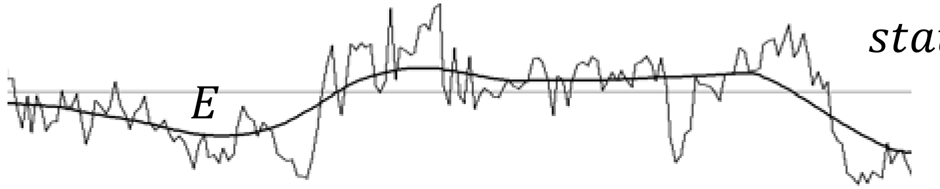
$$W_{mi}(thickness) = 1 + E_i/T_i$$

E = *smoothed (long wavelength) residual statics*

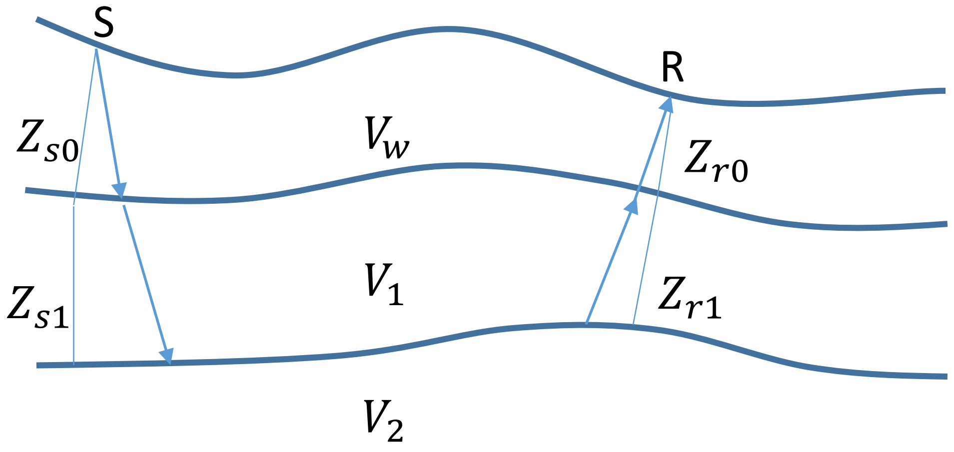
$$E_i = E \frac{z_i}{Total\ thickness}$$

Define $P_i = 1 / V_i$

W_{mi} = *slowness model weight for layer i*



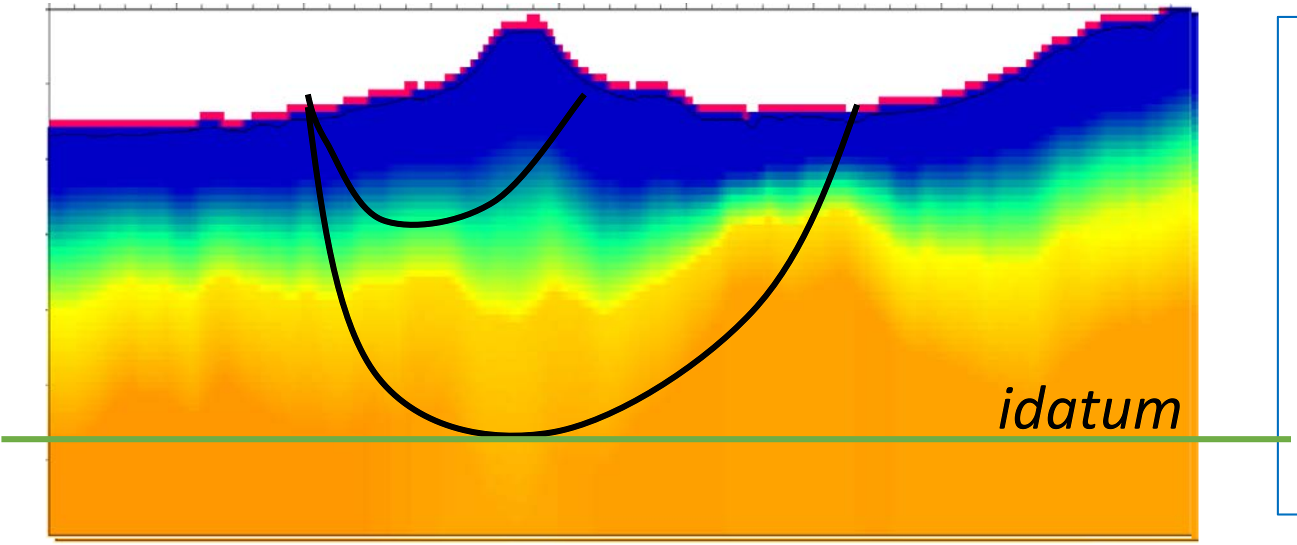
Computing model weight W_m



$$\boldsymbol{W_{mi} = 1 - E_i/(Z_i/V_i)}$$

$$\boldsymbol{W_{mi}}(thickness) = \boldsymbol{1 + E_i/T_i}$$

Refraction Tomography



Define : Weathering statics correction:

$$T = \sum_{iz=1}^{idatum} \left(\frac{1}{Vr} - m_{iz}\right) dz$$

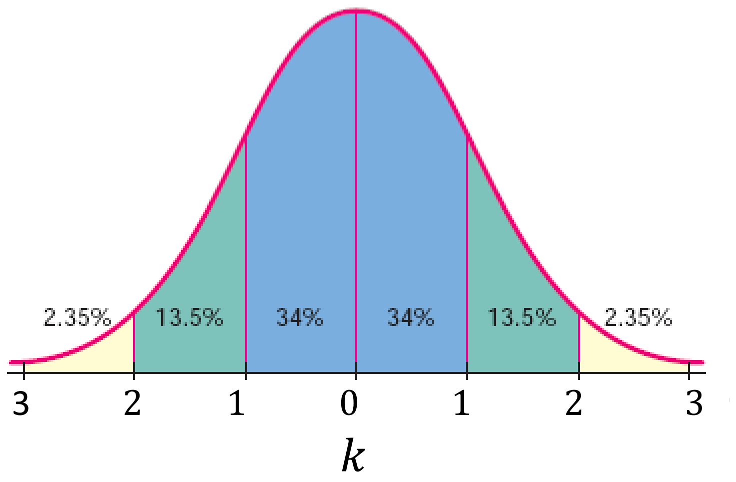
$$\boldsymbol{W_m = 1 - E/T} \quad , \quad \boldsymbol{W_m}(thickness) = \boldsymbol{1 + E/T}$$

$$E = \text{smoothed (long wavelength) residual statics}$$

Data weight W_d

$$J = ||\, \boldsymbol{W_d d} - \boldsymbol{W_d L W_m m} \,||^{2}$$

$$W_d = \begin{cases} 0 & \delta t > K \times std(\delta t) \\ 1 & otherwise \end{cases}$$



Where: $\delta t = d - LW_m m$

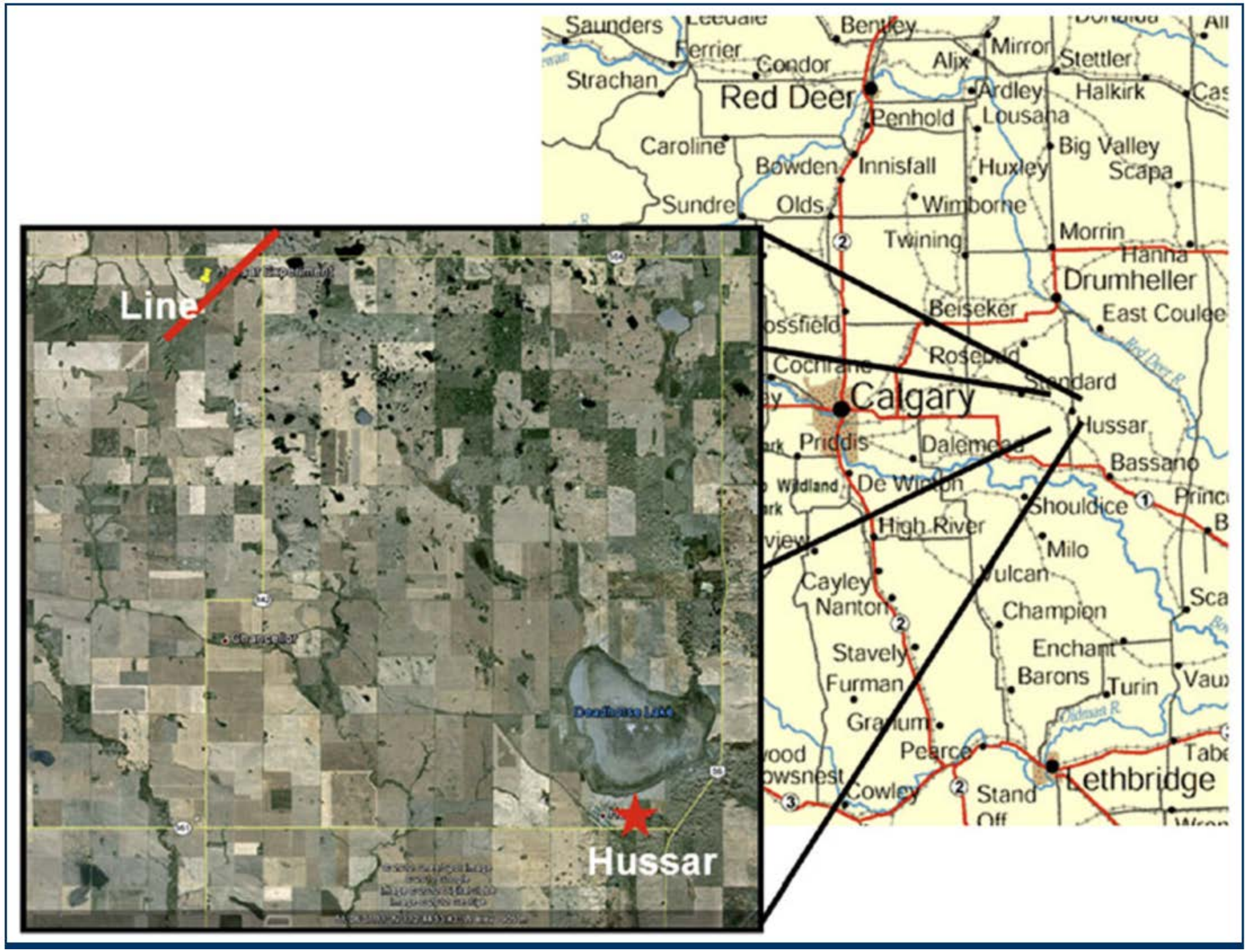
$std(\delta t)$ = standard deviation of δt

K = multiplier for standard deviation of first arrival residual

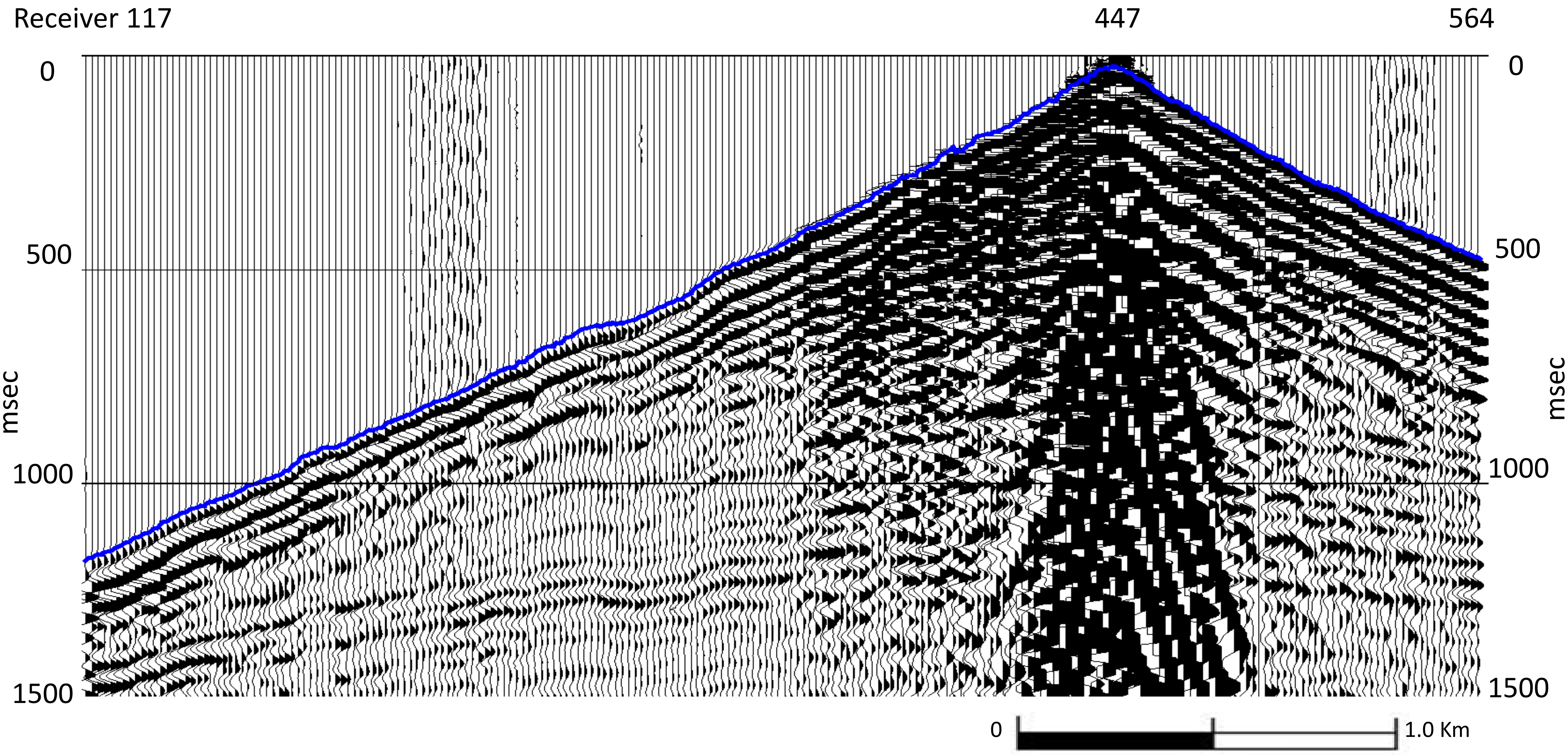
$K = 1$ keeps 68 % of data

$K = 2$ keeps 95 % of data

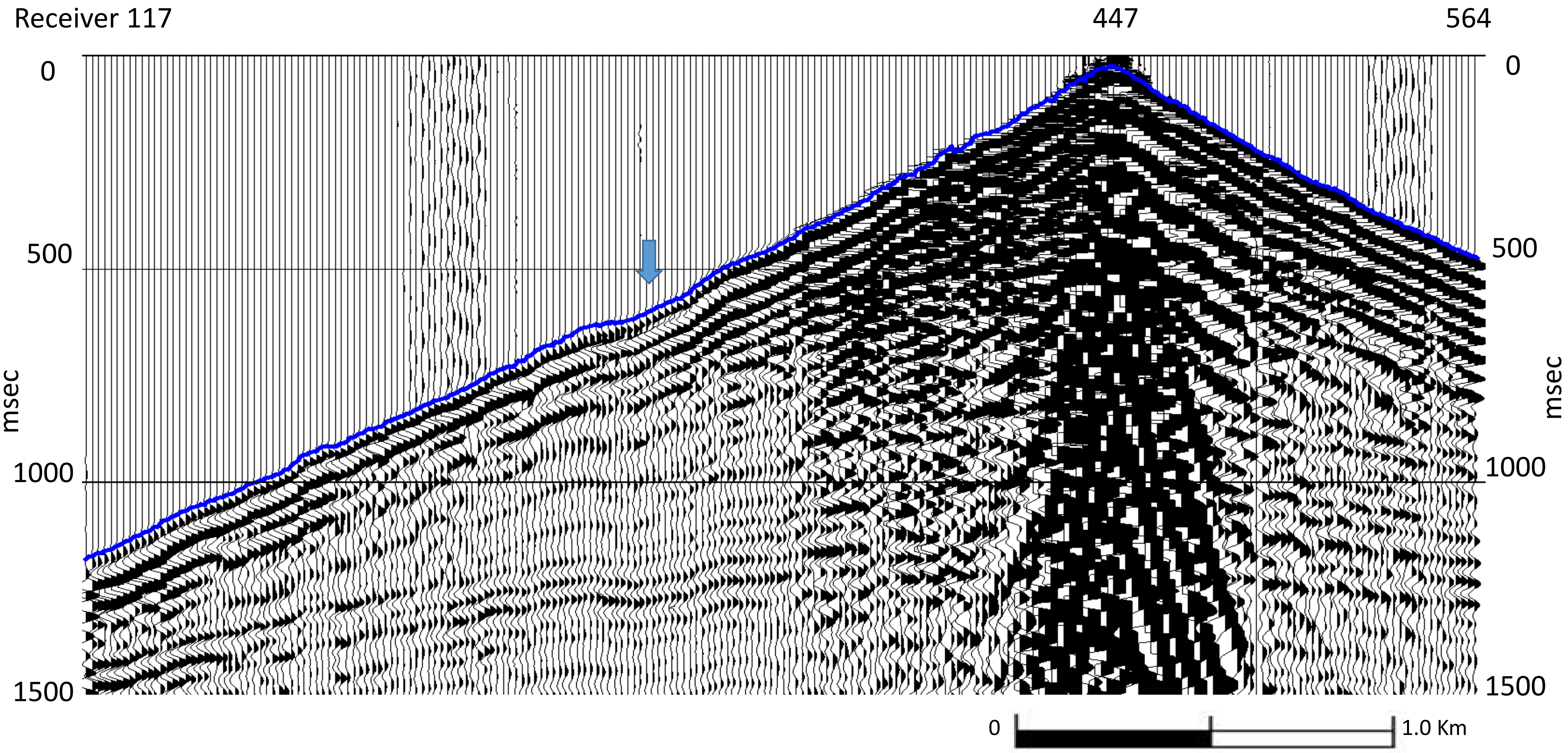
Field data example (Hussar 2D)



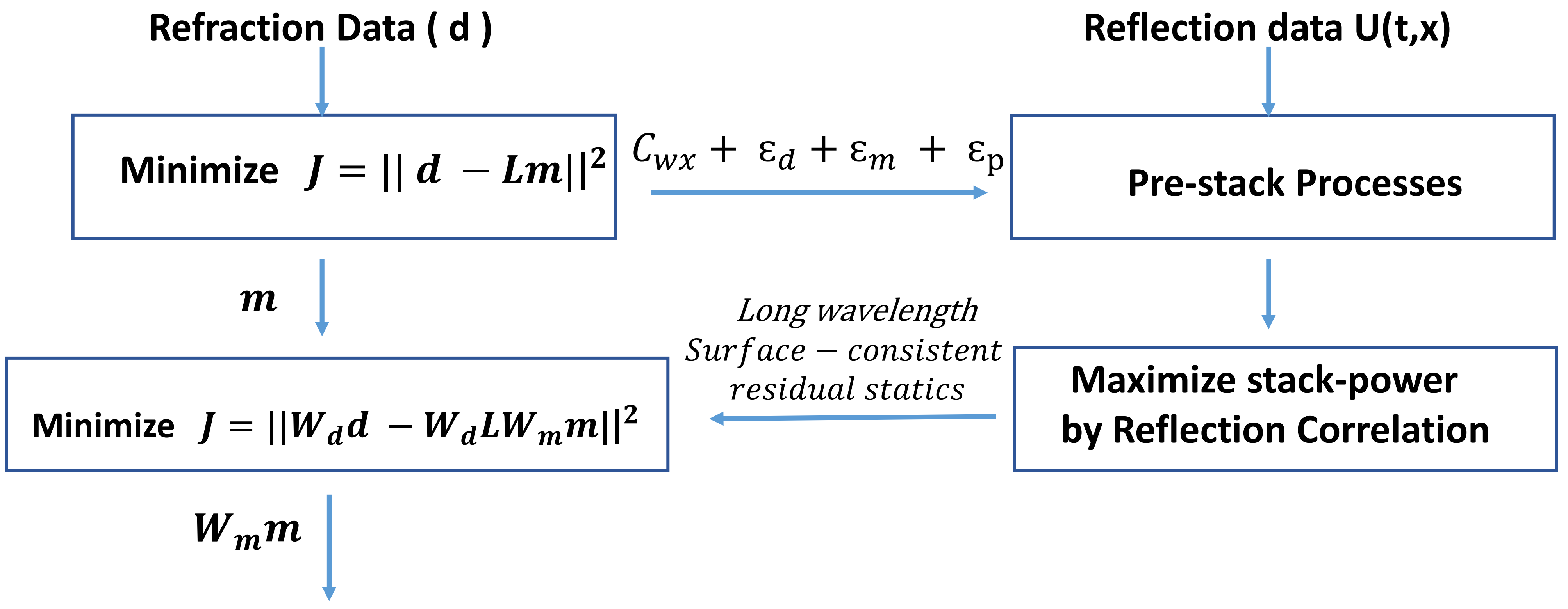
Shot record 447



Shot record 447

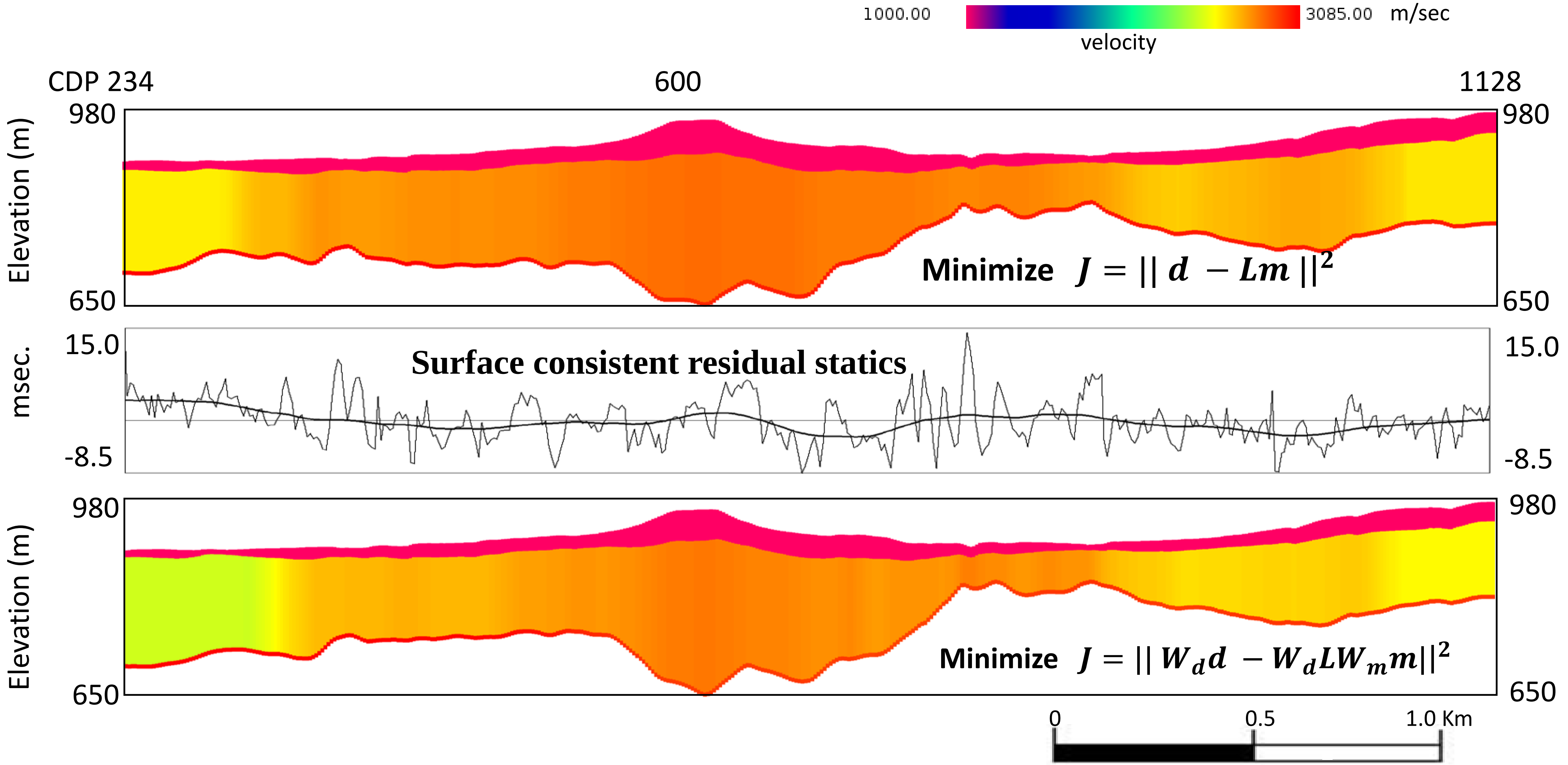


Non-linear optimization of near surface velocity model using reflection data

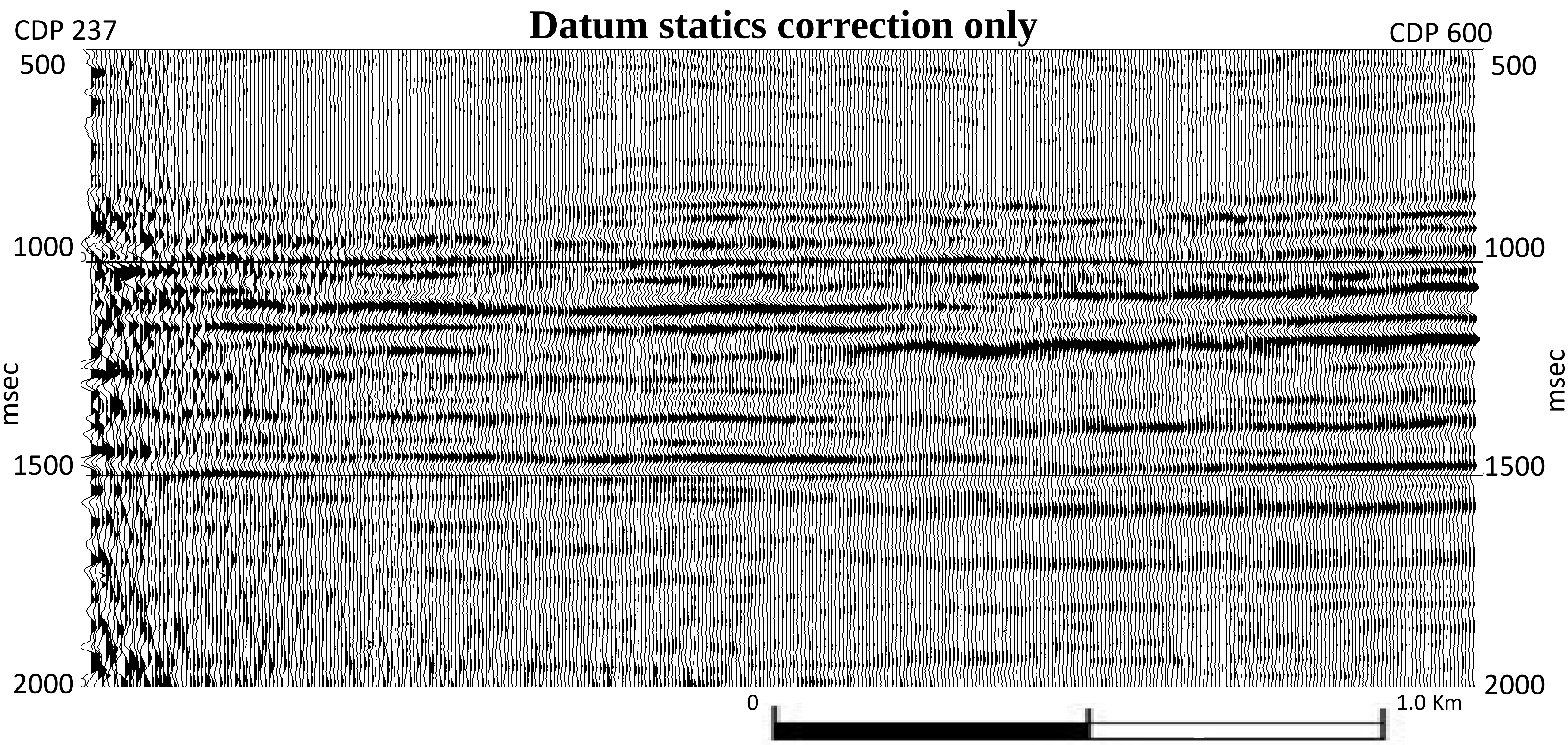


$$W_m = model\ weight$$
$$W_d = data\ weight$$

GLI solutions

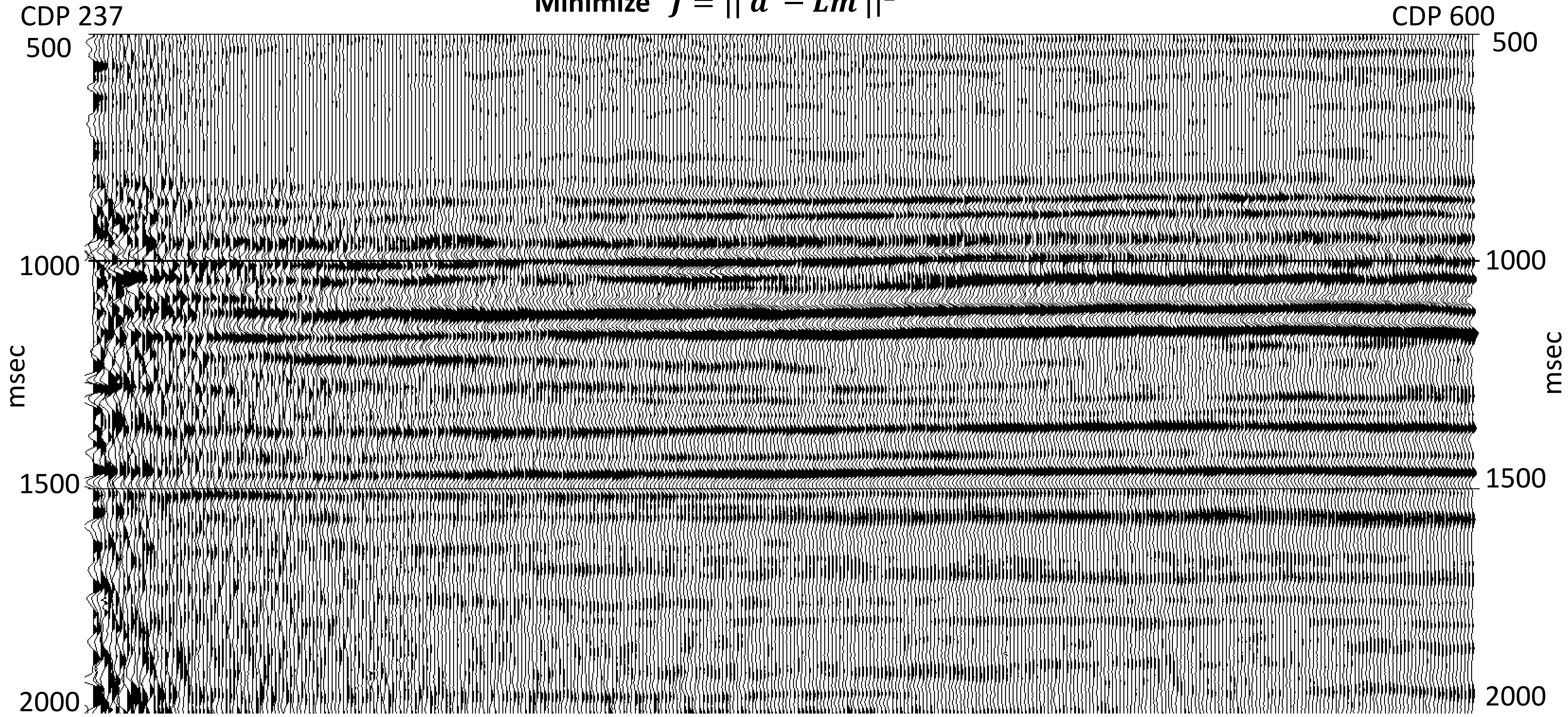


CDP stack


Datum statics correction only
CDP 237
CDP 600
500
1000
1500
2000
msec
500
1000
1500
2000
msec
0
1.0 Km

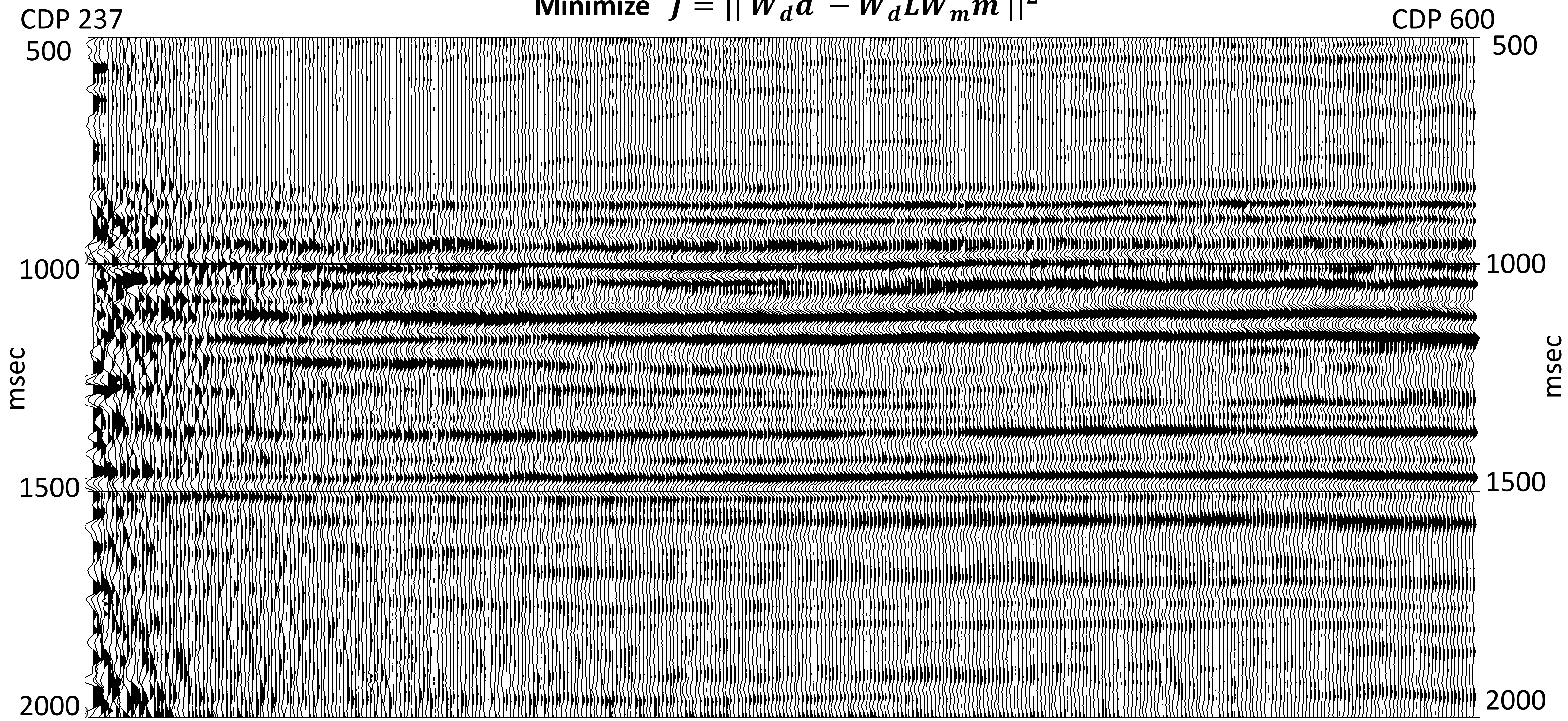
GLI Stack

Minimize $J = || \boldsymbol{d} - \boldsymbol{Lm} ||^2$

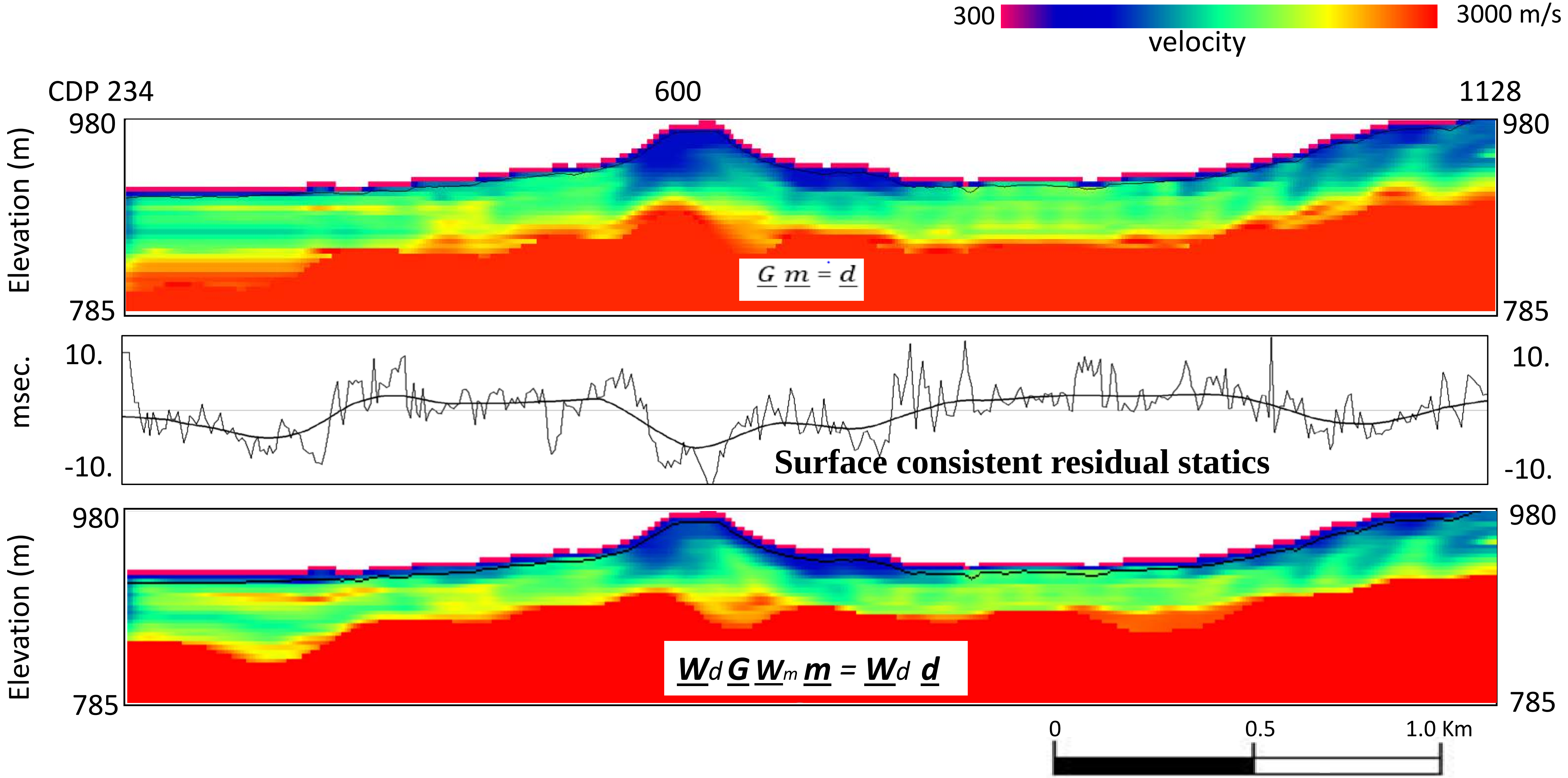


GLI Stack

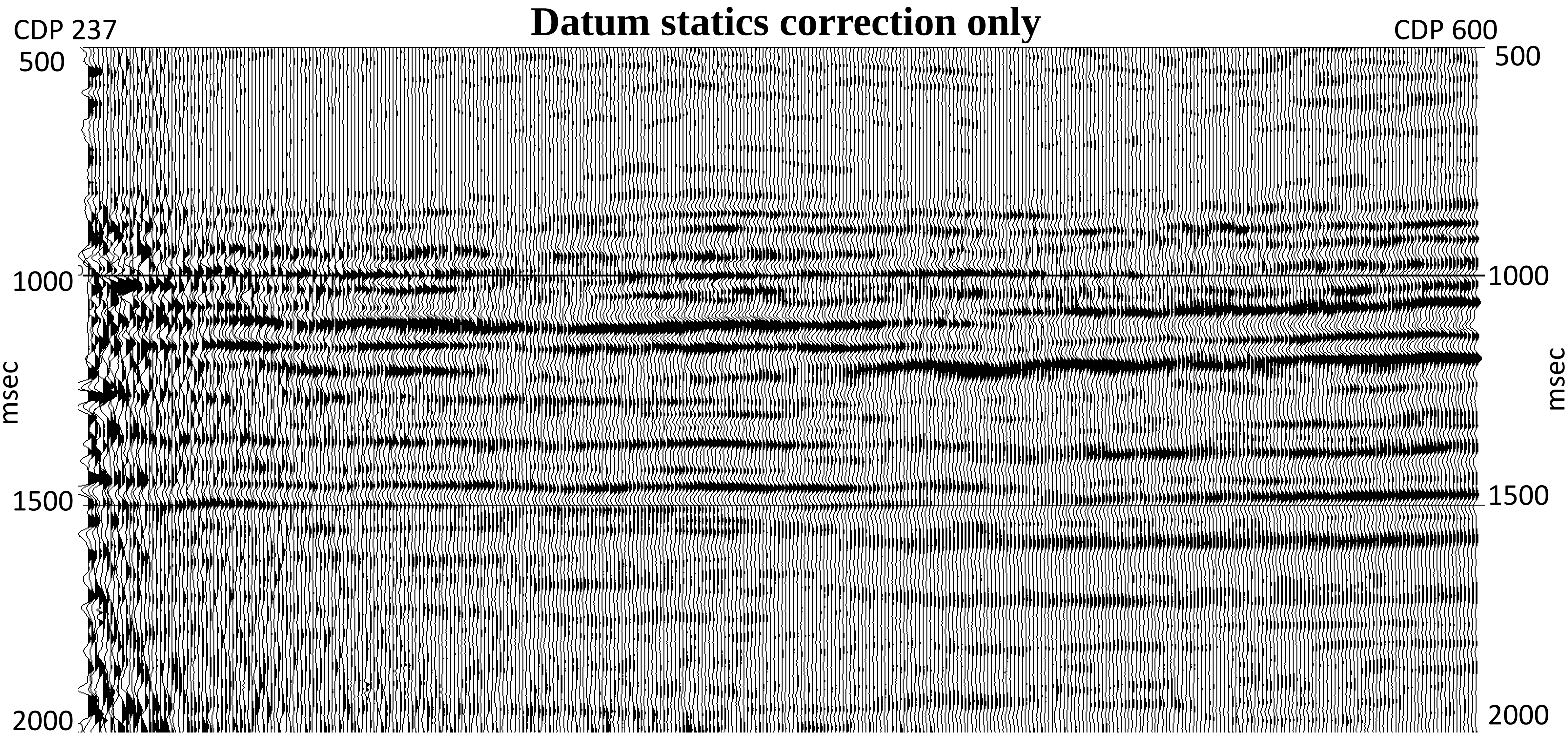
Minimize $J = ||\,W_d d \ - W_d L W_m m\,||^2$



Refraction tomography solutions

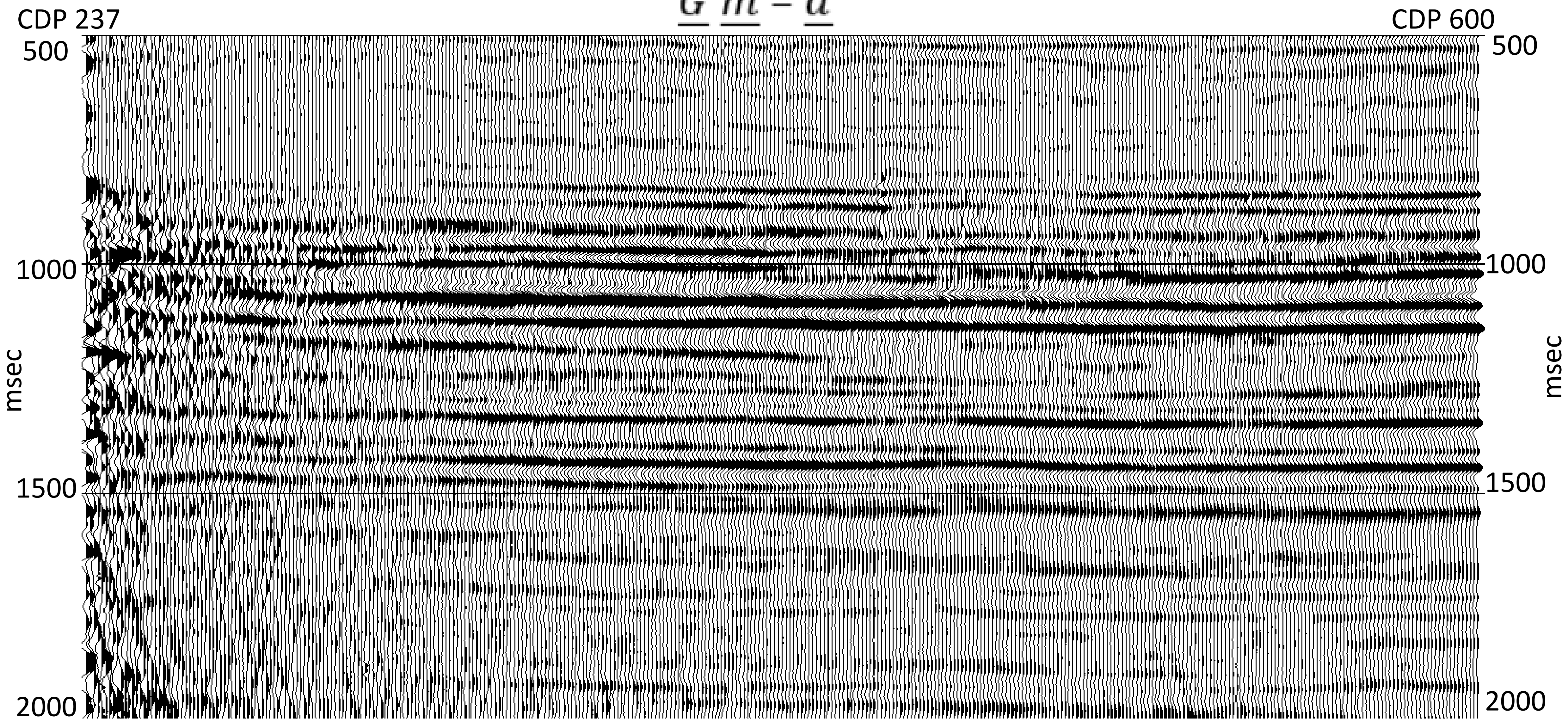


CDP Stack

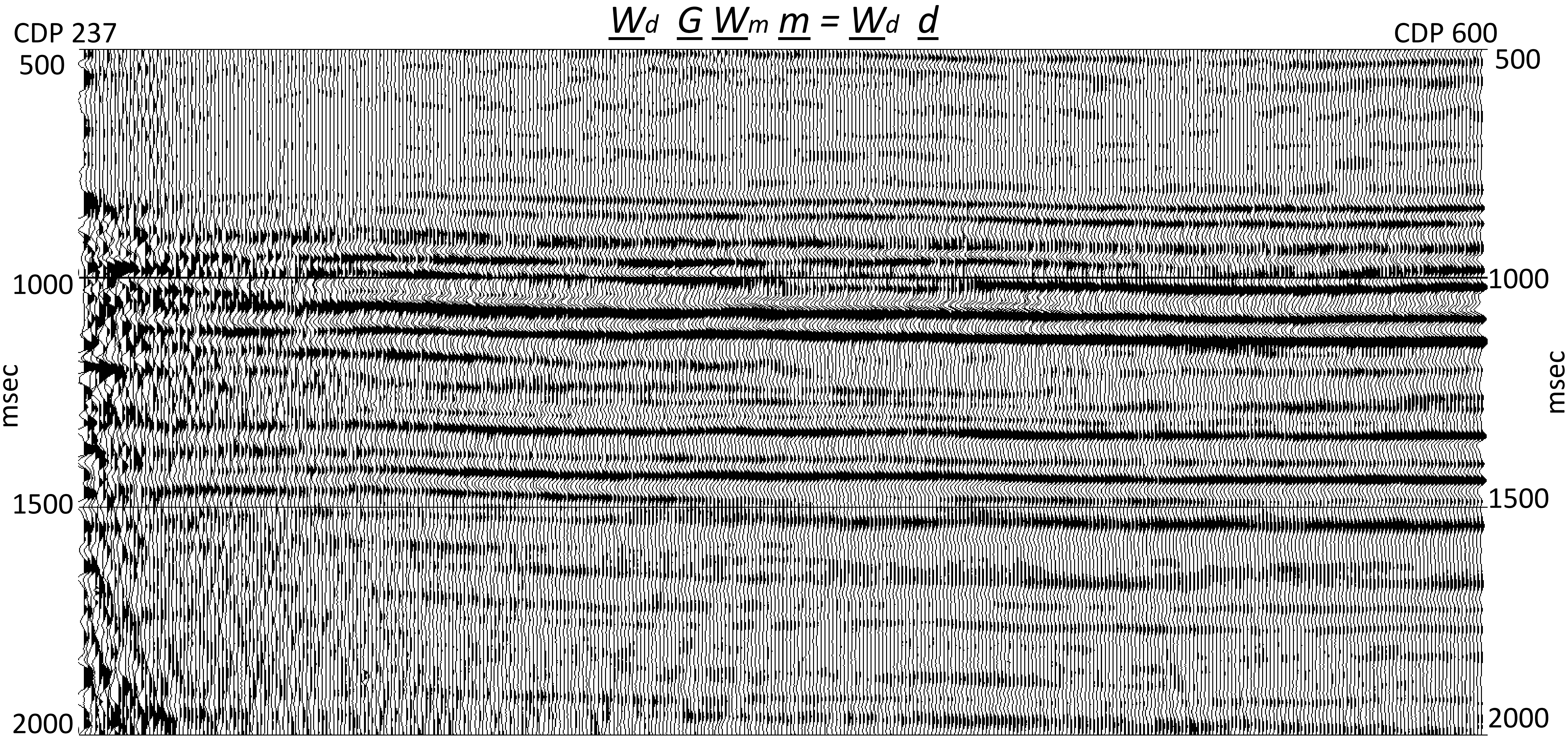

Datum statics correction only
CDP 237
CDP 600
500
1000
1500
2000
msec
500
1000
1500
2000
msec

Refraction tomography stack

$$\underline{G}\,\underline{m} = \underline{d}$$



Refraction tomography stack



FWI

- Misfit function

$$E(m) = \frac{1}{2}\sum_{r=1}^{ng}\sum_{s=1}^{ns}\int_0^{t_{max}} |\, p_{cal}(x_r, t; x_s) - p_{obs}(x_r, t; x_s)|^2$$

- Model update

$$\Delta m = -\left[\frac{\partial^2 E(m)}{\partial m^2}\right]^{-1}\frac{\partial E(m)}{\partial m}$$

- Hessian, gradient

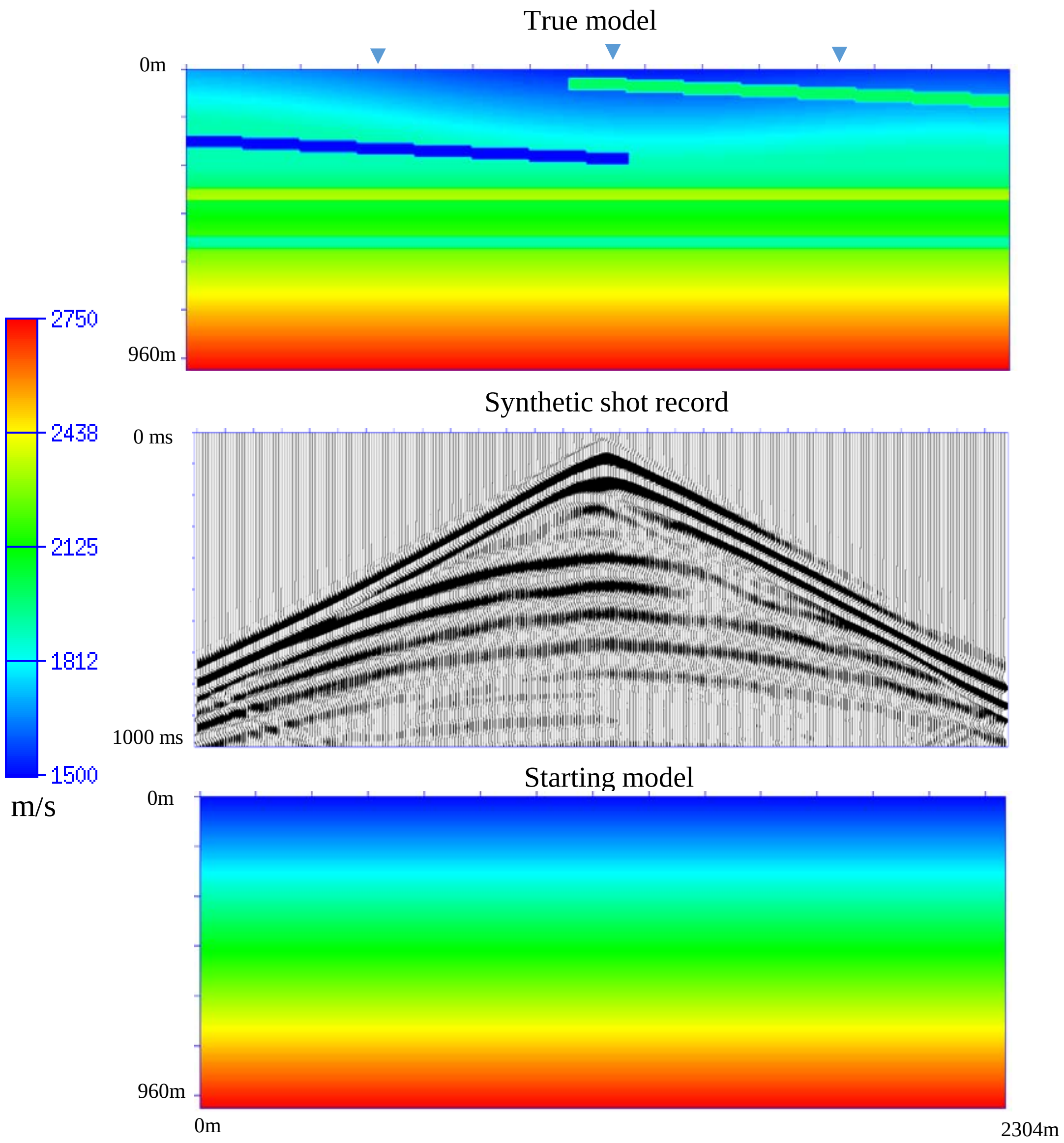
$$\Delta m = -H^{-1}\nabla E_m$$

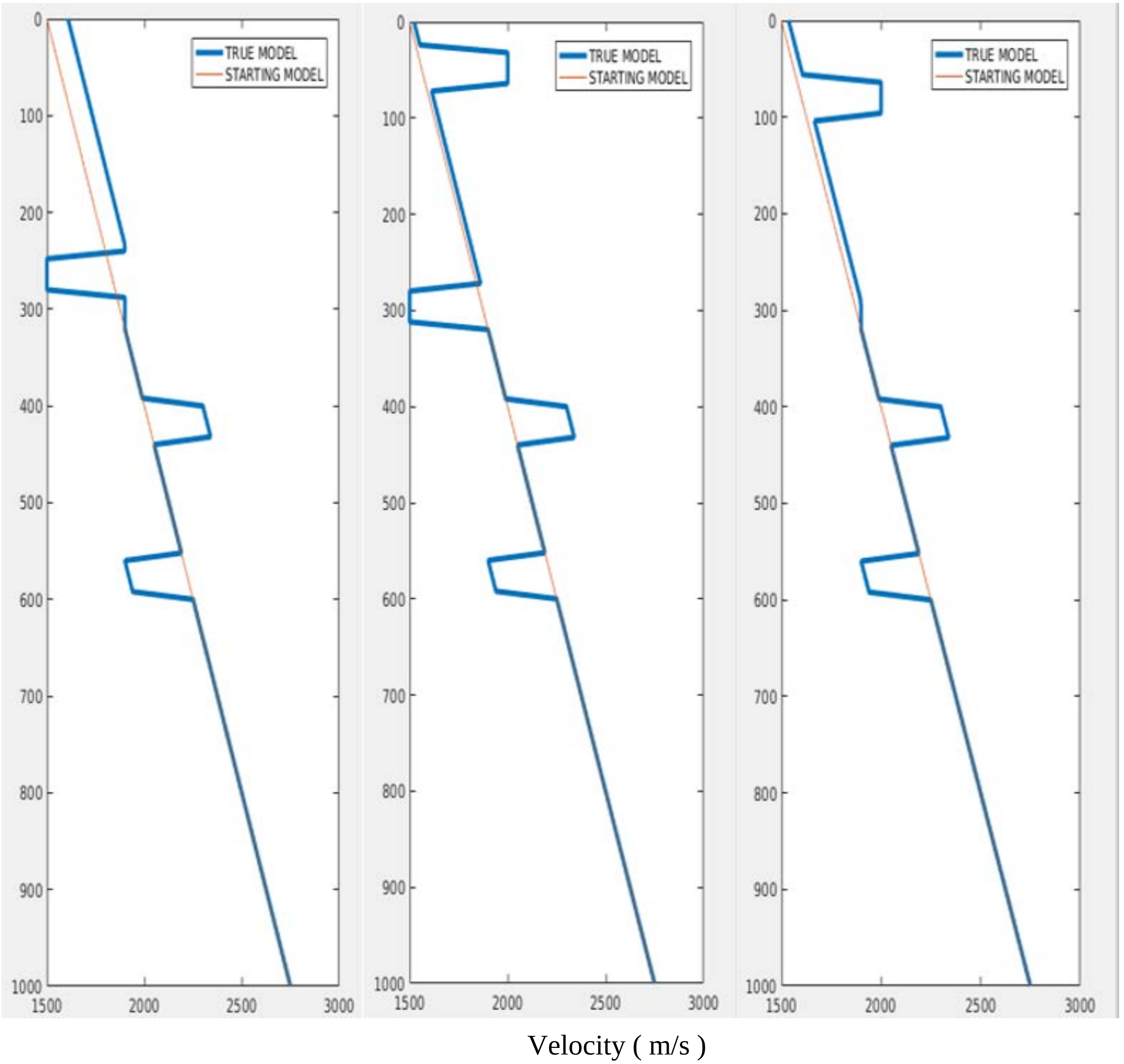
- Step length

$$\Delta m = -\alpha \nabla E_m$$

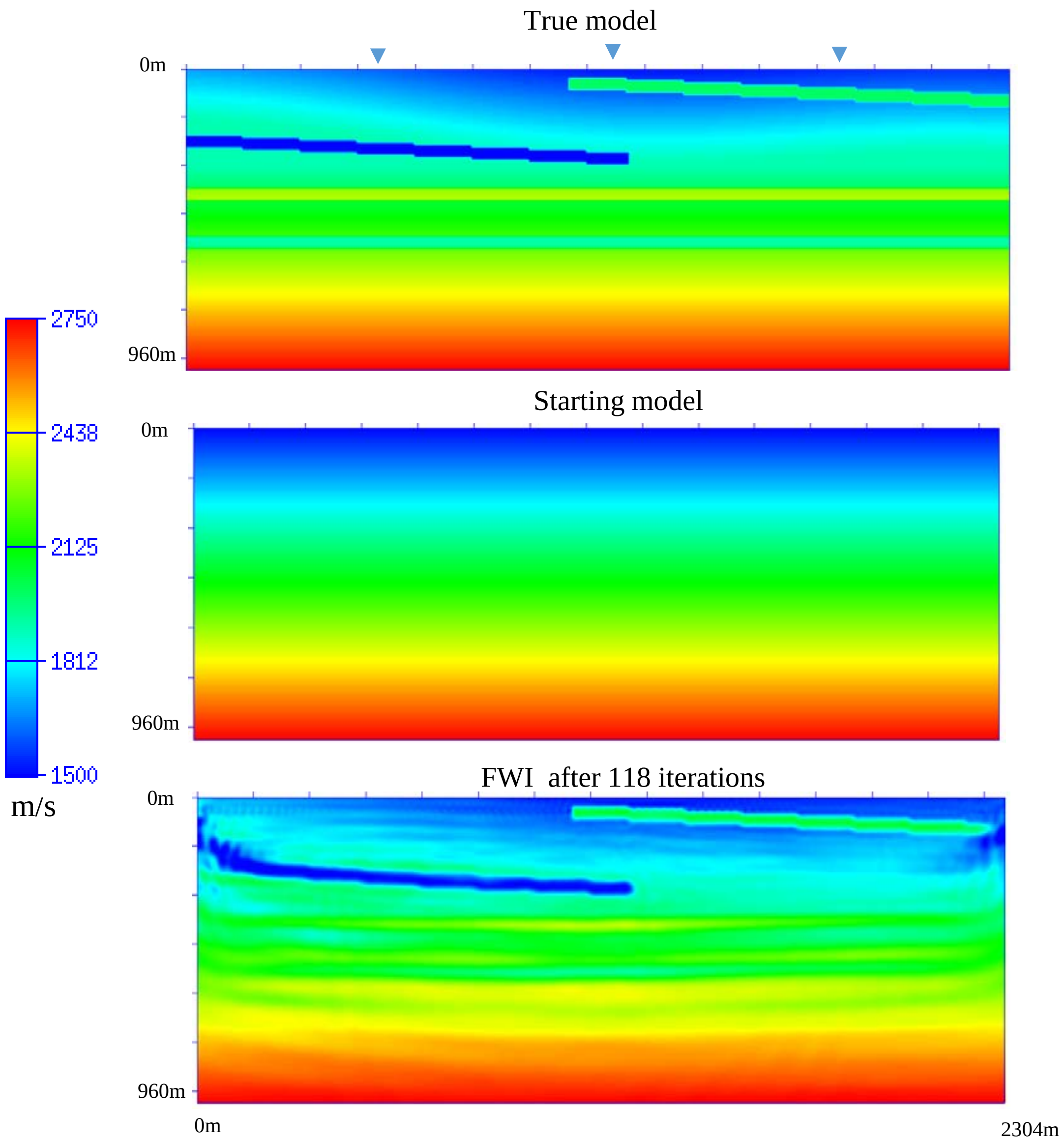
Yang 2015

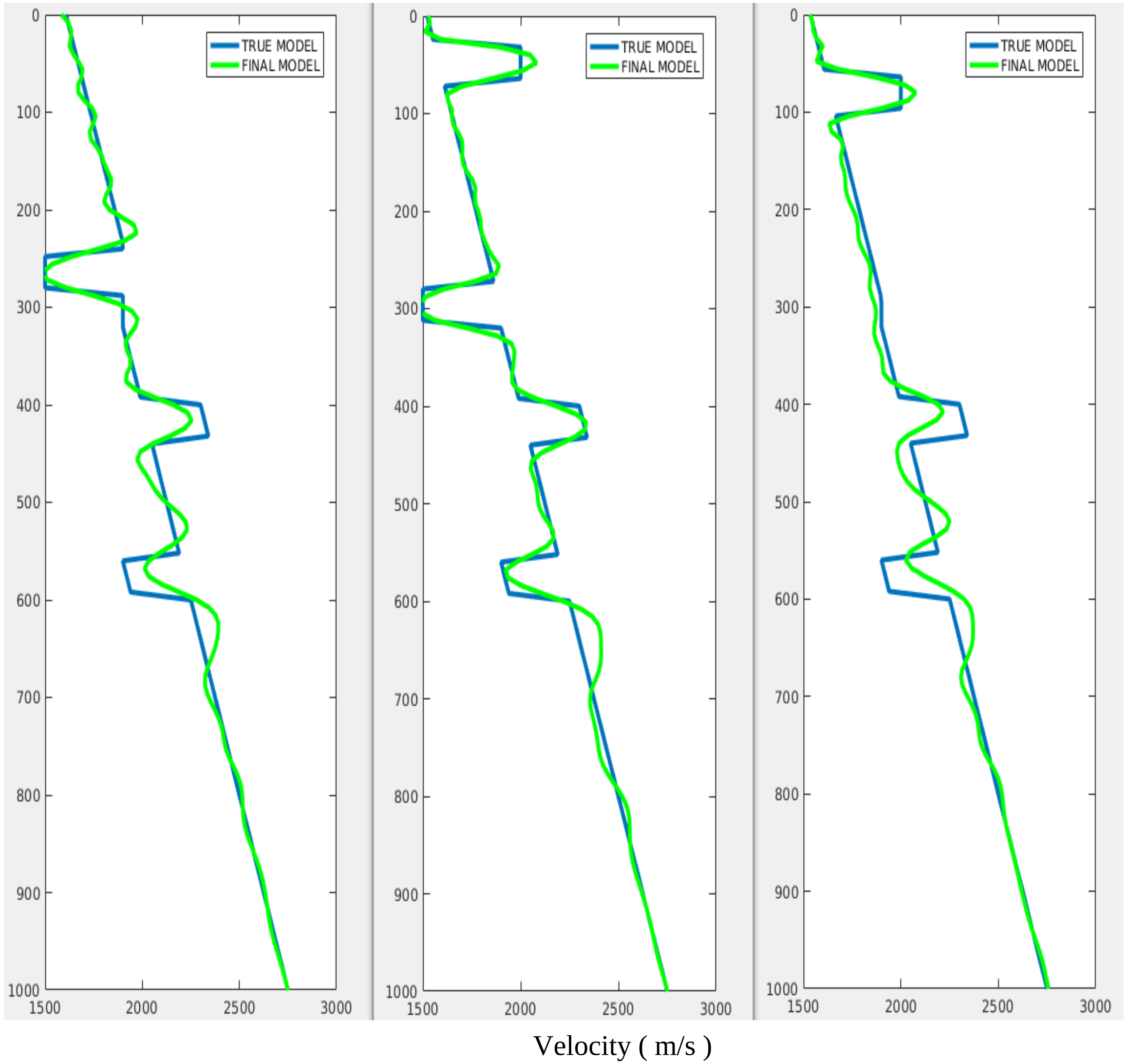
FWI numerical experiment





FWI numerical experiment





Conclusions

- We reviewed the GLI and refraction tomography methods and showed that reflection residual statics are often required.

- We modified the cost functions of GLI and refraction tomography to incorporate these reflection residual statics in the model space and data space regularization

- We used numerical experiment to demonstrate the resolving power of FWI. However, for real field data application careful preparation of the input data to ensure the input to FWI matches the assumption and physics of forward modeling is critical for successful FWI.

Acknowledgments

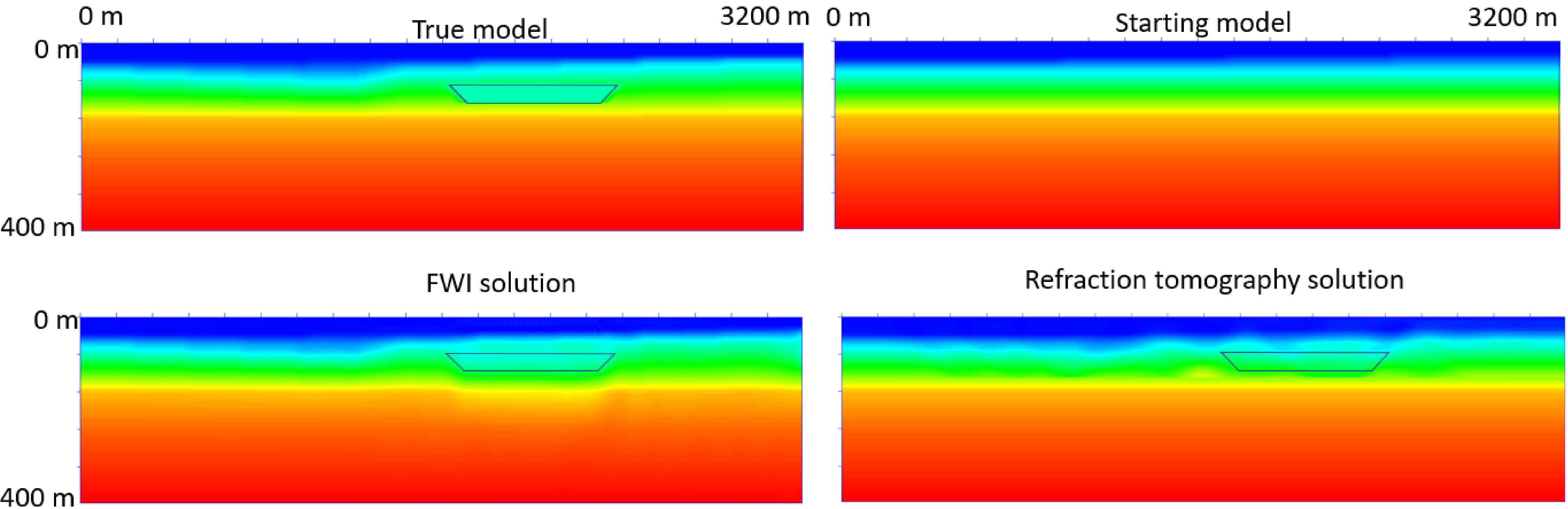
CREWES Sponsors

NSERC

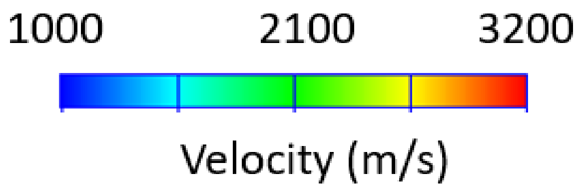
CREWES faculty, staff and students

Madagascar

FWI refraction test

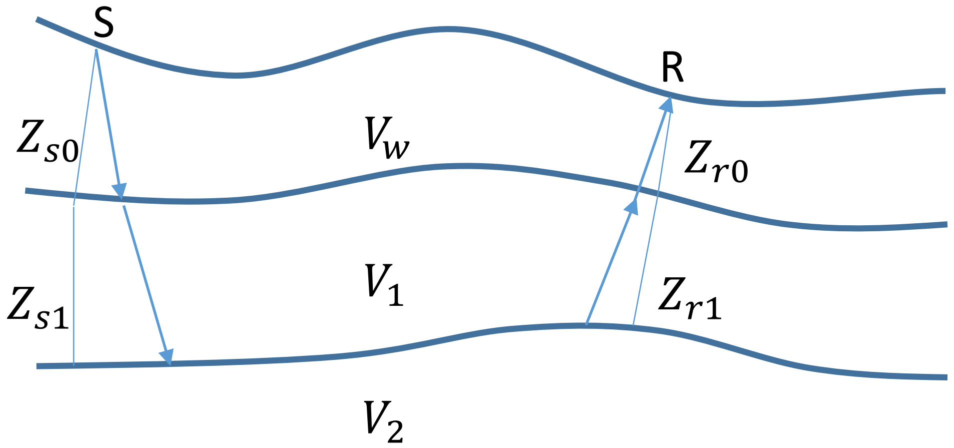


** velocity abnormally 1700 m/s
dimension 800m x 40m
depth 88m to 128m



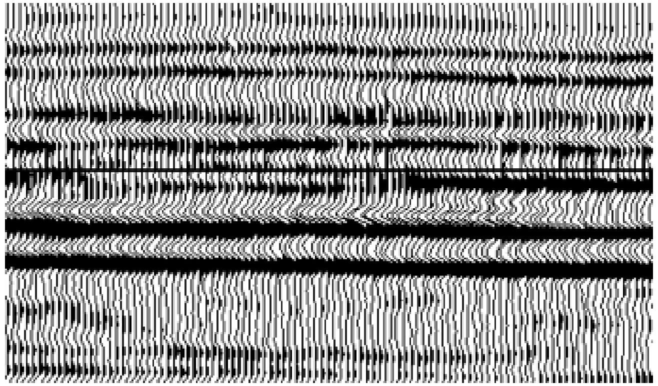
Computing model weight W_m

Refraction Inversion

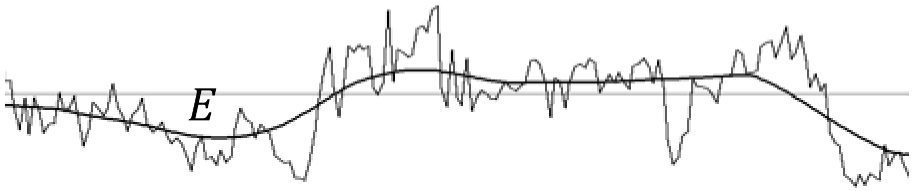


Weathering statics correction for layer i :

$$T_i = \frac{Z_i}{V_r} - \frac{Z_i}{V_i}$$



surface-consistent reflection residual statics



$$E = smoothed\ (long\ wavelength)\ residual\ statics$$

$$\boldsymbol{E_i} = E \frac{Z_i}{Total\ thickness}$$

Spatial resolution of FWI

Experimental setup of diffraction tomography

$$\boldsymbol{k} = \frac{2f}{c}\cos\left(\frac{\theta}{2}\right)\boldsymbol{n}$$

$\lambda = c/f$

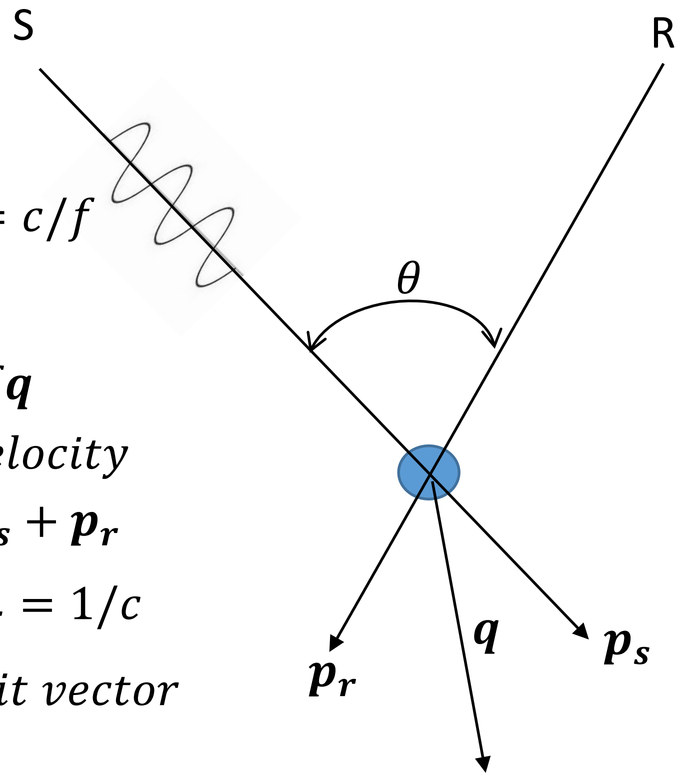
$\boldsymbol{k} = f\boldsymbol{q}$

$\mathbf{c} = velocity$

$\boldsymbol{q} = \boldsymbol{p_s} + \boldsymbol{p_r}$

$p_s = p_r = 1/c$

$\mathrm{n} = unit\ vector$

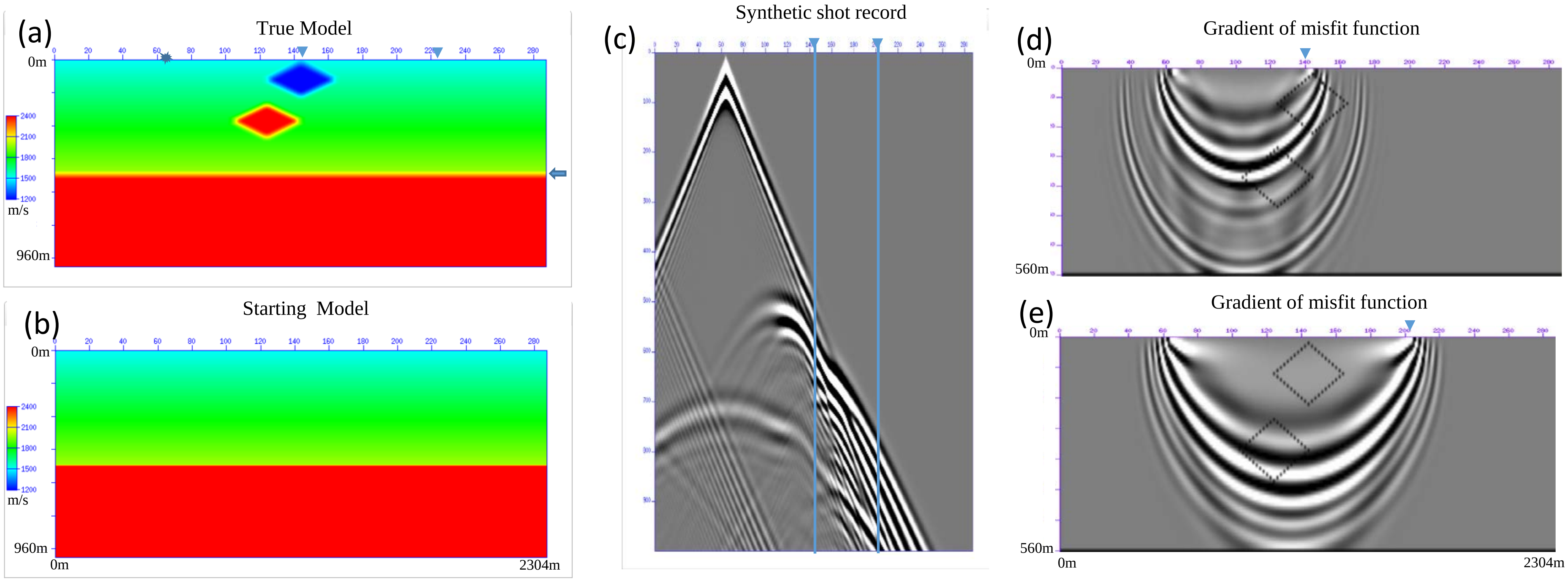


Virieux and Operto 2009

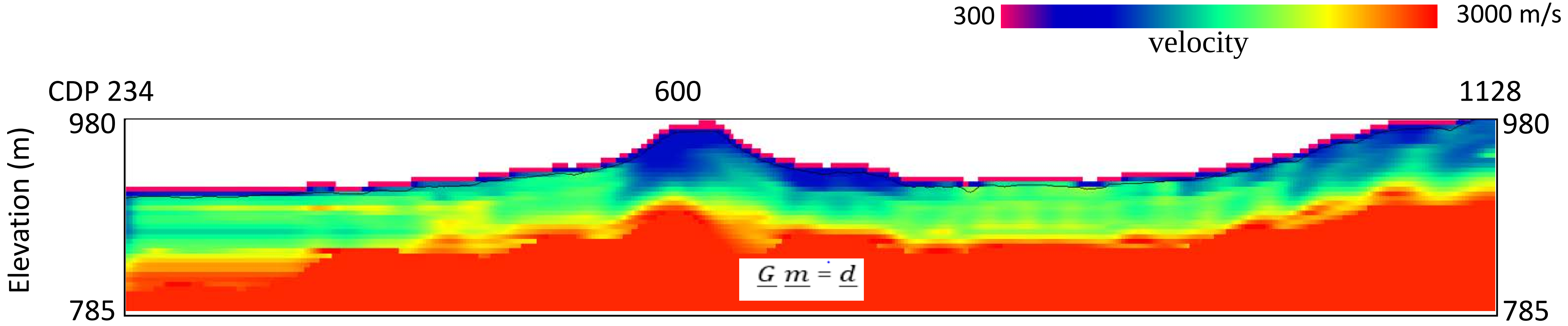
Spatial resolution of FWI

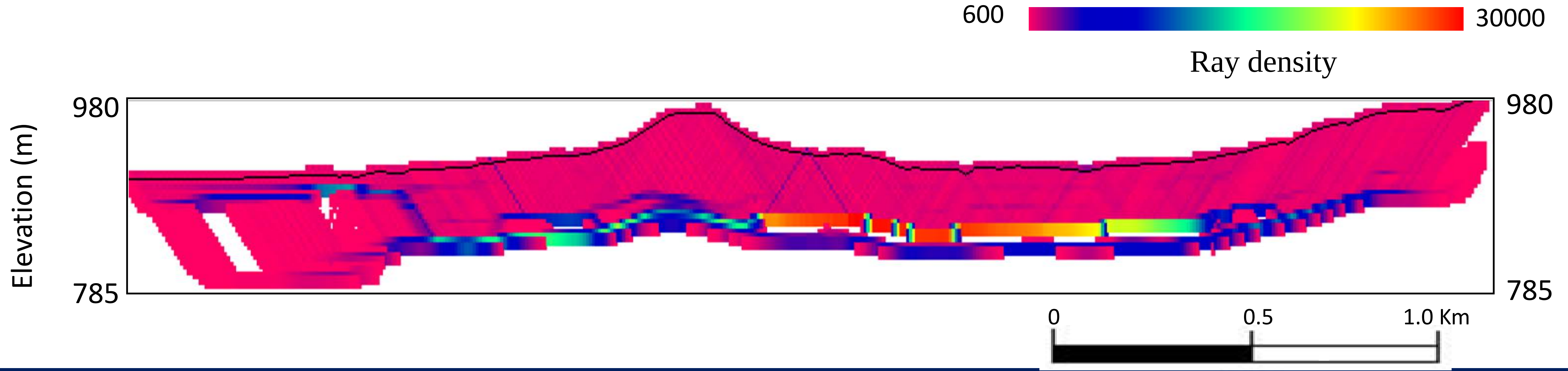
- Low frequency data and large aperture angle is required to resolve intermediate and low wavenumber (large wavelength) medium
- In shallow area where aperture angle is large, both low and high wavenumber medium can be reconstructed by direct waves, refraction and reflection in the shallow area
- In the deeper area where aperture angle is small, only high wavenumber (small wavelength) medium is reconstructed by reflection

FWI Gradient Experiment



Refraction tomography solutions





Refraction tomography solutions