A waveform inversion based on pure P- and S- wave separation

Hassan Khaniani, Shahpoor Moradi and Daniel Trad
CREWES sponsors meeting

December 1st , 2017

Outline

- **Introduction**
  - Isotropic elastic waves inversion
  - Polarization in elastic wave and crosstalk contributions
- **Crosstalk problems for full-wave equation migration and inversion**
- **Decoupled wave equation solutions**
  - Pure P- wave
  - Pure S- wave
- **Numerical experiments**
  - Simple models
  - Complex model (Marmousi II)
  - Field data 3D VSP program
- **Conclusions**

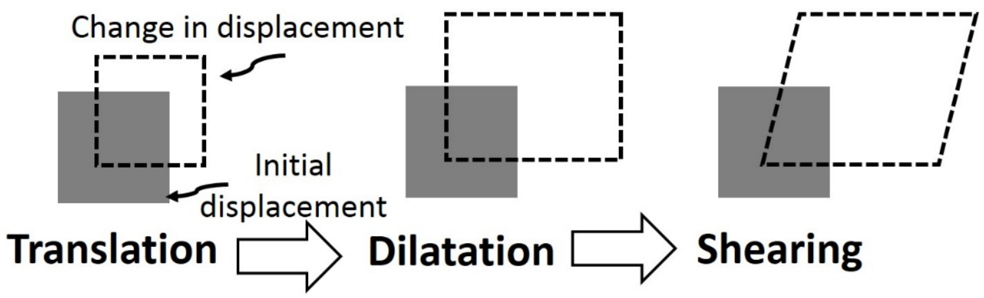
Isotropic elastic waves inversion

$$(a) : \delta\hat{\rho}(\mathbf{x}) = -\int dt \frac{\partial U_j^{\mathrm{F}}}{\partial t}\frac{\partial U_j^{\mathrm{B}}}{\partial t},$$

$$(b) : \delta\hat{\kappa}(\mathbf{x}') = -\int dt \sum_l \frac{\partial U_l^{\mathrm{F}}}{\partial x^l}\sum_k \frac{\partial U_k^{\mathrm{B}}}{\partial x^k},$$

$$(c) : \delta\hat{\mu}(\mathbf{x}') = -\int dt \sum_{lm}\left(\frac{\partial U_l^{\mathrm{F}}}{\partial x^m}+\frac{\partial U_m^{\mathrm{F}}}{\partial x^l}\right)\left(\frac{\partial U_l^{\mathrm{B}}}{\partial x^m}+\frac{\partial U_m^{\mathrm{B}}}{\partial x^l}\right) - \frac{2}{3}\int dt \sum_l \frac{\partial U_l^{\mathrm{F}}}{\partial x^l}\sum_k \frac{\partial U_k^{\mathrm{B}}}{\partial x^k}.$$

Particle displacement



ρ **: Density**

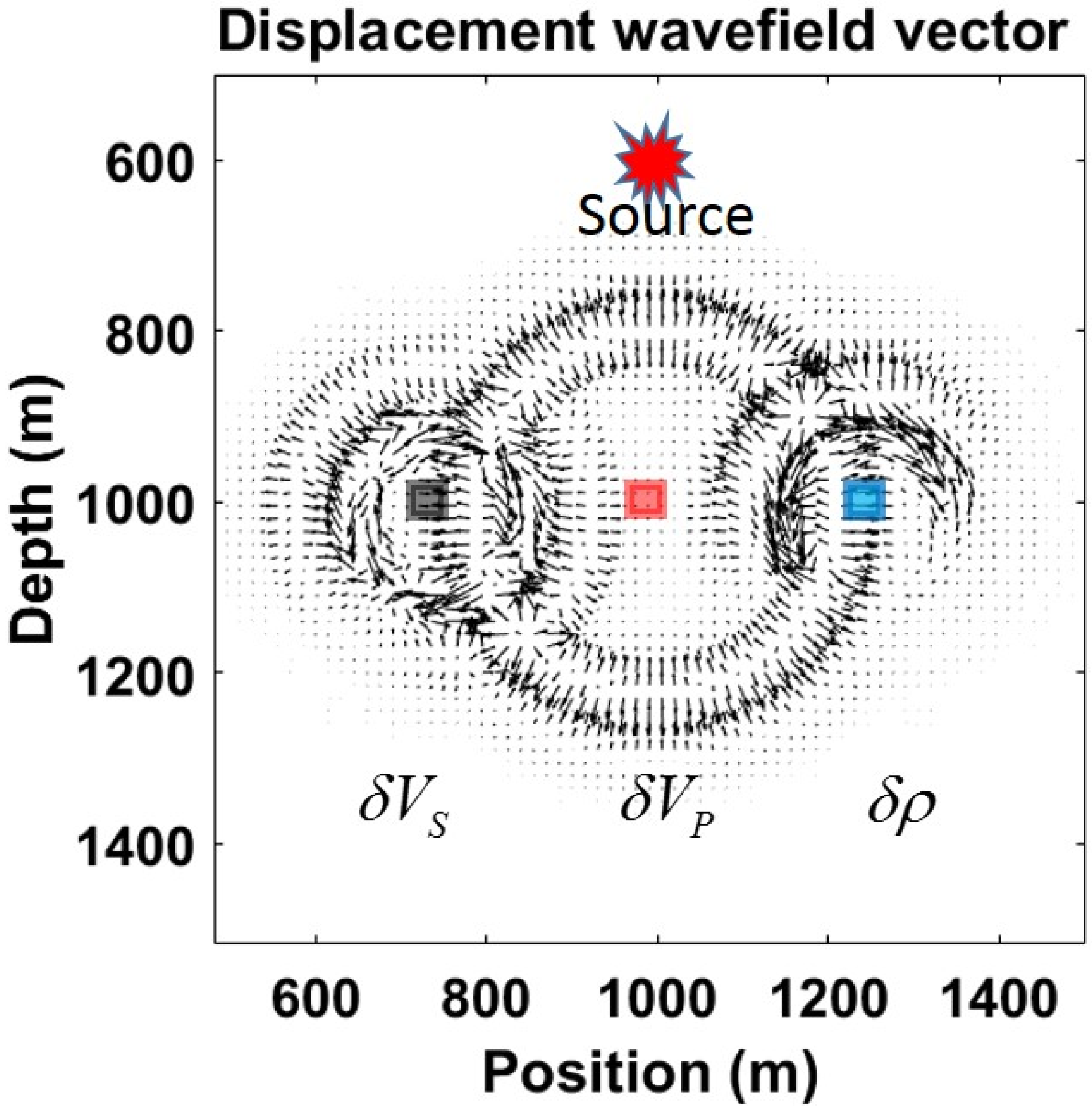
κ **: Bulk modulus**

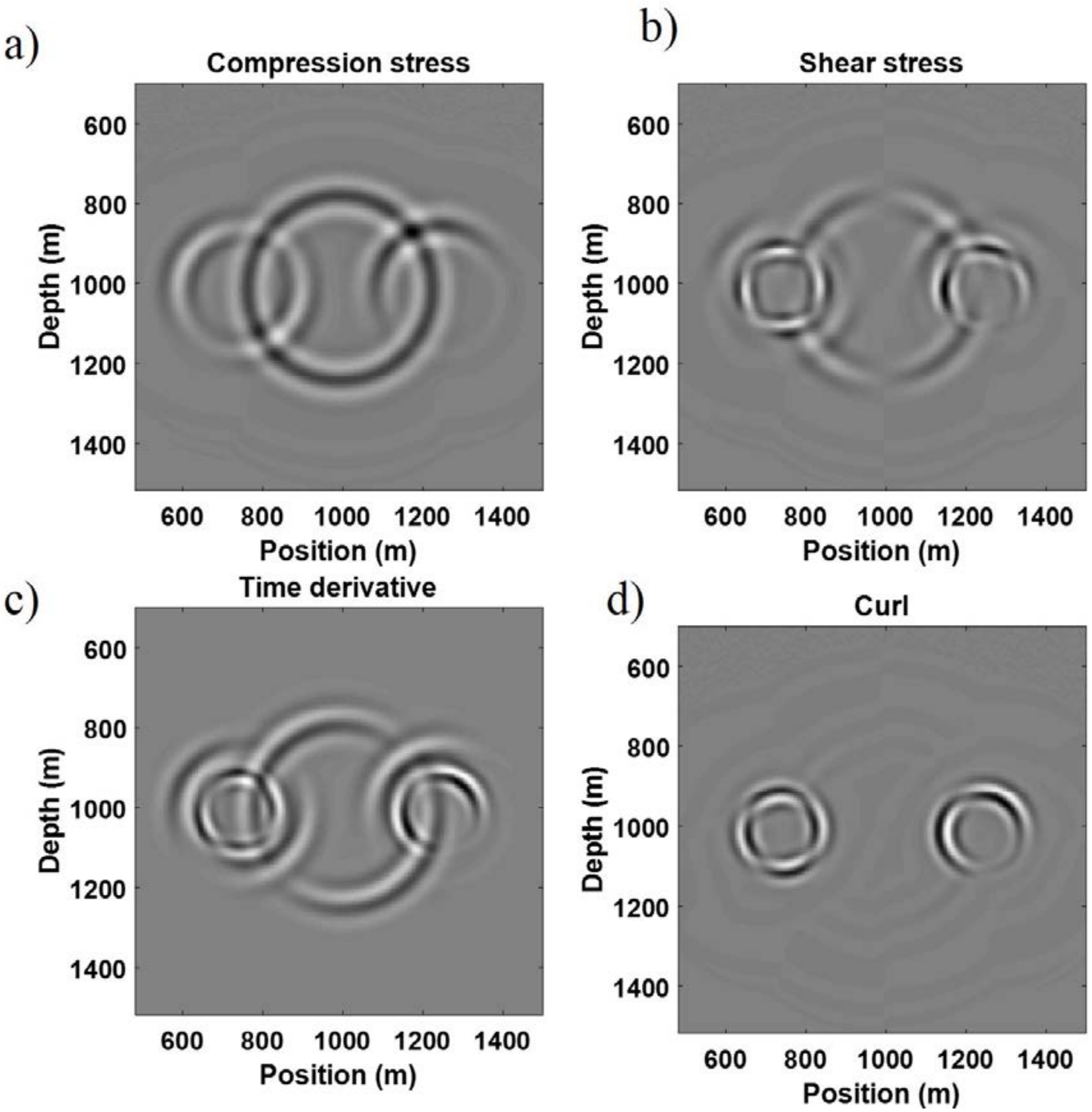
μ **: Shear modulus**

U^F **: Forward propagated displacement field from source**

U^B **: Backward propagated displacement field from receivers**

Efficiency of inversion formula





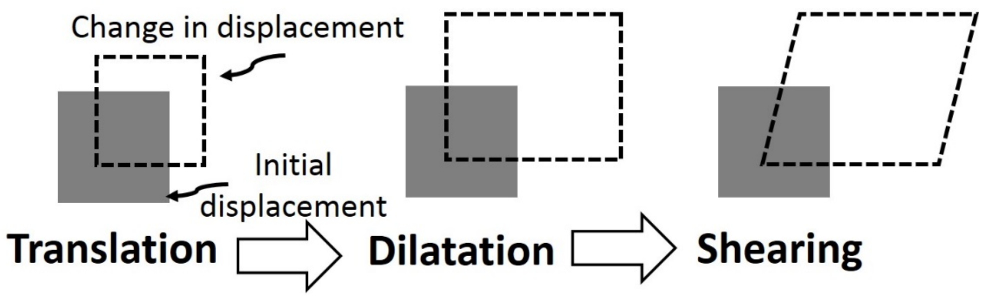
Isotropic elastic waves inversion

$$(a) : \delta\hat{\rho}(\mathbf{x}) = -\int dt \frac{\partial U_j^{\mathrm{F}}}{\partial t}\frac{\partial U_j^{\mathrm{B}}}{\partial t},$$

$$(b) : \delta\hat{\kappa}(\mathbf{x}') = -\int dt \sum_l \frac{\partial U_l^{\mathrm{F}}}{\partial x^l} \sum_k \frac{\partial U_k^{\mathrm{B}}}{\partial x^k},$$

$$(c) : \delta\hat{\mu}(\mathbf{x}') = -\int dt \sum_{lm} \left(\frac{\partial U_l^{\mathrm{F}}}{\partial x^m} + \frac{\partial U_m^{\mathrm{F}}}{\partial x^l}\right)\left(\frac{\partial U_l^{\mathrm{B}}}{\partial x^m} + \frac{\partial U_m^{\mathrm{B}}}{\partial x^l}\right) - \frac{2}{3}\int dt \sum_l \frac{\partial U_l^{\mathrm{F}}}{\partial x^l} \sum_k \frac{\partial U_k^{\mathrm{B}}}{\partial x^k}.$$

Particle displacement



ρ **: Density**

κ **: Bulk modulus**

μ **: Shear modulus**

U^F **: Forward propagated displacement field from source**

U^B **: Backward propagated displacement field from receivers**

: both of P- and S- waves has the main contribution

:P- wave has the main contribution

:S- wave has the main contribution

Problems…

- **Computational costs**
  - Optimization (forward modeling)
  - Migration and inversion
- **Unexpected artifacts due to the crosstalk effect**
  - Colocation of various wave mode
- **Wavefield separation (or transformations) and decomposition**
  - Divergence and curl operators on the wavefield (Dellinger and Etgen, 1990)
  - Projection of the wavefield along the polarization vectors (Yan and Sava, 2009; Zhang and McMechan, 2010; Ren and Liu, 2016; Wang and Cheng, 2017)
  - Subtracting the pure P- wave from full-wave equation (Wang et al., 2015)

They require full-wave equation solution

Solution by pure mode P- and S- wave separation

- **Pure mode P- and S- wave equation**

  (Chen, 2014, Cheng and Kang, 2014, 2016 and Yuan, 2017)

- **Benefits**
  - Preserve the multicomponent features
  - Reduce the uncertainty of migration/inversion by avoiding colocation of P- and S-waves images
  - Computationally faster that full-wave equation

Pure P- and S- wave equation

Isotropic media

$$\nabla \cdot \left(\tilde{\Gamma} \tilde{\boldsymbol{U}} - \rho \omega^2 \tilde{\boldsymbol{U}} \right) = 0 \quad \text{and} \quad \nabla \times \left(\tilde{\Gamma} \tilde{\boldsymbol{U}} - \rho \omega^2 \tilde{\boldsymbol{U}} \right) = 0$$

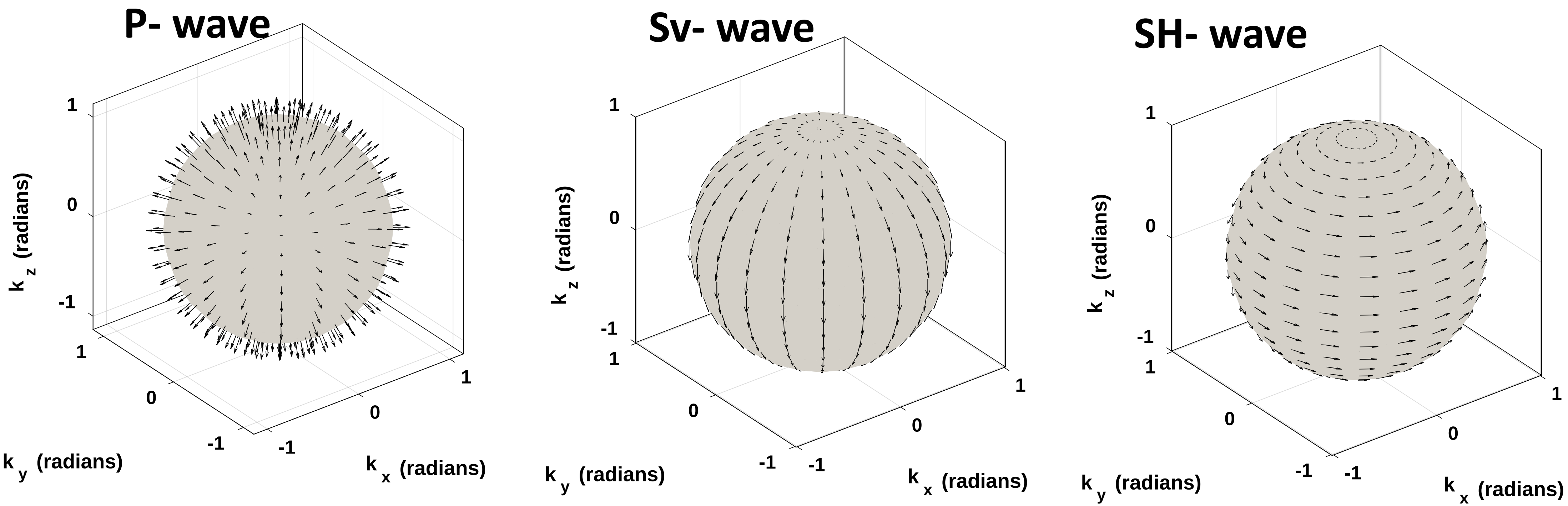
Anisotropic media

$$\left(M \tilde{\Gamma} M^{-1} \right) M \, \tilde{\boldsymbol{U}} = \rho \omega^2 M \tilde{\boldsymbol{U}};$$

(Cheng and Kang, 2016)

$\tilde{\Gamma}$: **Christoffel matrix** M: **Polarization diagonal matrix** ω: **Angular frequency**

Polarization



Pure mode P- and S- wave equation in isotropic media

Velocity-stress method in staggered grid finite difference time domain

P- wave

$$\frac{\partial v_1}{\partial t} = V_P^2 \frac{\partial D}{\partial x_1},$$
$$\frac{\partial v_2}{\partial t} = V_P^2 \frac{\partial D}{\partial x_2},$$
$$\frac{\partial v_3}{\partial t} = V_P^2 \frac{\partial D}{\partial x_3},$$
$$\frac{\partial D}{\partial t} = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3},$$

$\boldsymbol{v}$ **: Particle velocity**

D **: Compressional stress**

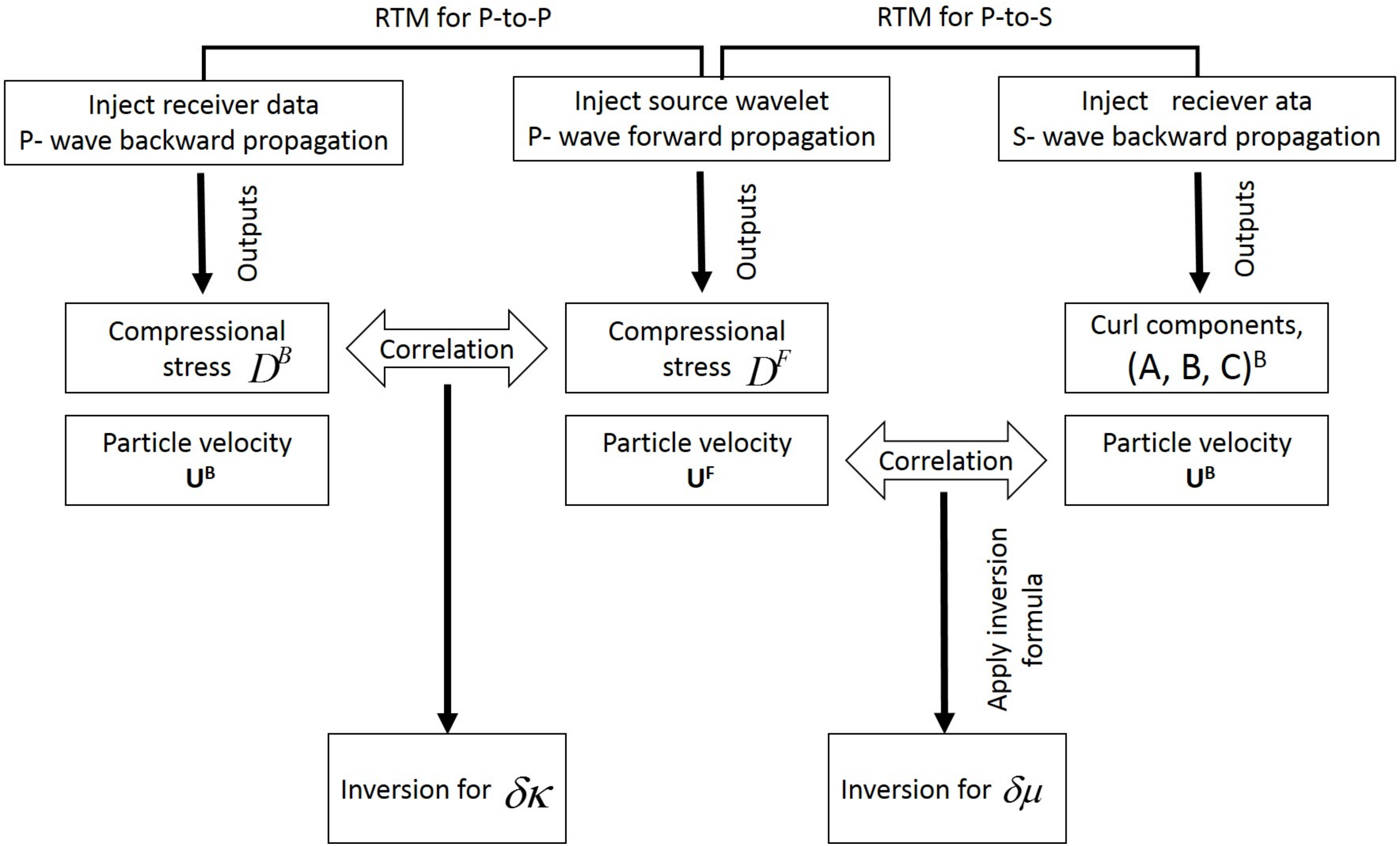
S- wave

$$\frac{\partial v_1}{\partial t} = -V_S^2 \left(\frac{\partial C}{\partial x_2} + \frac{\partial B}{\partial x_3} \right),$$
$$\frac{\partial v_2}{\partial t} = V_S^2 \left(\frac{\partial C}{\partial x_1} - \frac{\partial A}{\partial x_3} \right),$$
$$\frac{\partial v_3}{\partial t} = V_S^2 \left(\frac{\partial A}{\partial x_2} + \frac{\partial B}{\partial x_1} \right),$$
$$\frac{\partial A}{\partial t} = \frac{\partial v_2}{\partial x_3} - \frac{\partial v_3}{\partial x_2},$$
$$\frac{\partial B}{\partial t} = \frac{\partial v_1}{\partial x_3} - \frac{\partial v_3}{\partial x_1},$$
$$\frac{\partial C}{\partial t} = \frac{\partial v_1}{\partial x_2} - \frac{\partial v_2}{\partial x_1},$$

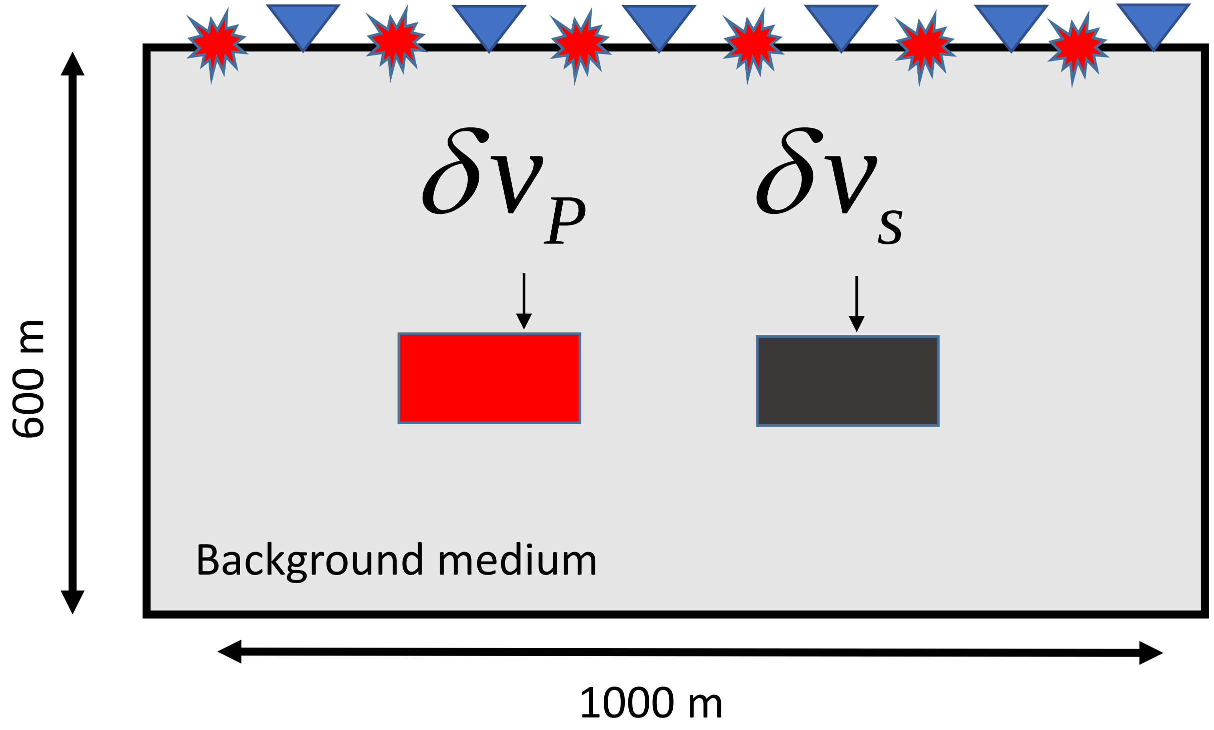
$(A,$ B and $C)$

Curl components

Description of algorithm for P-to-P and P-to-S

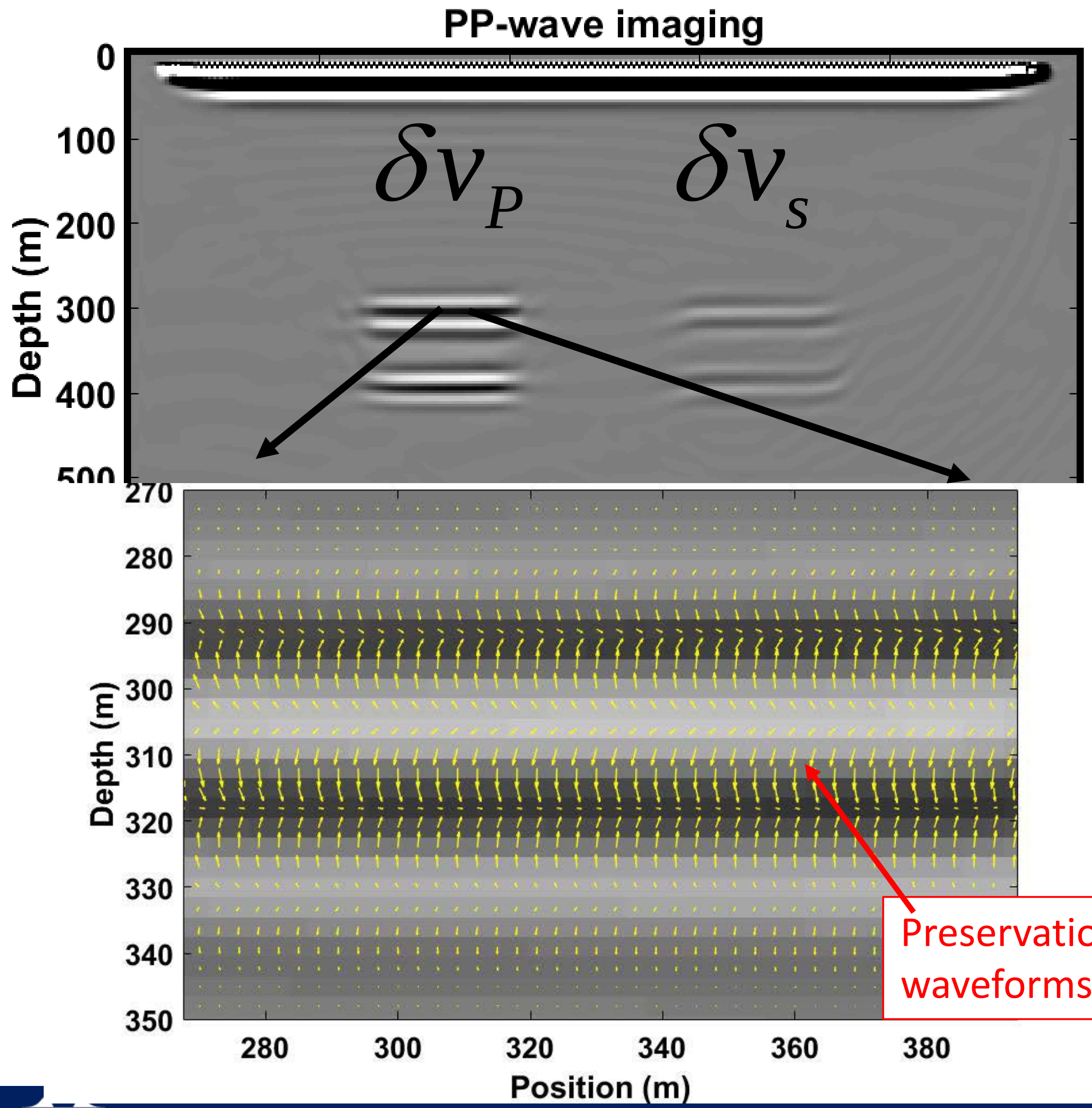


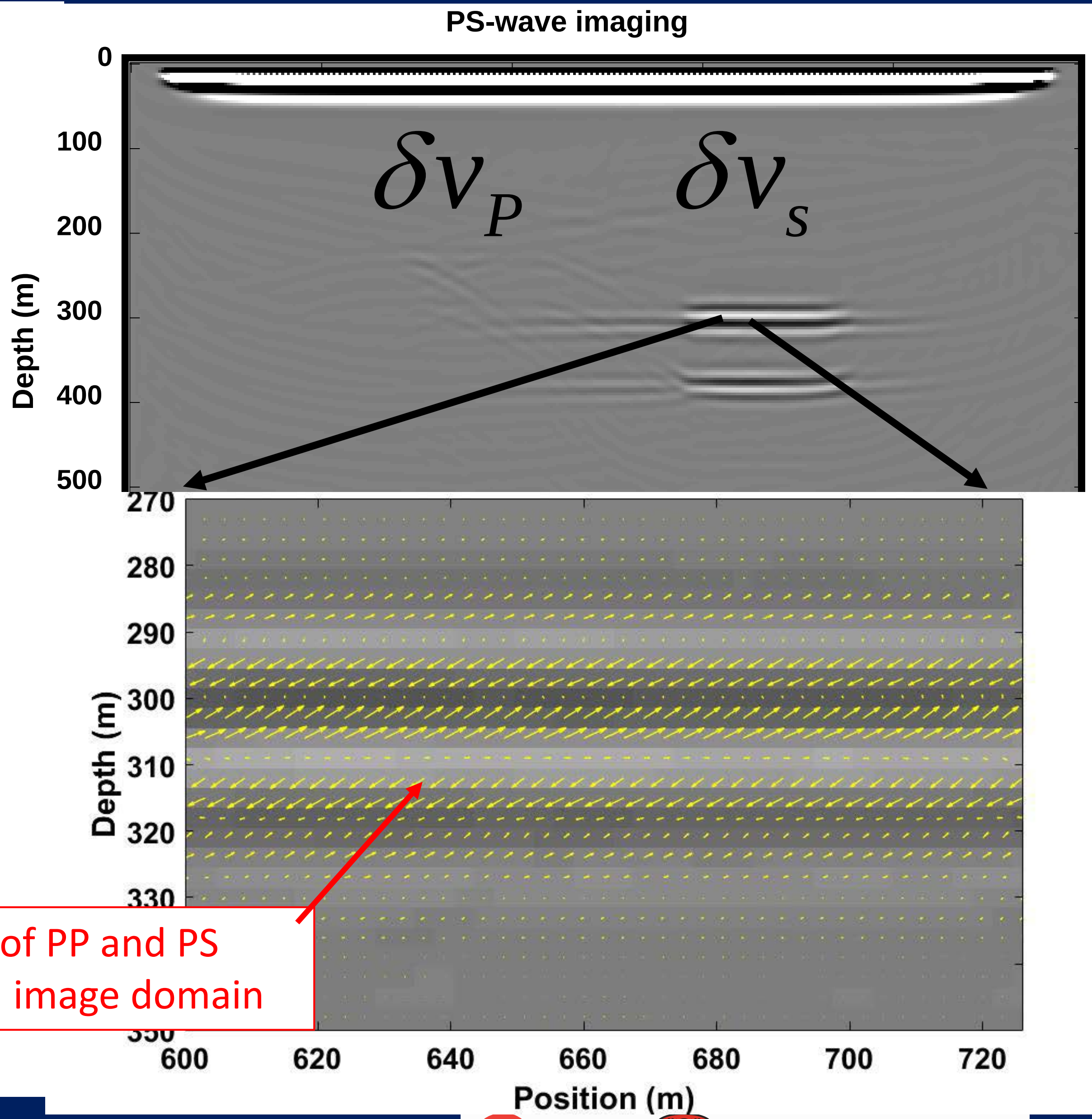
Simple model experiments



151 shots records are simulated and migrated/inverted

Numerical examples



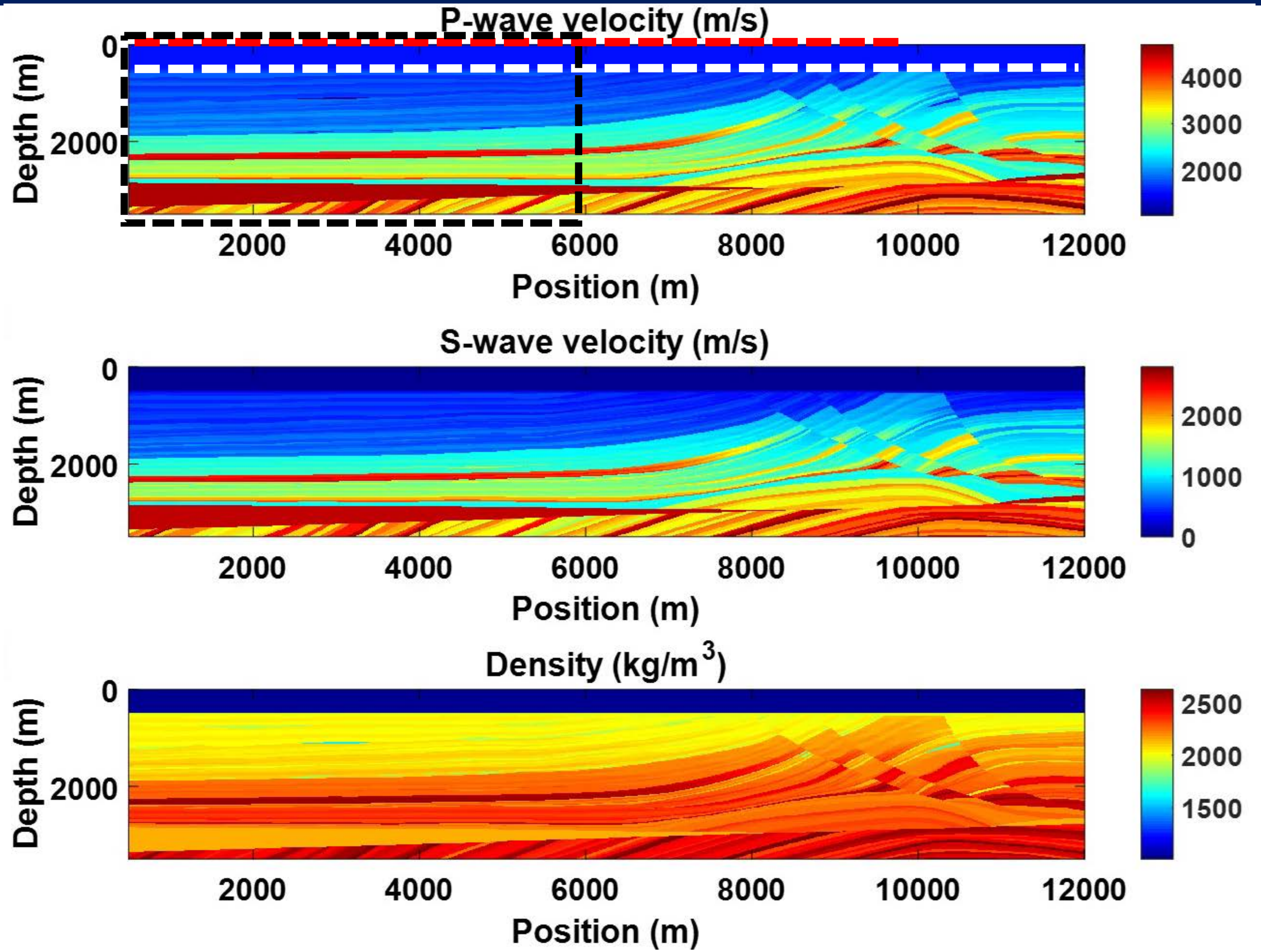


Preservation of PP and PS waveforms in image domain

Marmousi II marine environment

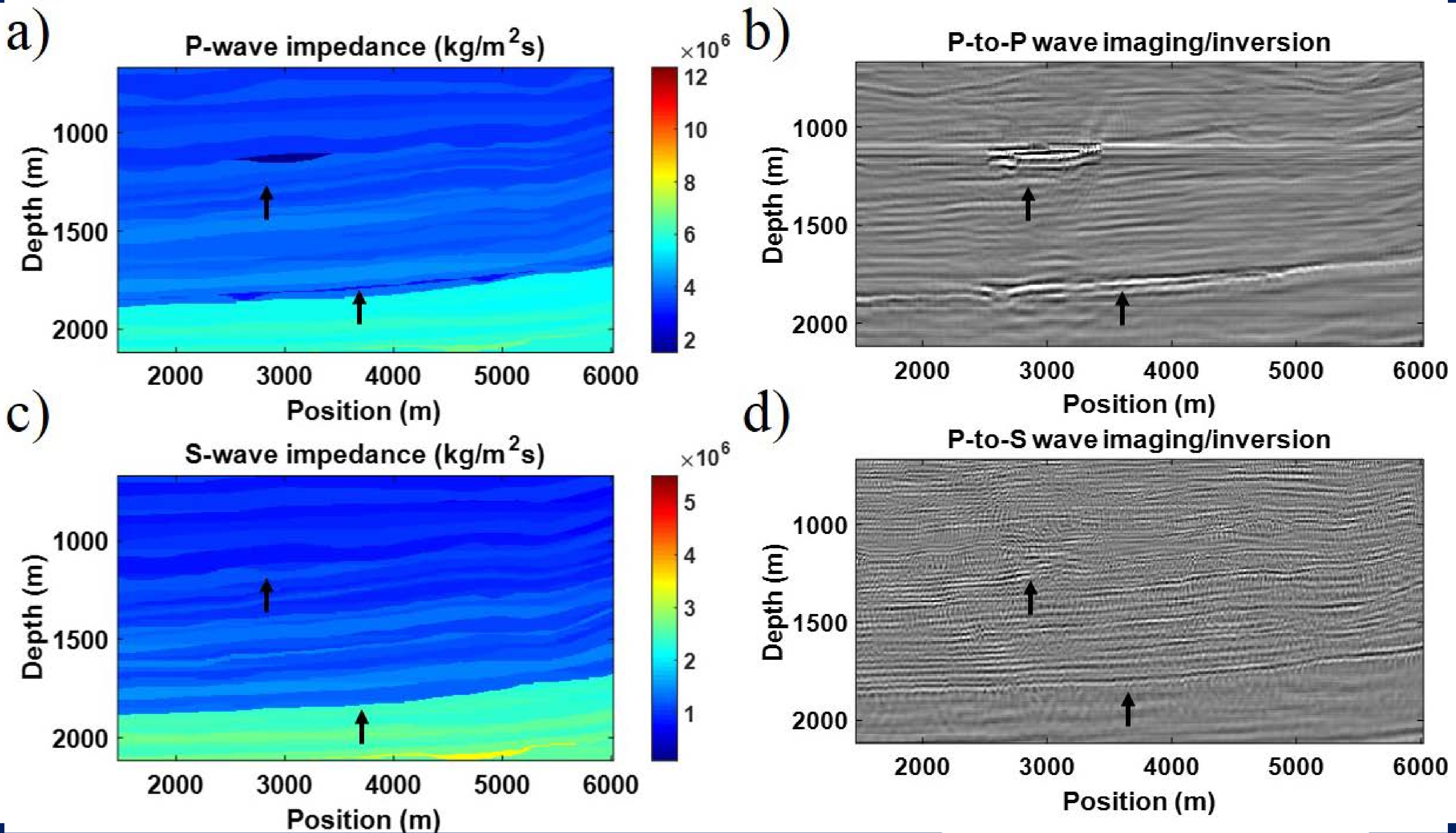
Image quality

- **Acquisition limitations**
  - **Shot position**
  - **Receiver length (10 km in split spread configuration)**
- **Amplitude radiation patterns (AVO effects)**



Marmousi II marine environment

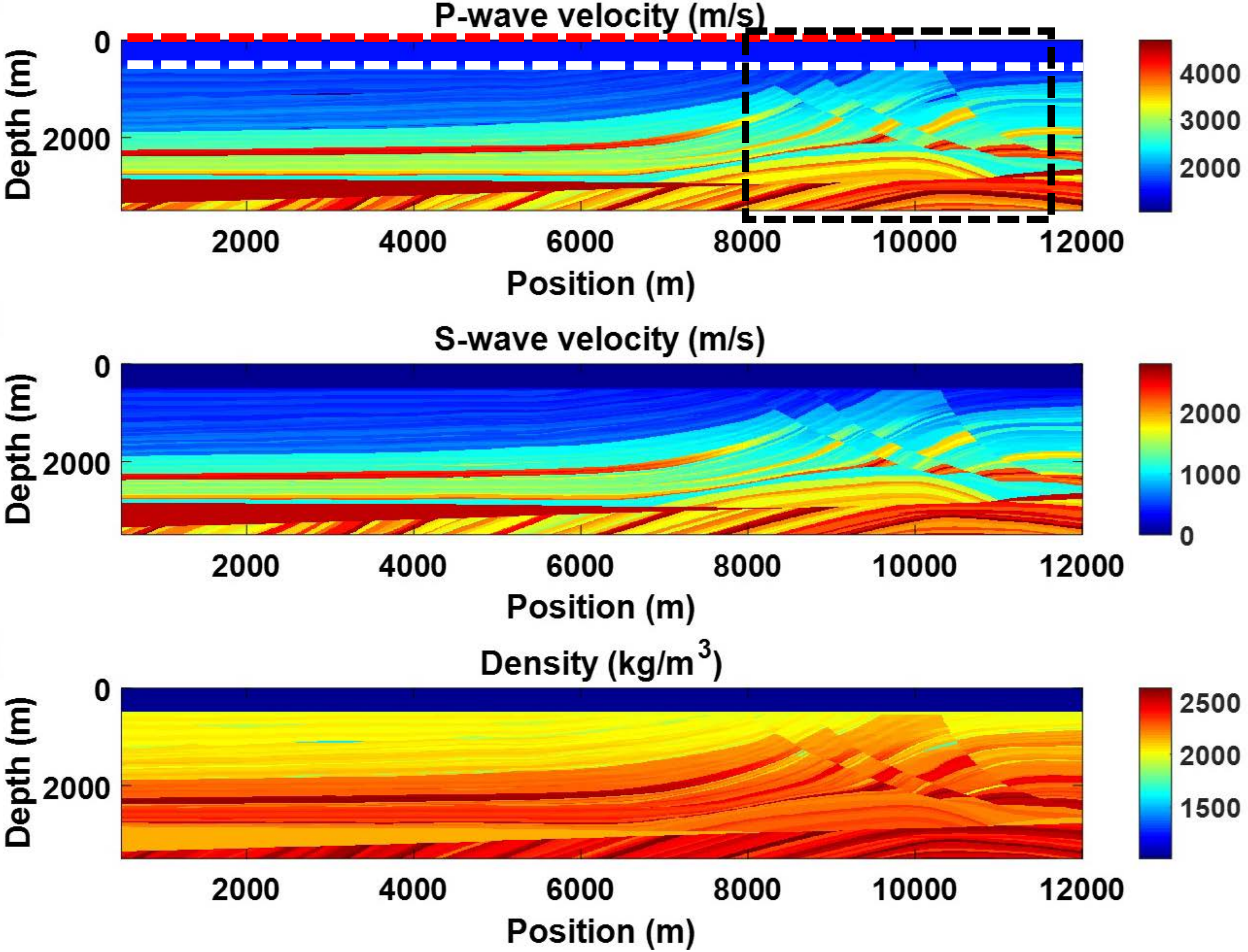
Smooth lateral variation in elastic properties



Marmousi II marine environment

Image quality

- **Acquisition limitations**
  - **Shot position**
  - **Receiver length (10 km in split spread configuration)**
- **Amplitude radiation patterns (AVO effects)**

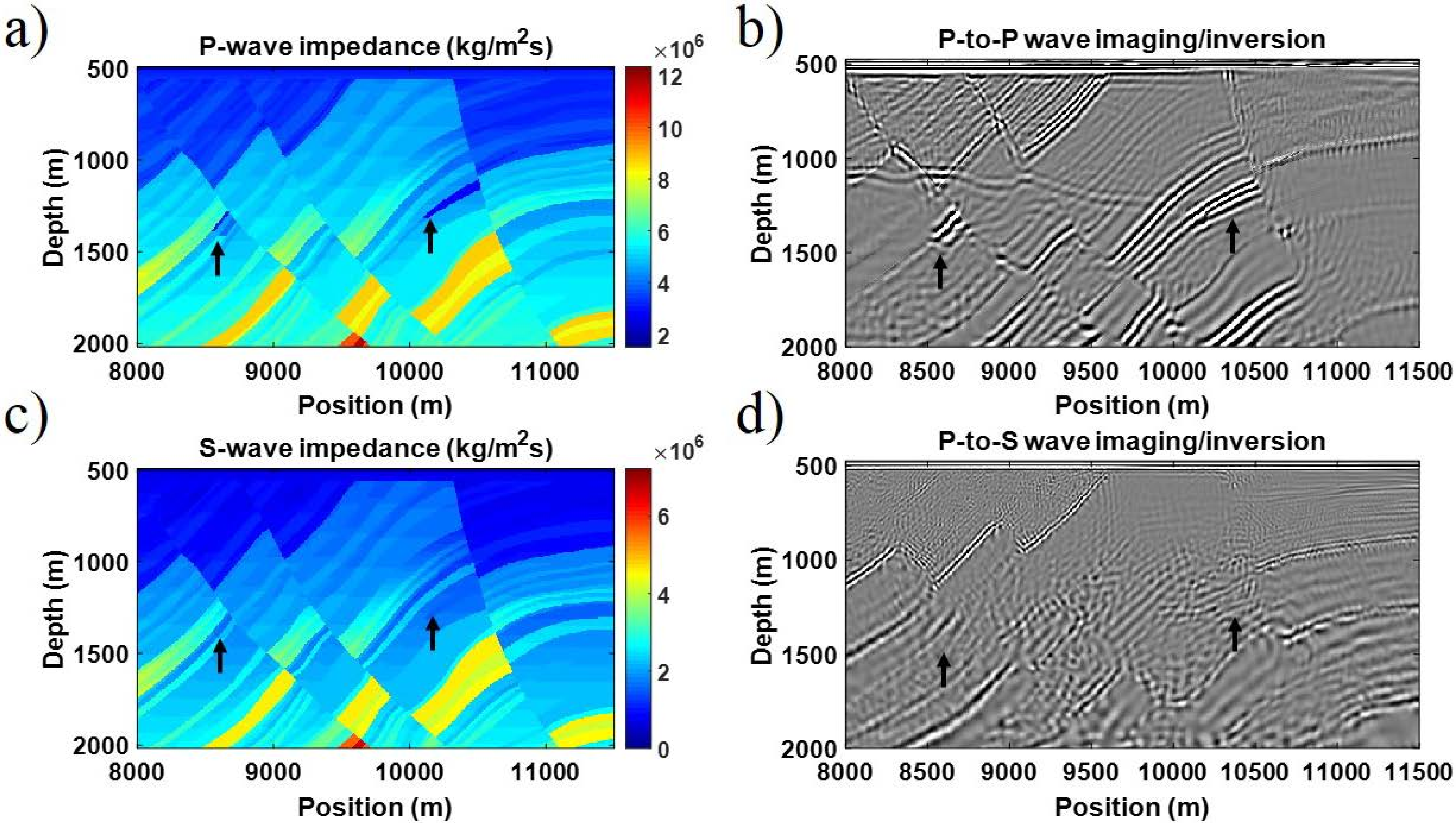


CREWES

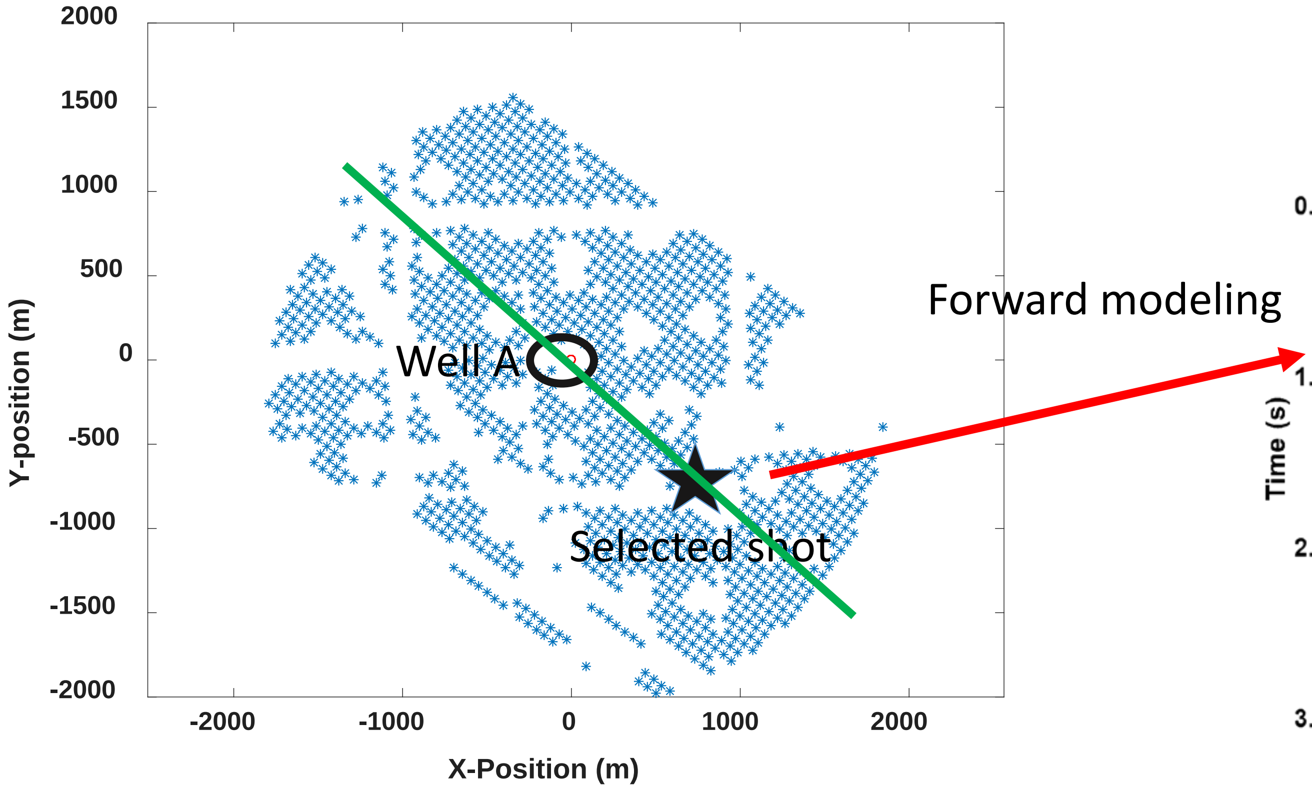
NSERC CRSNG

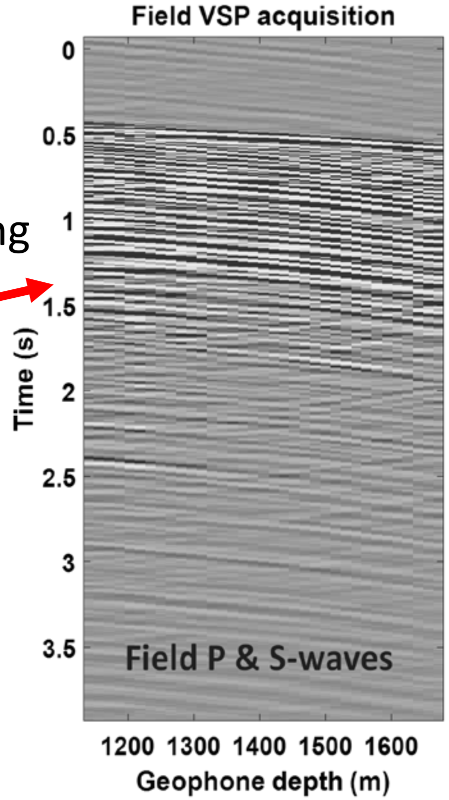
UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
Department of Geoscience

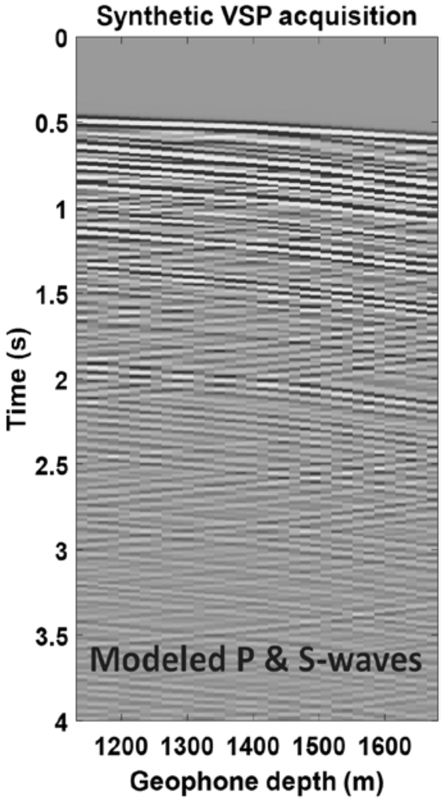
Complex structure



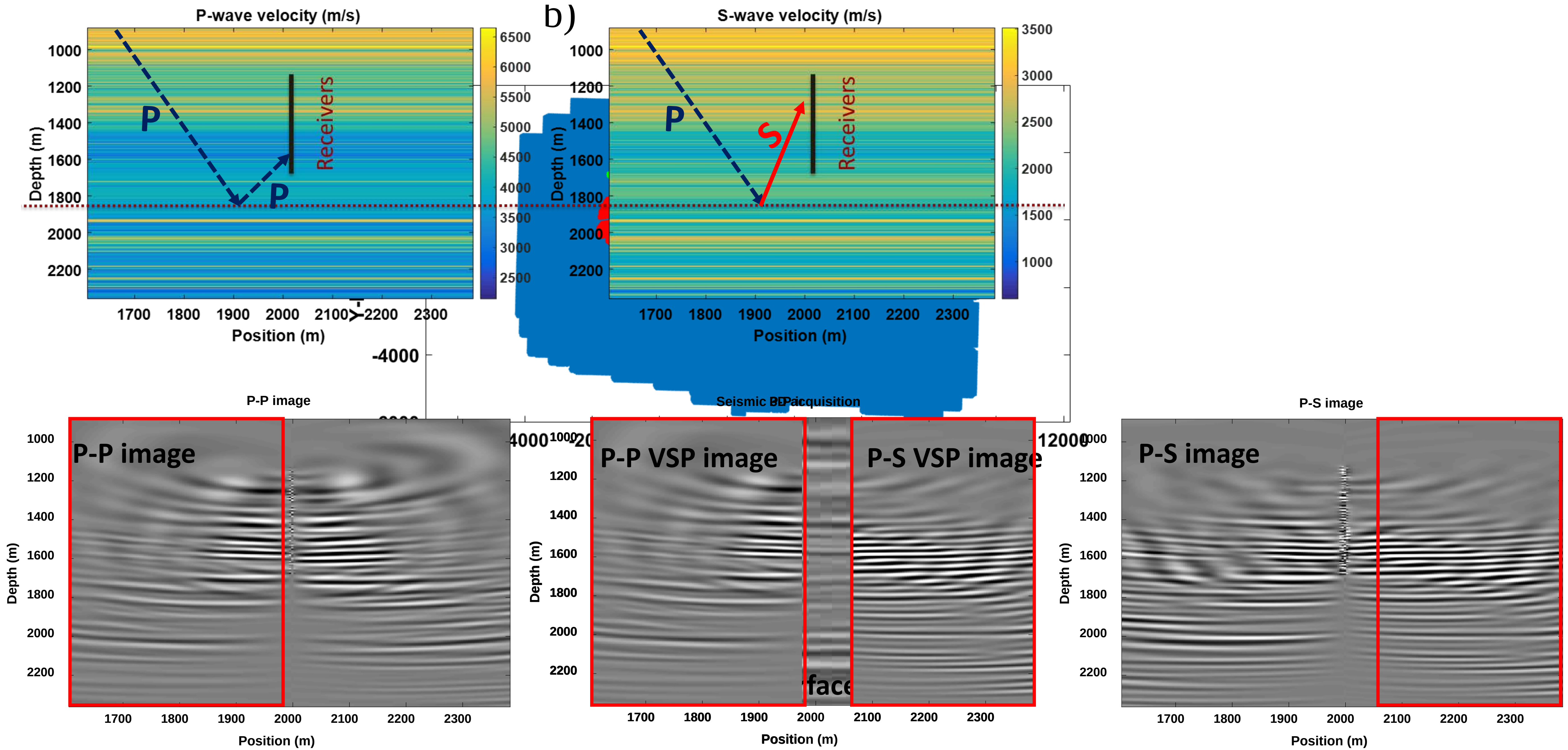
Field data example







3D VSP RTM Results vs 3D surface seismic imaging



Conclusion

- ✓**An algorithm for migration and inversion is developed by**
  - ✓Pure P- and S- wave separation based on decoupled wave equation
  - ✓It is a multicomponent migration and inversion
  - ✓Mitigates the computation time of full-wave equation modeling, RTM and FWI
  - ✓Primary reflection data
- ✓**The method is applicable to**
  - ✓PP, PS, SP, SH-SH
  - ✓Anisotropic medium
  - ✓Surface seismic, downhole imaging
  - ✓and P- and S- wave polarization in global seismology

Acknowledgments

- CREWES faculty and sponsors
- NSERC (CRDPJ 461179-13)
- University of Southern California and U.S. Department of Energy (National Energy Technology Laboratory) for field data provision
- Kris Innanen
- Ali Fathalian, Khaled Dulaijan and Bernie Law

Thank you