Viscoacoustic VTI and TTI wave equations and their application for anisotropic reverse time migration: Constant-Q approximation

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Outline

- Generalized standard linear solid model (GSlS)
  - Approximation constant-Q
- Anisotropic viscoacoustic wave equation
  - VTI and TTI media
  - Suppressing shear wave artifacts
- Anisotropic viscoacoustic reverse-time migration
  - Constructing a regularized equation
  - Stability condition
  - Numerical example
- Conclusion

Generalized standard linear solid model (GSlS)

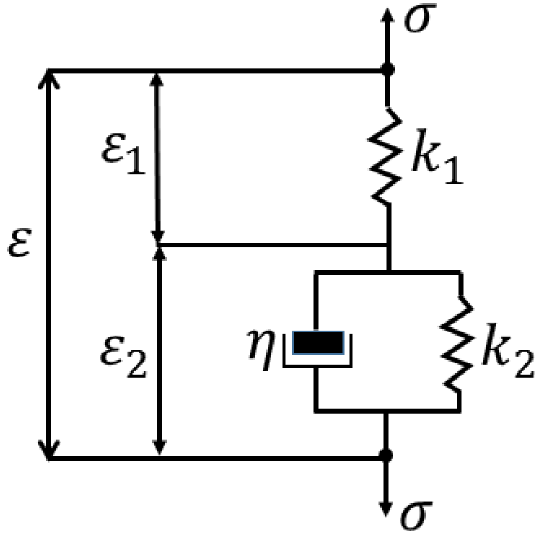
Single standard linear solid (SLS)

$$M(\omega) = M_R \frac{1 + i\omega\tau_\varepsilon}{1 + i\omega\tau_\sigma}$$

Complex modulus

Relaxed modulus

Relaxation times

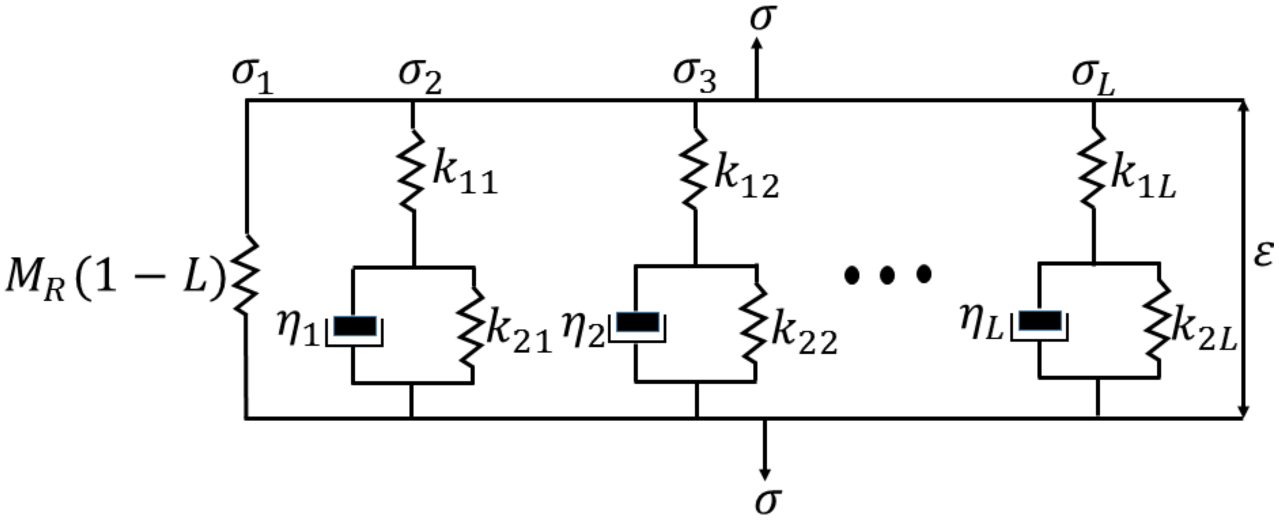


Generalized standard linear solid model (GSlS)

$$M(\omega) = M_R \left[1 - L + \sum_{l=1}^{L} \frac{1 + \omega\tau_{\varepsilon l}}{1 + \omega\tau_{\sigma l}} \right]$$

L:The number of single standard linear elements

L=?



(Liu et al. 1976)

Approximation constant-Q for GSLS model

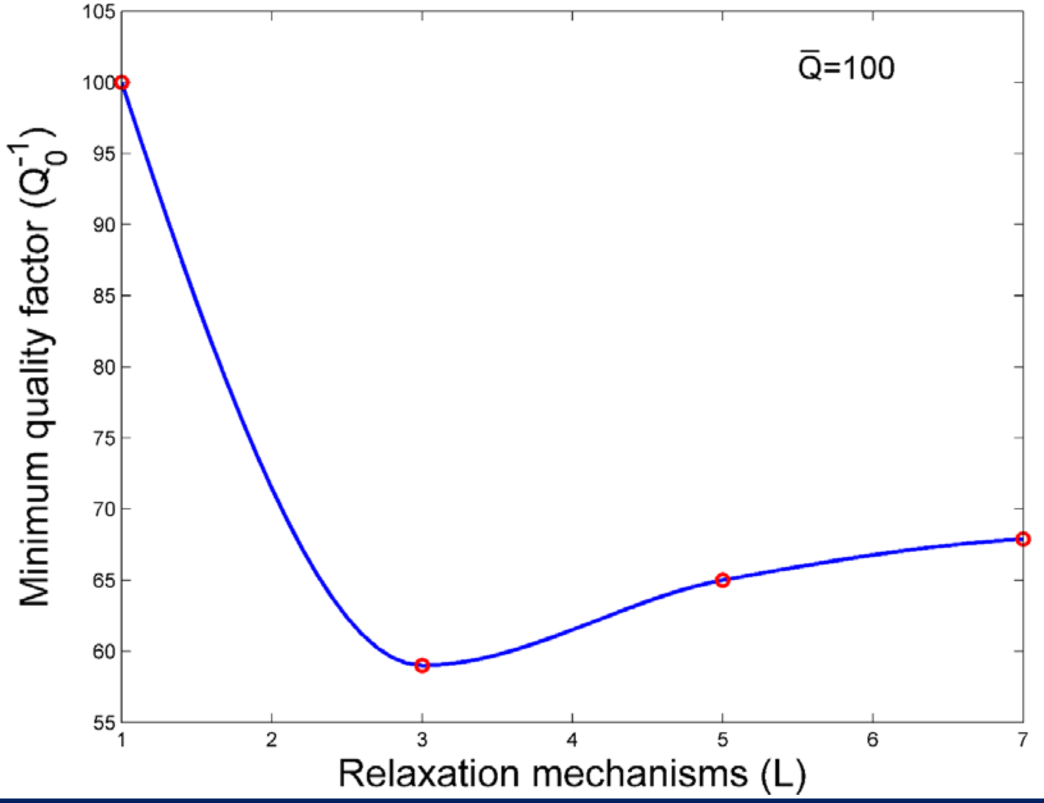
Frequency-dependent phase velocity: $v_P(\omega) = \left(Re\left[\sqrt{\rho/M(\omega)}\right]\right)^{-1}$

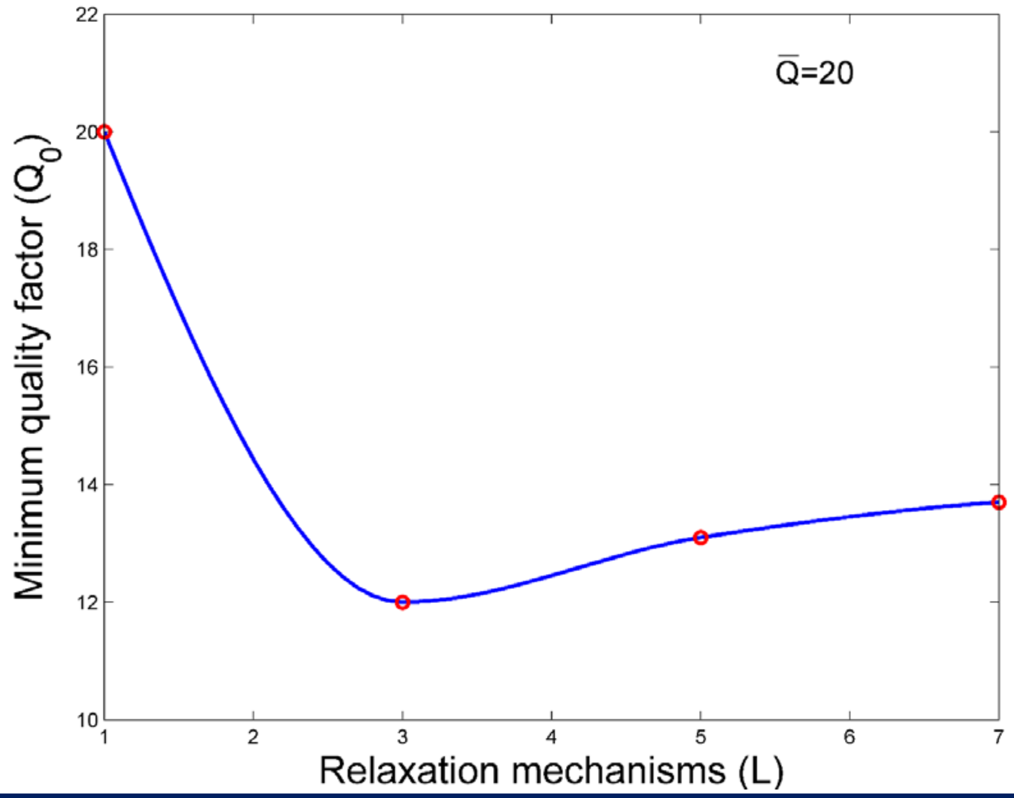
Quality factor: $Q(\omega) = Re[M(\omega)]/Im[M(\omega)]$

Nearly constant quality factor: $\bar{Q} = Q_0 L \left(\sum_{l=1}^{L} \frac{2\omega_0 \tau_{0l}}{1 + \omega_0^2 \tau_{0l}^2} \right)^{-1}$

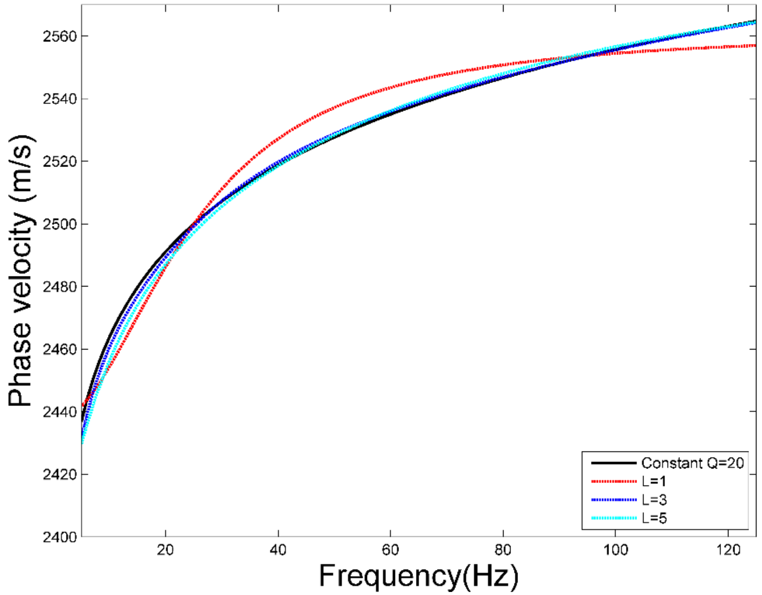
Q_0: Value of Q at ω_0

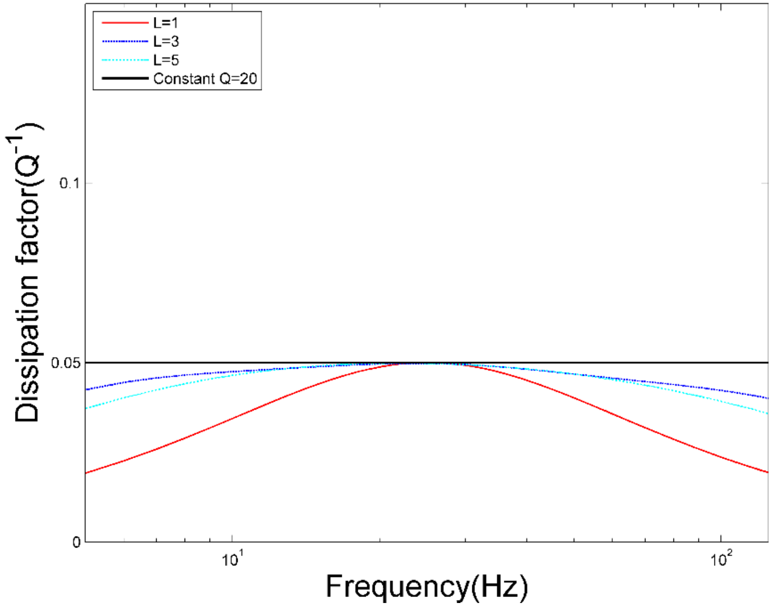
$\tau_0^2 = \tau_\varepsilon \tau_\sigma$

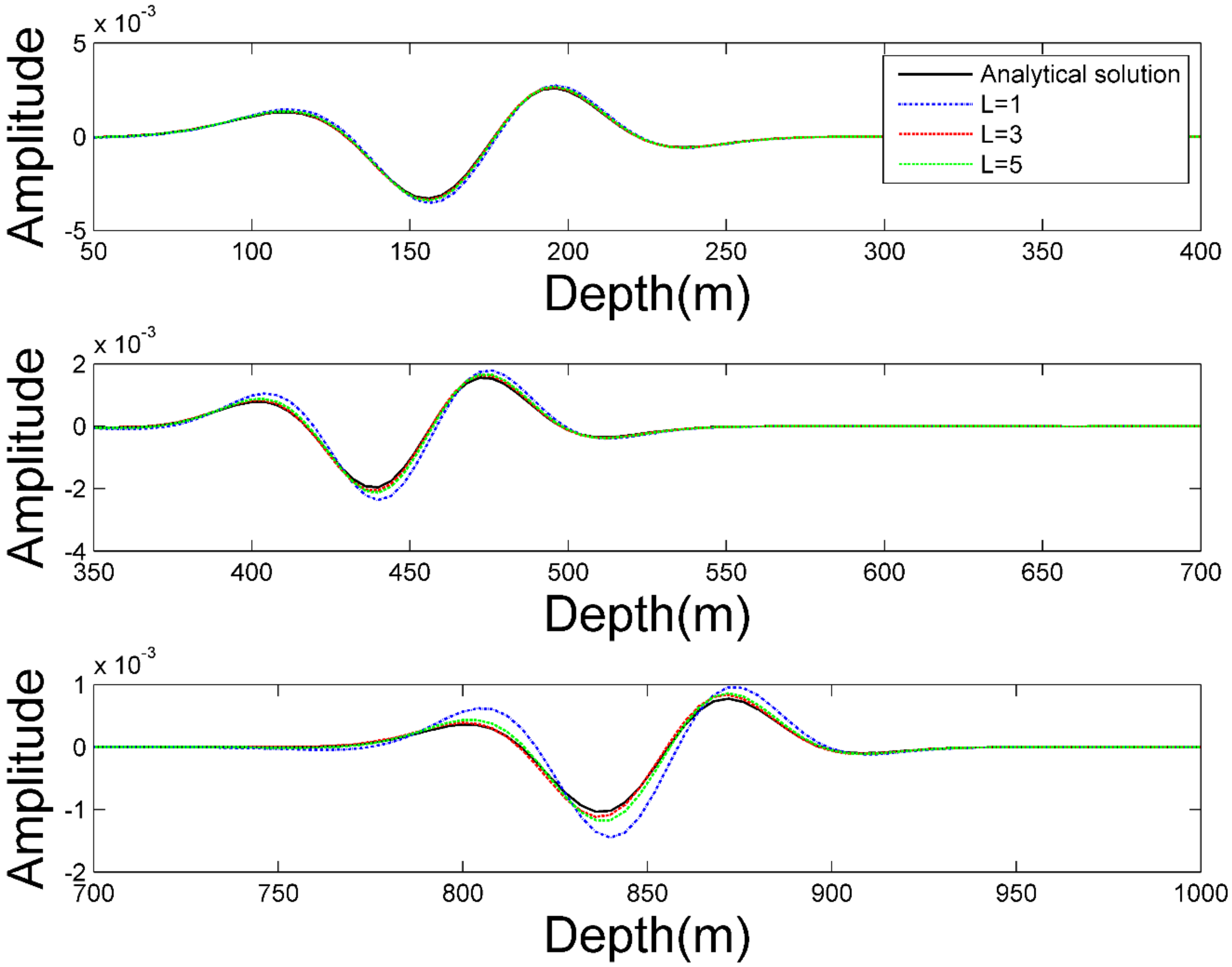




Approximation constant-Q over a broad frequency range







Viscoacoustic wave equation in VTI media

General linear stress-strain relationship (Hooke's law) reads: $\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$

Acoustic TI approximation in Hooke's law (setting $V_S = 0$)

$$c_{ijkl} = \begin{pmatrix} c_{11} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{11} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$c_{11} = \rho V_P^2 (1 + 2\varepsilon)$

$c_{13} = \rho V_P^2 \sqrt{1 + 2\delta}$

$c_{33} = \rho V_P^2$

$c_{44} = 0$

$c_{66} = 0$

Hooke's law simplified and reduced to two independent equations linking stresses and strains

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix} = \rho V_P^2 \begin{pmatrix} (1+2\varepsilon) & (1+2\varepsilon) & \sqrt{1+2\delta} & 0 & 0 & 0 \\ (1+2\varepsilon) & (1+2\varepsilon) & \sqrt{1+2\delta} & 0 & 0 & 0 \\ \sqrt{1+2\delta} & \sqrt{1+2\delta} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{pmatrix}$$

$$\sigma_H = \rho V_P^2 \left[(1+2\varepsilon)(\varepsilon_{11} + \varepsilon_{22}) + \sqrt{1+2\delta}\,\varepsilon_{33} \right]$$

$$\sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}(\varepsilon_{11} + \varepsilon_{22}) + \varepsilon_{33} \right]$$

→ $\sigma_H = \sigma_{11} = \sigma_{22}$ Horizontal stress component

$\sigma_V = \sigma_{33}$ Vertical stress component

(Alkhalifah. 1998, and Duveneck et al. 2011)

Viscoacoustic wave equation in VTI media

For one relaxation mechanism (L = 1), the split-field PML formulation in VTI media can be written as

$$\partial_t u_x = \frac{1}{\rho}\partial_x \sigma_H - d(x)u_x$$

$$\partial_t u_z = \frac{1}{\rho}\partial_z \sigma_V - d(z)u_z$$

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon)\left[\left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_x\left(u_x + d(z)u_x^{(1)}\right)\right] - r_H\right] + \sqrt{1+2\delta}\left[\partial_z\left(u_z + d(x)u_z^{(1)}\right)\right]\right] - (d(x)+d(z))\sigma_H - d(x)d(z)\sigma_H^{(1)}$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}\left[\partial_x\left(u_x + d(z)u_x^{(1)}\right)\right] + \left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_z\left(u_z + d(x)u_z^{(1)}\right)\right] - r_V\right] - (d(x)+d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}$$

u: Particle velocity

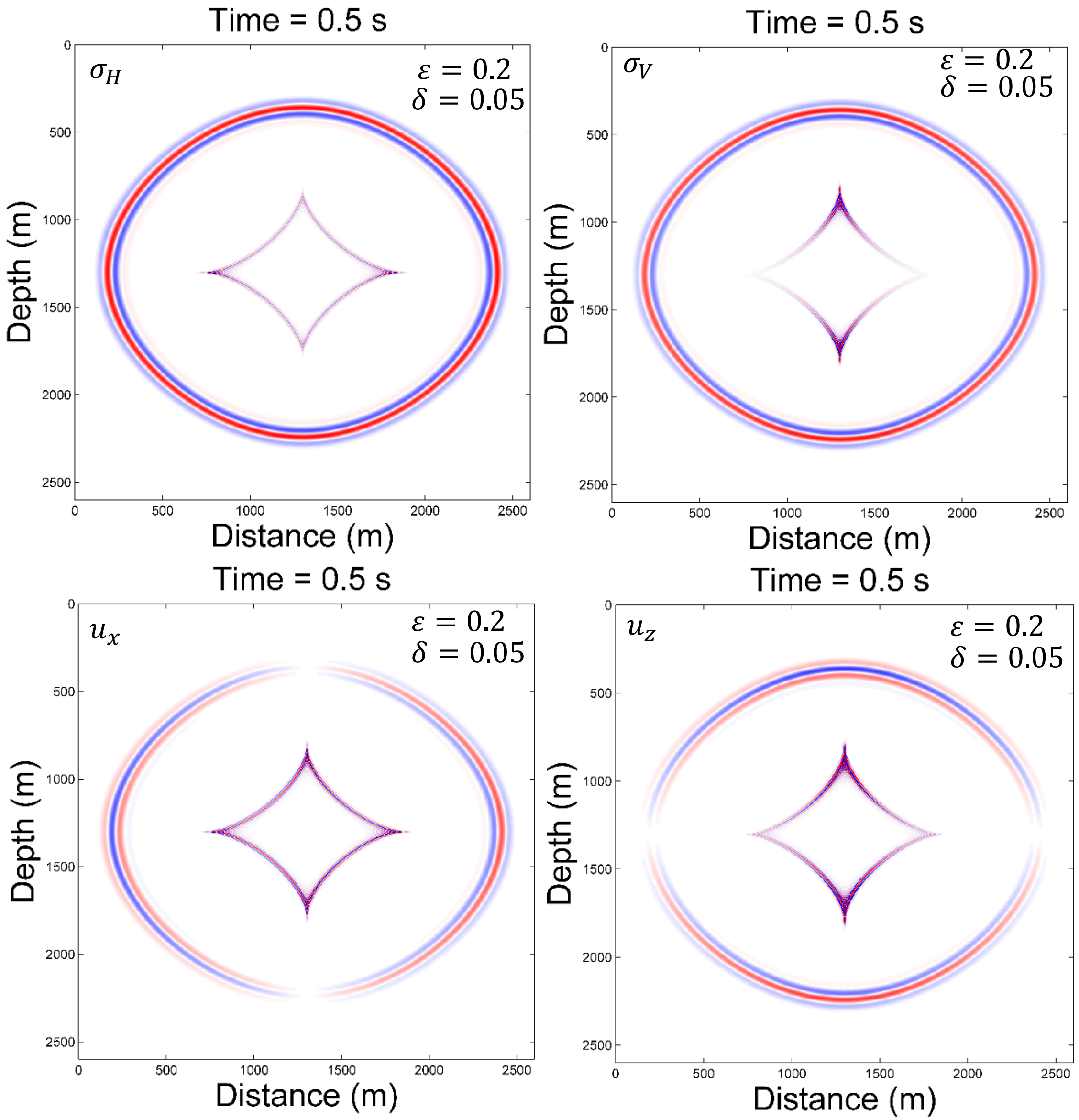
σ: Stress component

r: Memory variable

Split-field PML formulation in TTI media:

Spatial derivative in a rotated coordinate system: $\begin{pmatrix}\partial_{x'}\\ \partial_{y'}\\ \partial_{z'}\end{pmatrix} = R\begin{pmatrix}\partial_{x}\\ \partial_{y}\\ \partial_{z}\end{pmatrix} \rightarrow \begin{cases}\partial_{x'} = \cos\theta\cos\varphi\,\partial_x - \sin\theta\,\partial_z \\ \partial_{z'} = \cos\varphi\sin\theta\,\partial_x + \cos\theta\,\partial_z\end{cases}$

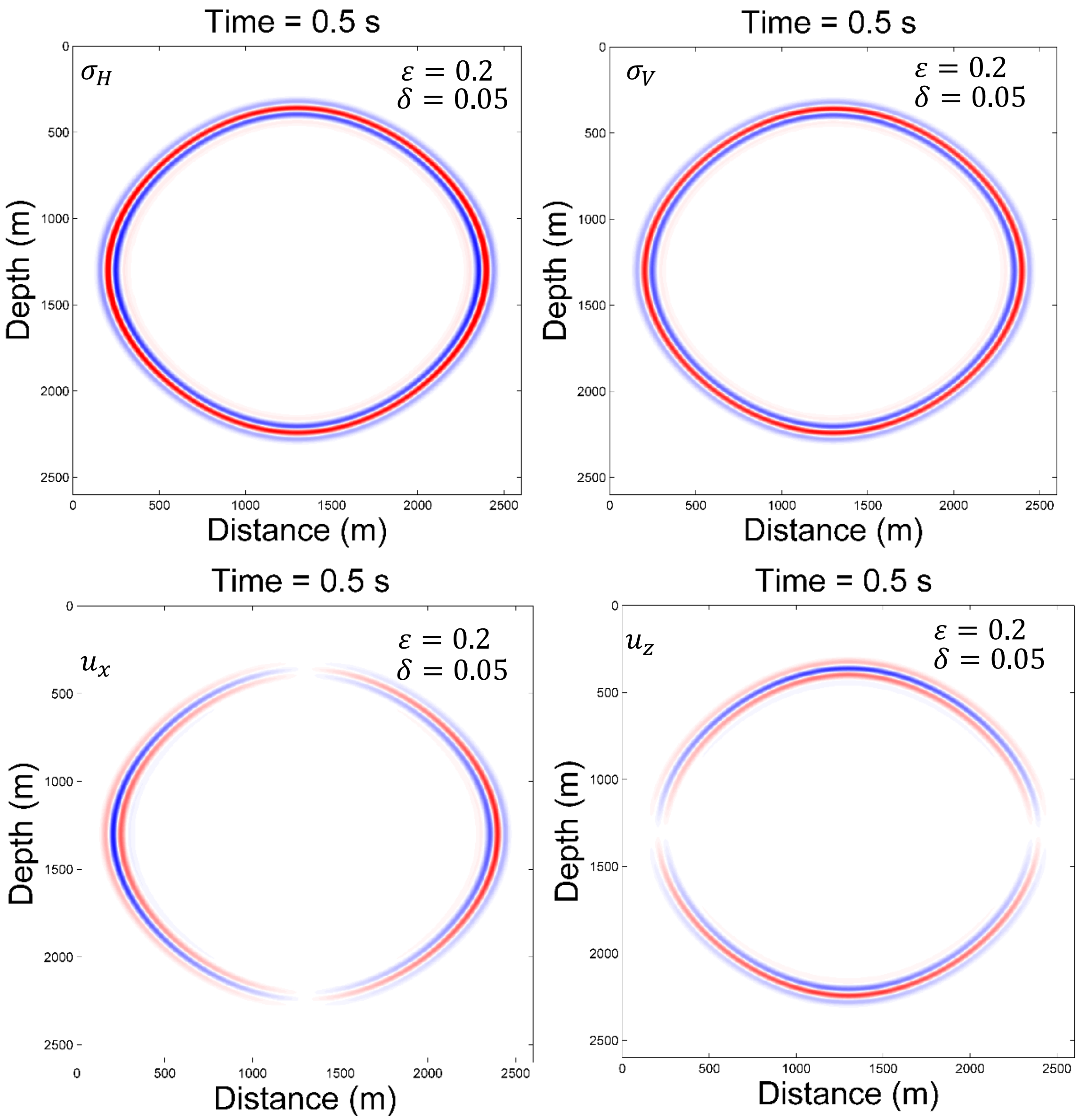
2D wavefield snapshots in a viscoacoustic VTI medium



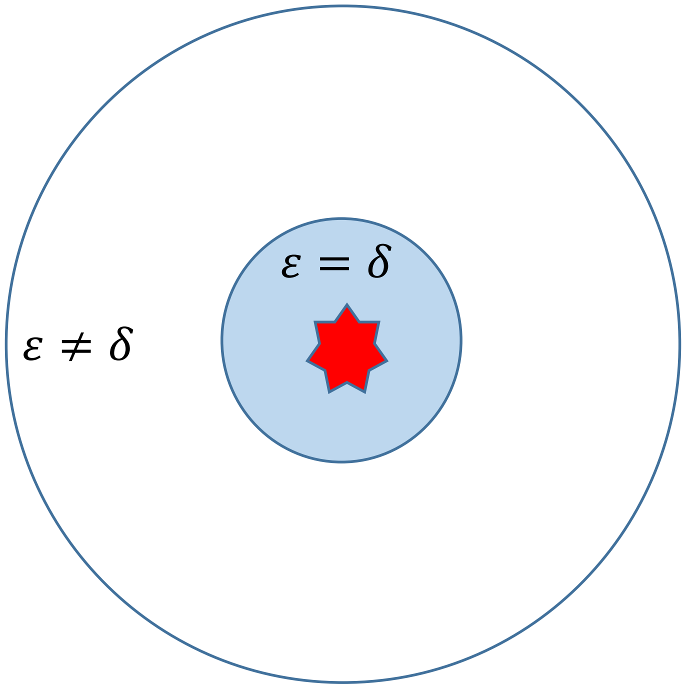
- The anisotropic viscoacoustic constant velocity model

- Source signature: zero-phase Ricker wavelet with central frequency of 25 Hz

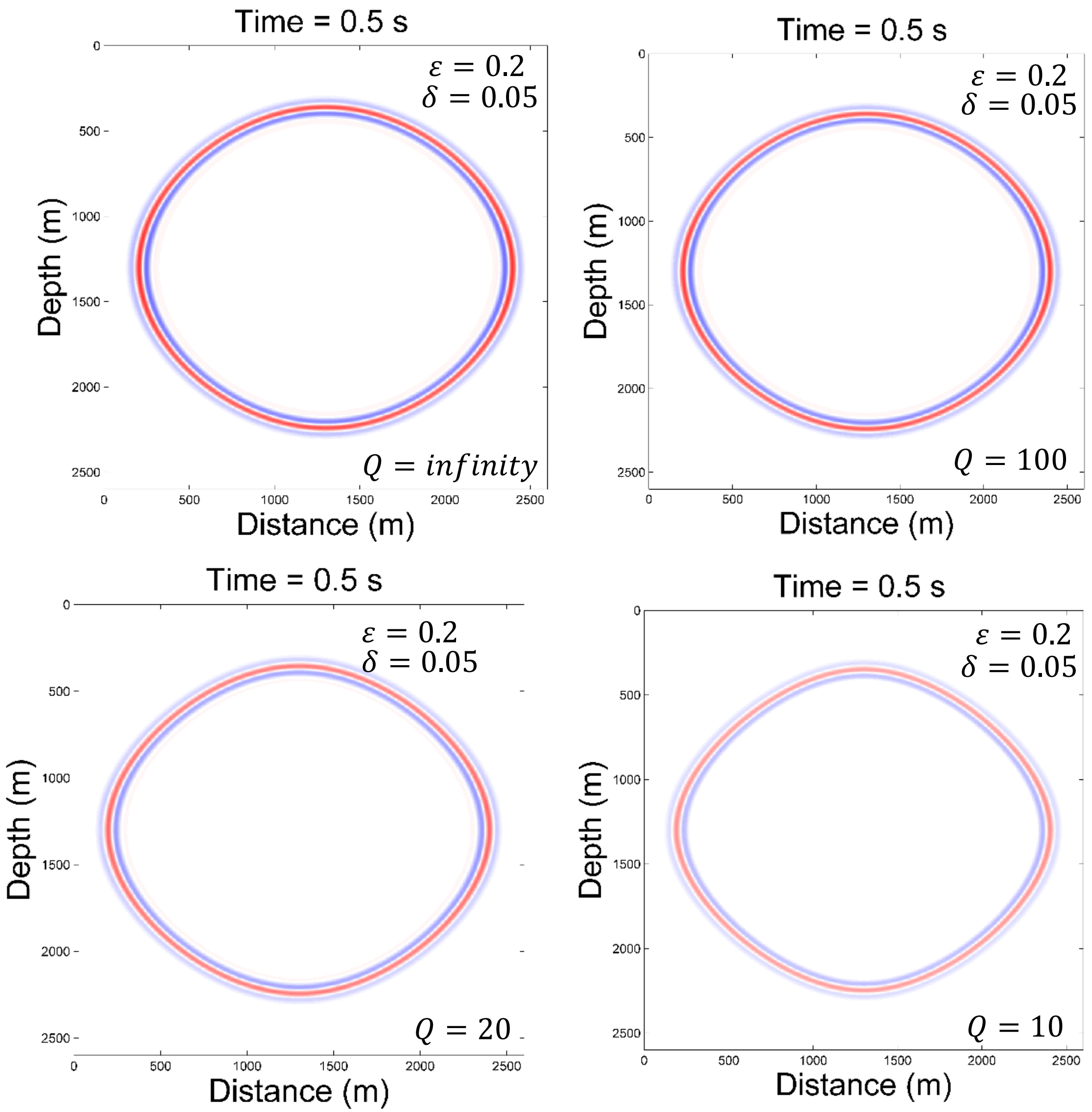
2D wavefield snapshots in a viscoacoustic VTI medium

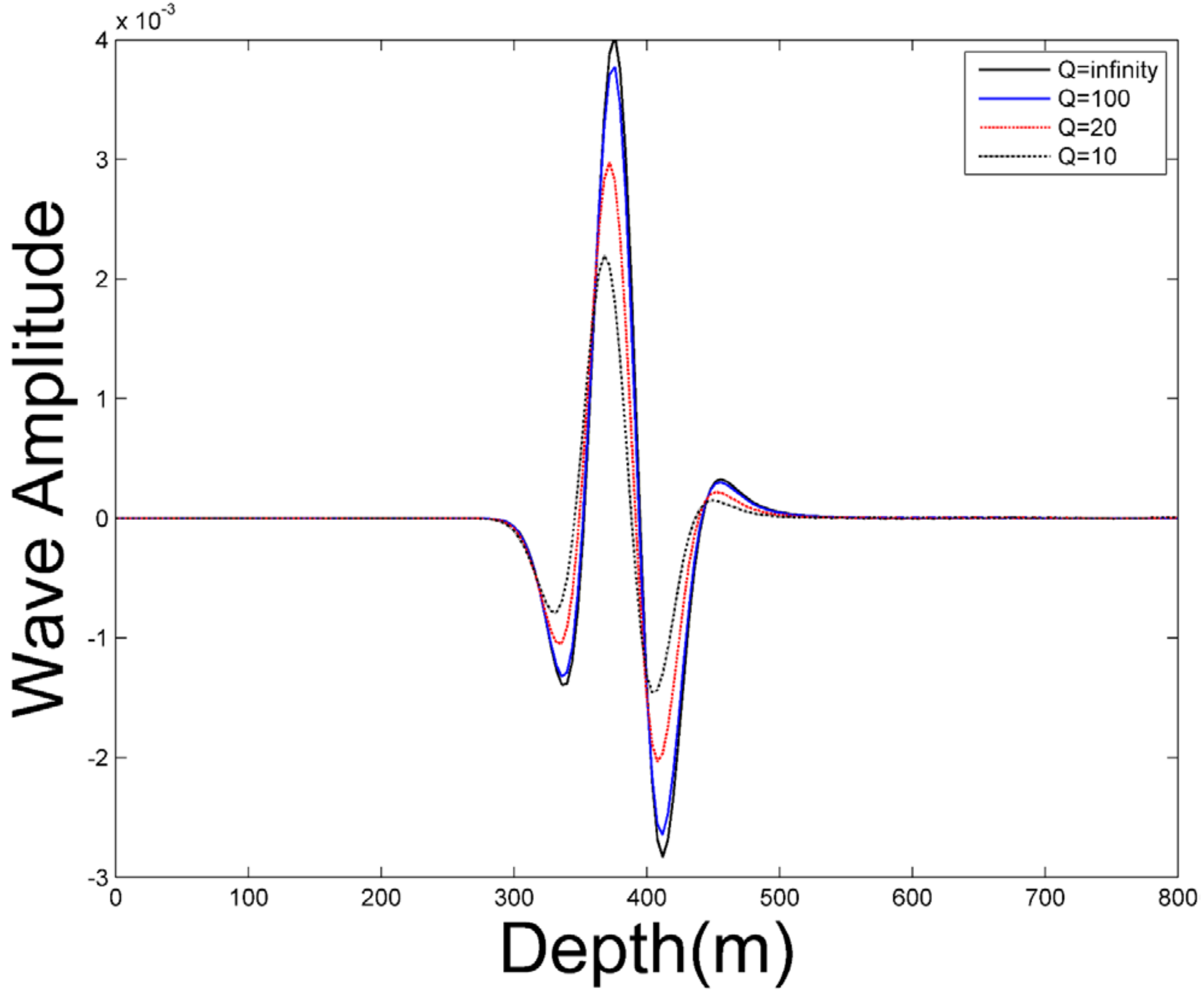


- Shear wave artifacts are generated in an elliptic medium ($\varepsilon \neq \delta$); can be suppressed at the source by designing a small smoothly tapered circular region with $\varepsilon = \delta$ around the source.

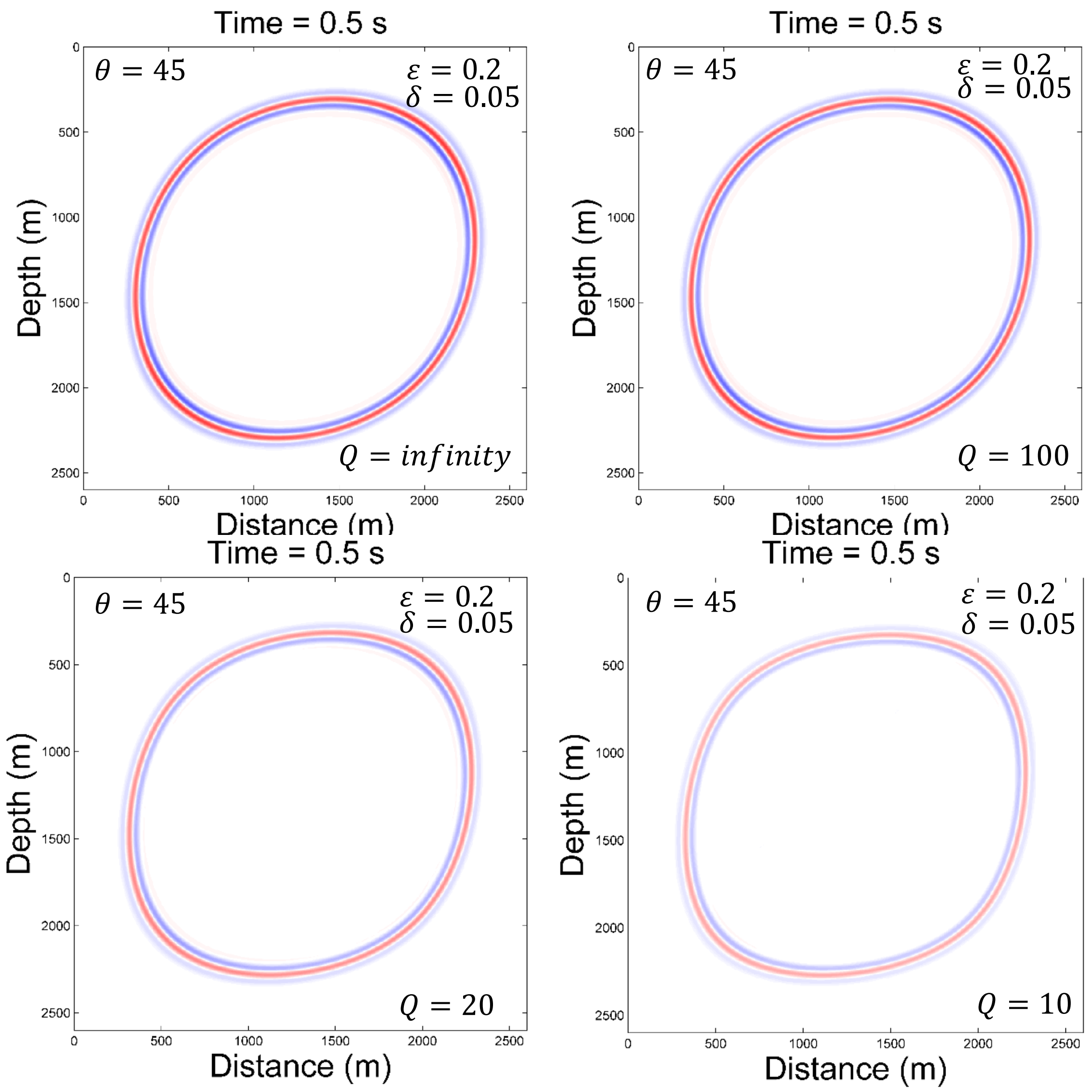


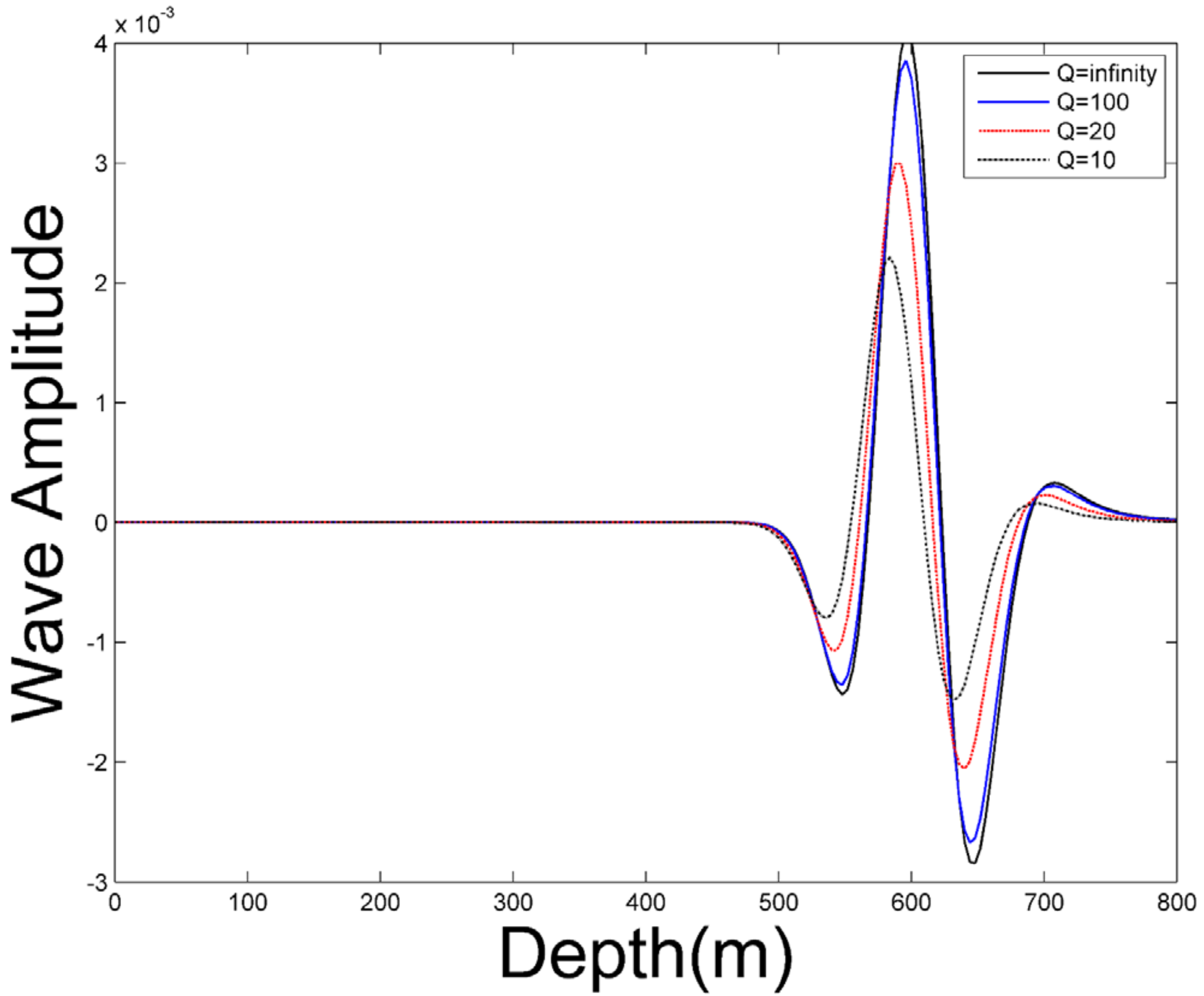
2D wavefield snapshots in a viscoacoustic VTI medium





2D wavefield snapshots in a viscoacoustic TTI medium

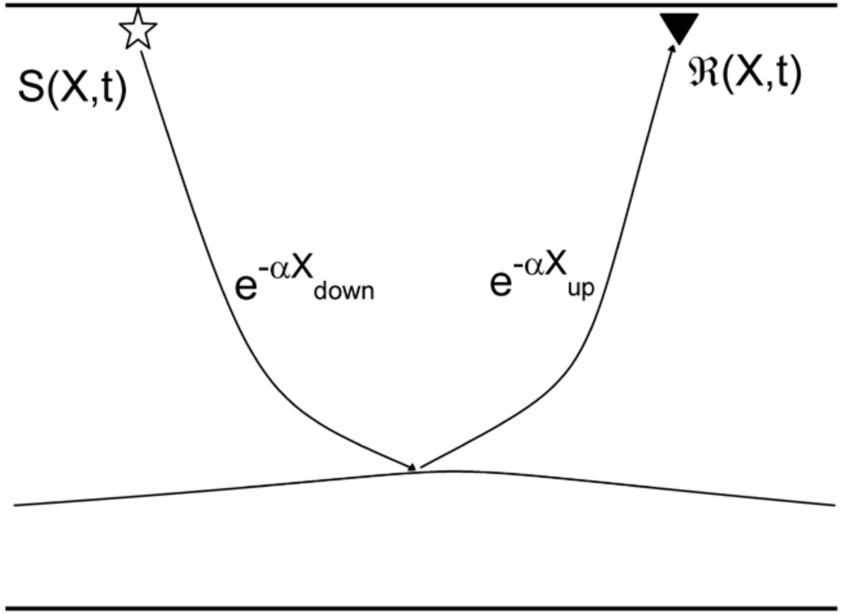




Viscoacoustic RTM (theory and method)

- Source normalized cross-correlation imaging condition is more suitable. Only backward receiver wavefield is needed to compensate.

- Postulate: the wavelets of forward and backward wavefields will match well at the reflection point; better-resolved images, with no regularization in the forward wavefield.



The backward receiver wavefield

$$\Re(x,z,t) = R(x,z,t)e^{-\alpha X_{down}}e^{-\alpha X_{up}}$$

Source normalized cross-correlation imaging condition:

$$I(x,z) = \frac{\int e^{-\alpha X_{down}} S(x,z,t) e^{+\alpha X_{up}} e^{-\alpha X_{down}} e^{-\alpha X_{up}} R(x,z,t)dt}{\int e^{-2\alpha X_{down}} S^2(x,z,t)}$$

$$= \frac{\int R(x,z,t)dt}{S(x,z,t)}$$

Viscoacoustic RTM (Construct a regularized equation)

- In seismic wave simulation, high-frequencies lead to instability.
- To avoid high-frequency effects in RTM, regularization must be considered.
- We add a regularization term $\epsilon\rho V_{Px,z}\partial_t u_{x,z}$

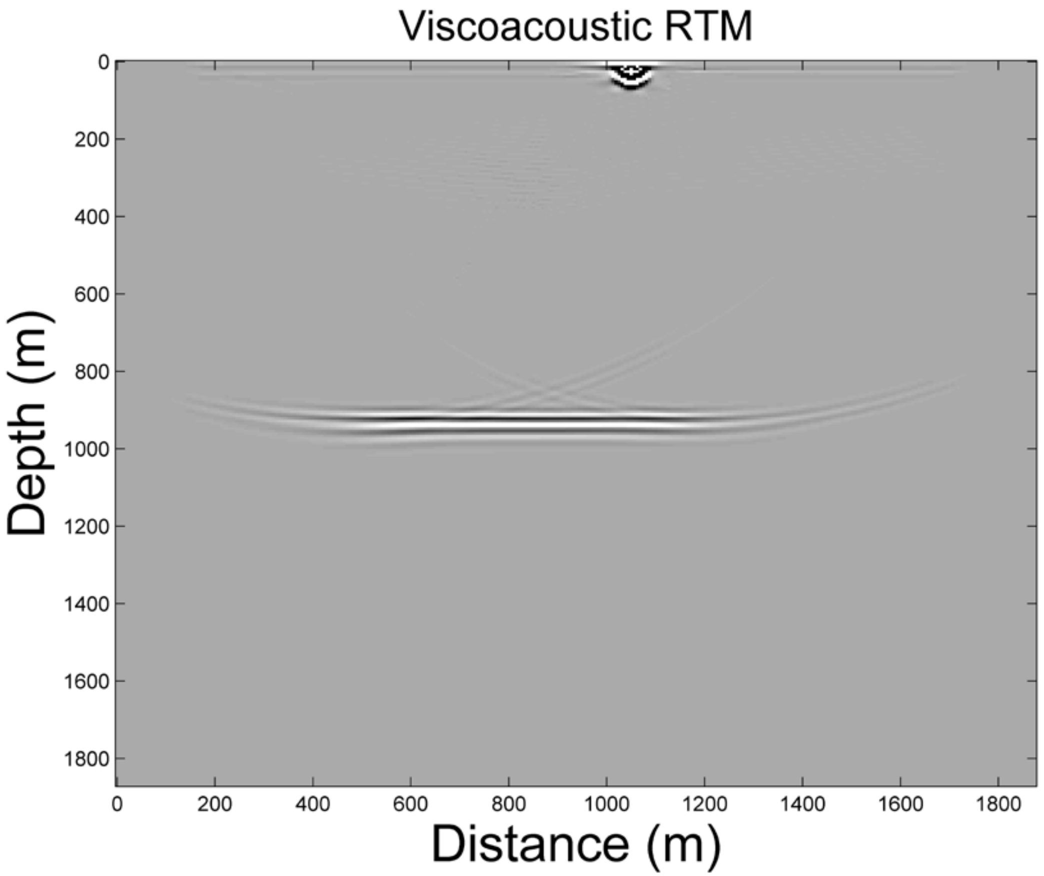
Regularization term

$$\partial_t\sigma_H = \rho V_P^2\left[(1+2\varepsilon)\left[\left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_x\left(u_x + d(z)u_x^{(1)}\right)\right] - r_H\right] + \sqrt{1+2\delta}\left[\partial_z\left(u_z + d(x)u_z^{(1)}\right)\right]\right]$$
$$- \left[\epsilon\rho V_P\sqrt{1+2\varepsilon}\left[\partial_t\left(u_x + d(z)u_x^{(1)}\right)\right]\right] - (d(x)+d(z))\sigma_H - d(x)d(z)\sigma_H^{(1)}$$

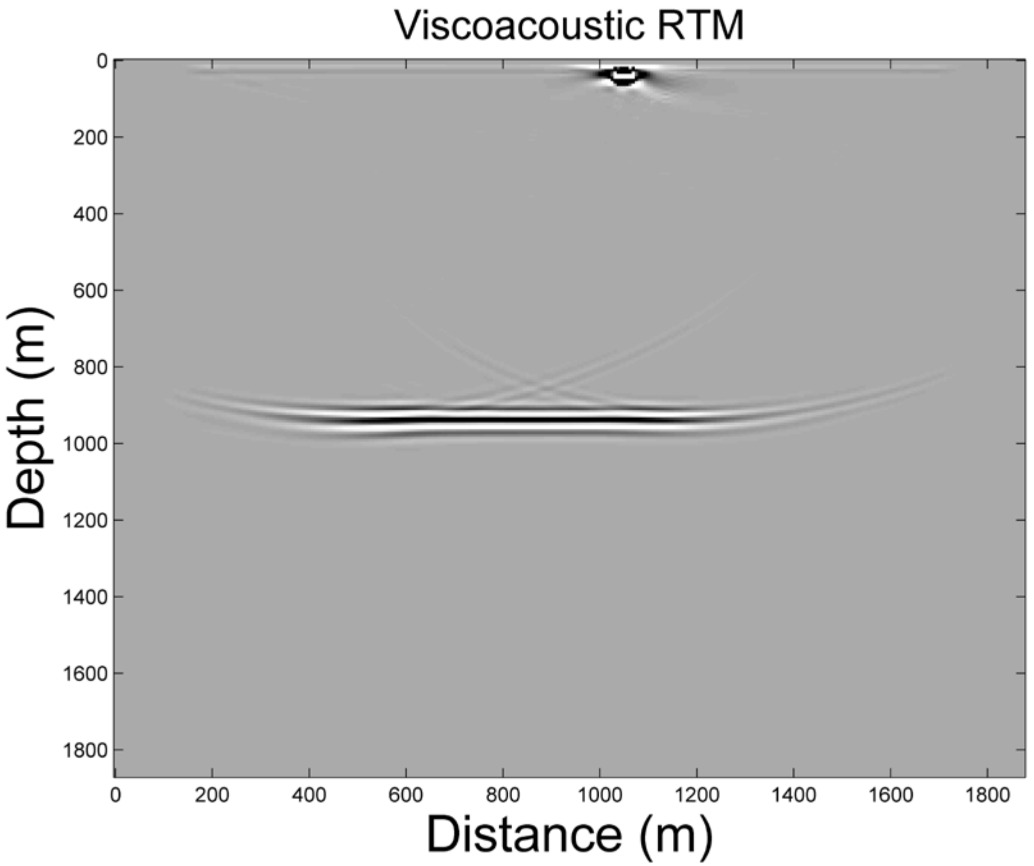
$$\partial_t\sigma_V = \rho V_P^2\left[\sqrt{1+2\delta}\left[\partial_x\left(u_x + d(z)u_x^{(1)}\right)\right] + \left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_z\left(u_z + d(x)u_z^{(1)}\right)\right] - r_V\right]$$
$$- \left[\epsilon\rho V_P\left[\partial_t\left(u_z + d(x)u_z^{(1)}\right)\right]\right] - (d(x)+d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}$$

Regularization term

Viscoacoustic RTM with regularization term



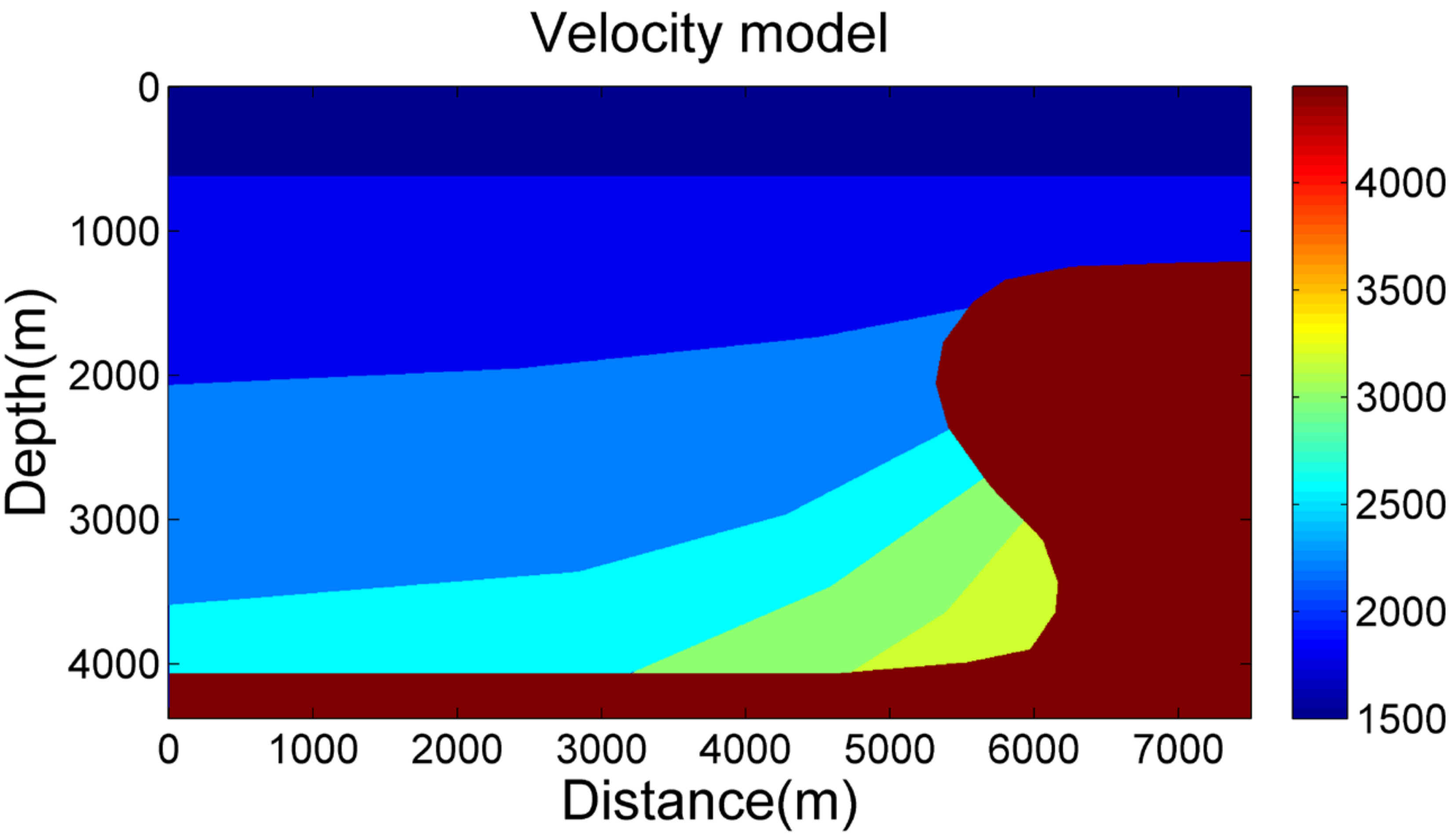
Q-RTM with filtering



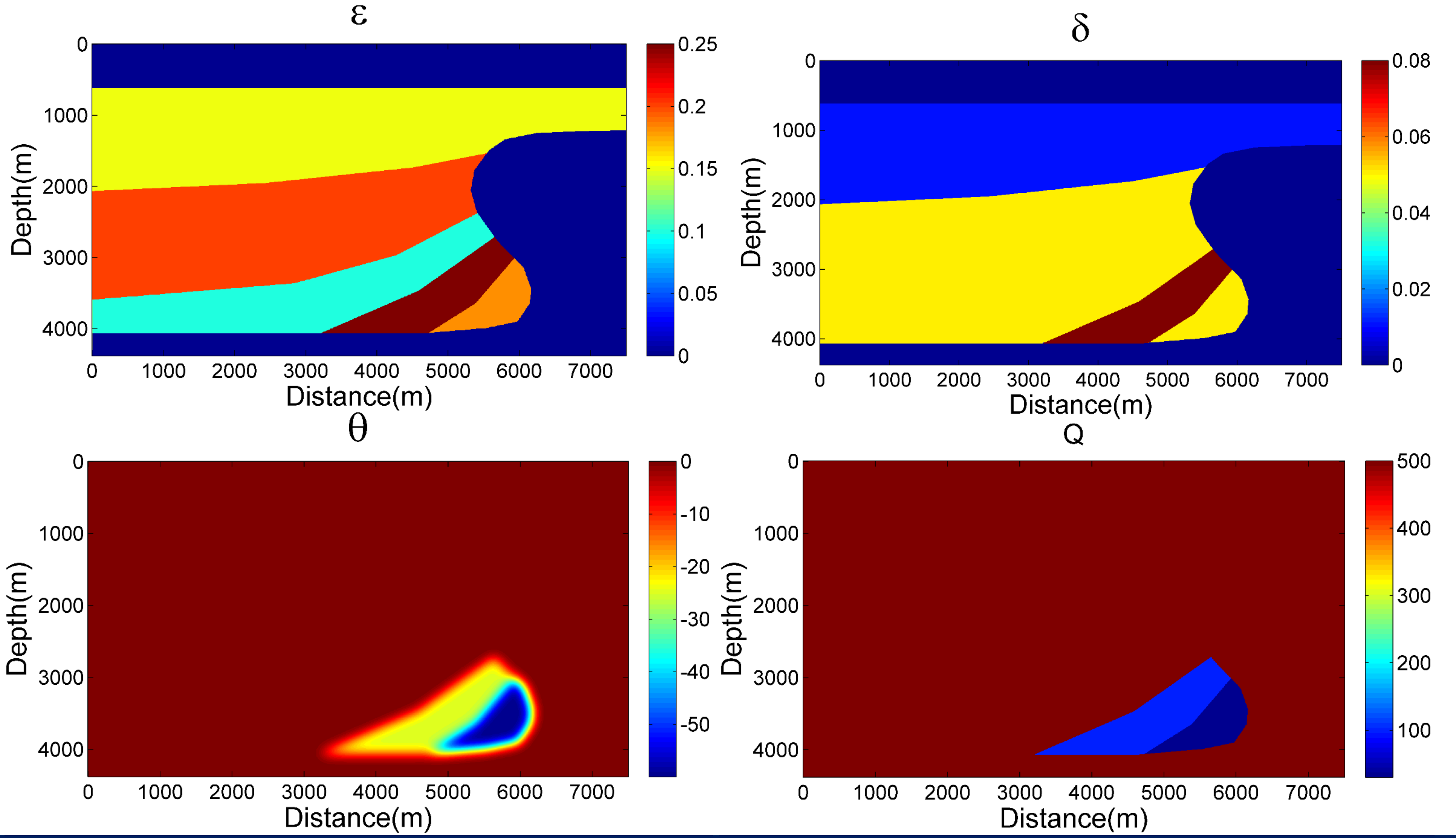
Q-RTM with regularization term

Numerical example

- Velocity model: a salt dome and dipping anisotropic layers terminating against salt
- Rapid variation of the tilt angle around the salt presents challenges to TTI RTM (Duveneck et al. 2011, Zhang et al. 2011)

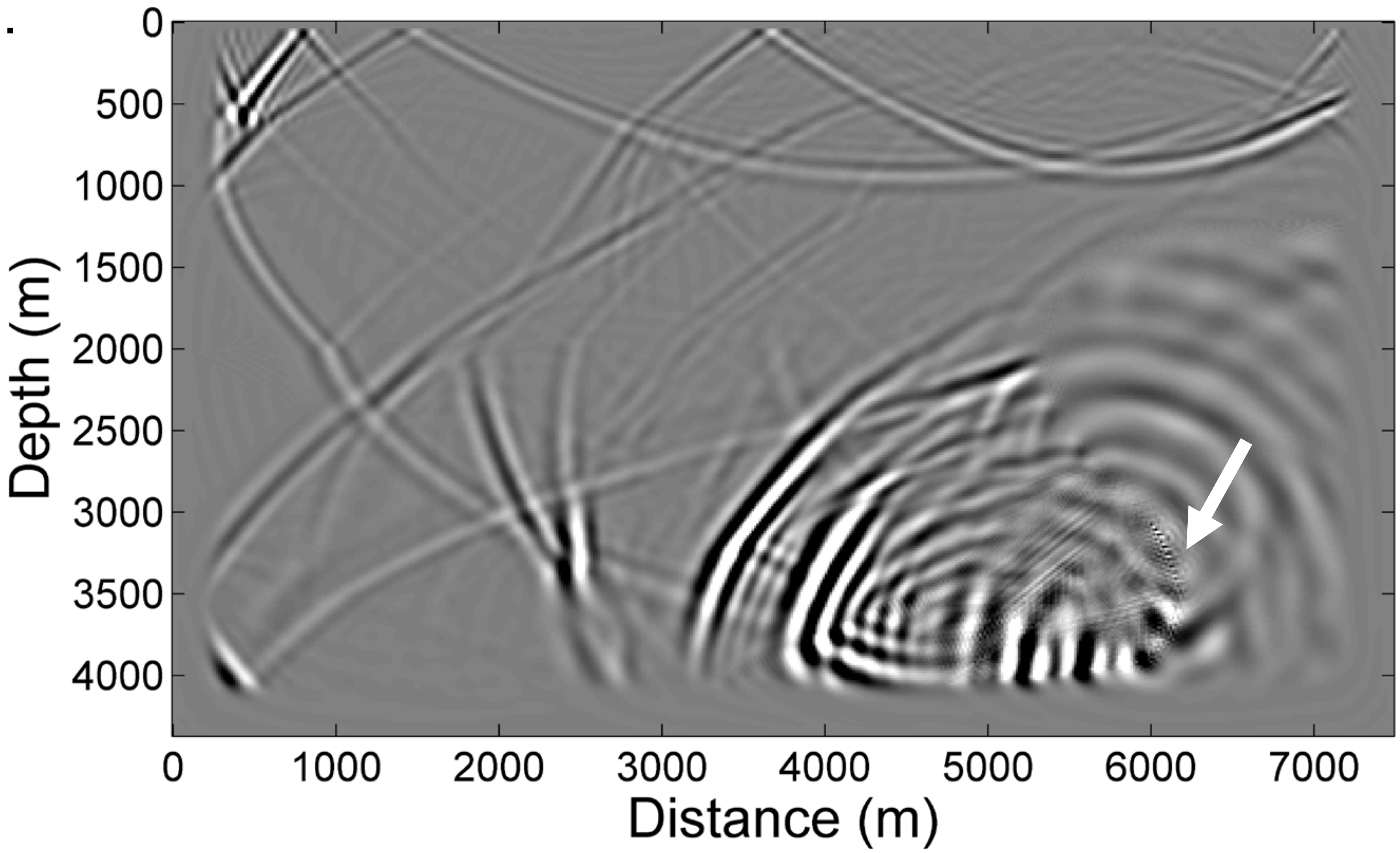


Numerical example



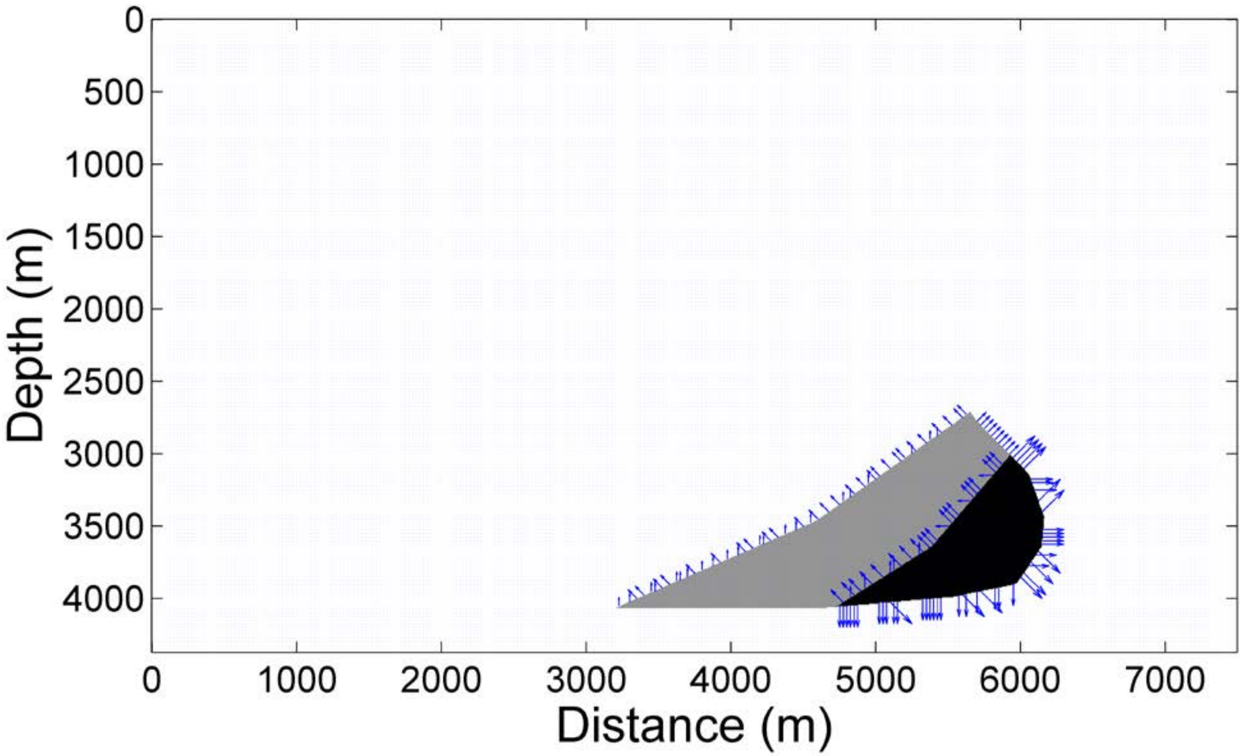
Numerical example

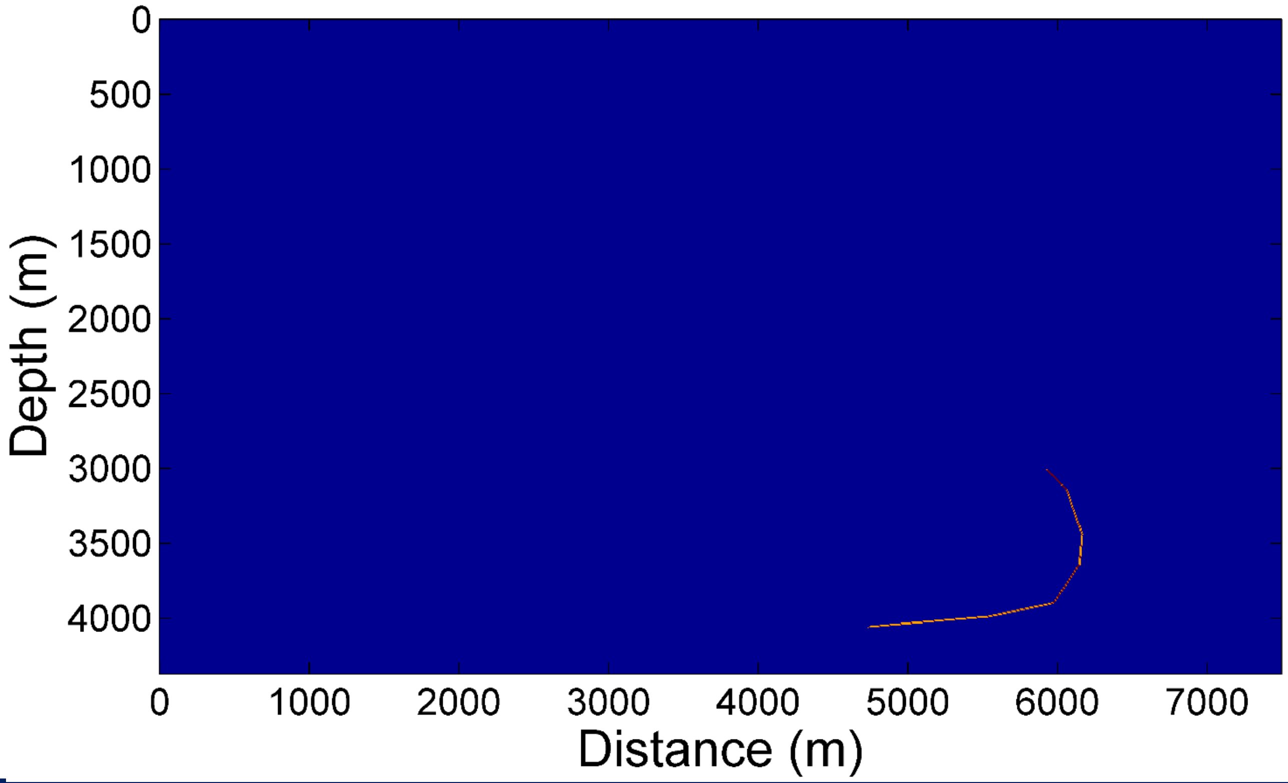
Variation of the tilt angle around the salt flank causes instability in the simulated wavefield.



Numerical example

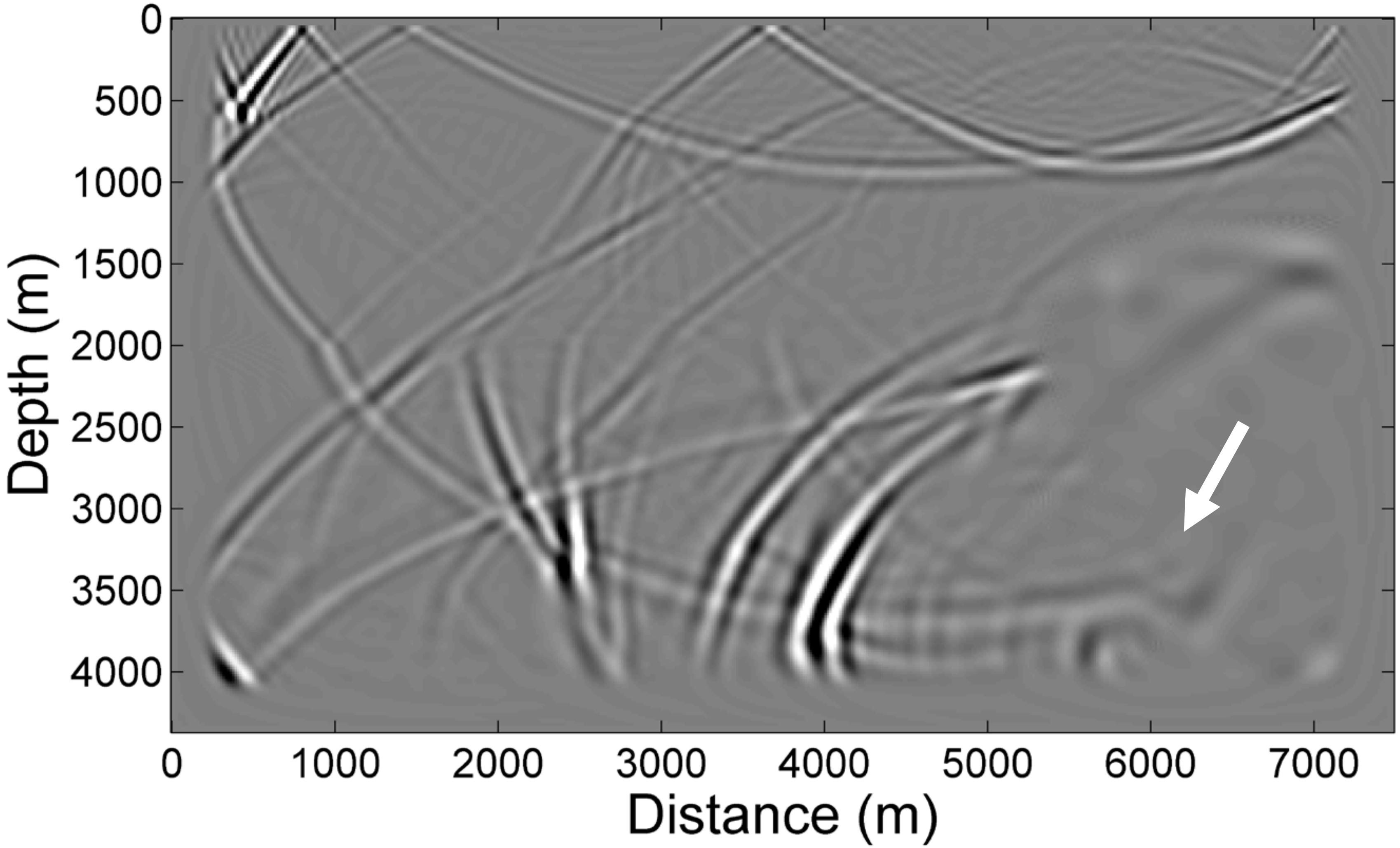
- Regions of large gradients excite these instabilities.
- We first identify high gradient points with a threshold
- Then equate $\varepsilon = \delta$ in a region around the selected high gradient points.



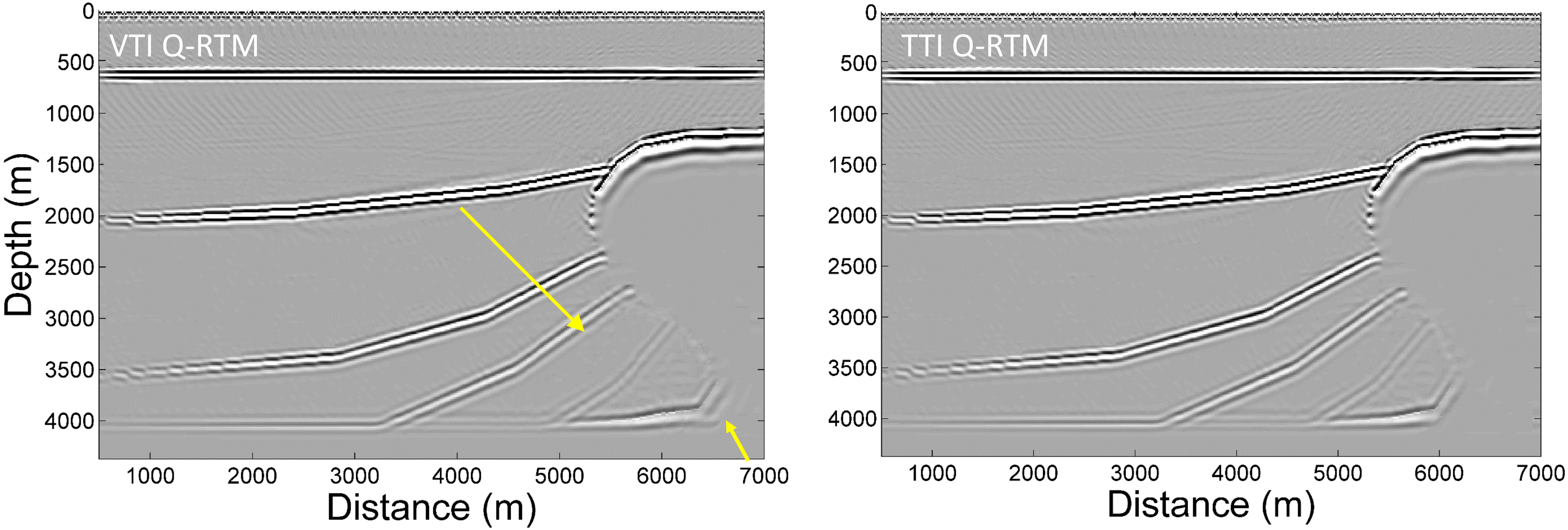


Numerical example

Stable snapshot after parameter equating is employed



Numerical example



Conclusions and future work

- Time-domain approximate constant-Q / SLS wave propagation is investigated. One SLS element is sufficient.

- Viscoacoustic VTI and TTI wave equation are solved numerically; a regularization operator is introduced to eliminate high-frequency instability problems.

- A stable TTI RTM is achieved by suppressing anisotropy in areas of rapid changes in the symmetry axes.

- TTI RTM has the ability to produce a more highly resolved, accurate image than VTI RTM, especially in the areas with strong variations of dip angle along the tilted symmetry-axis.

- Application of anisotropic equations to 3D RTM, field data; reducing computation time remains a challenge
- Applicable to time-domain viscoacoustic FWI

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