Nonlinear Inversion for Fluid and Effective-Stress Parameters

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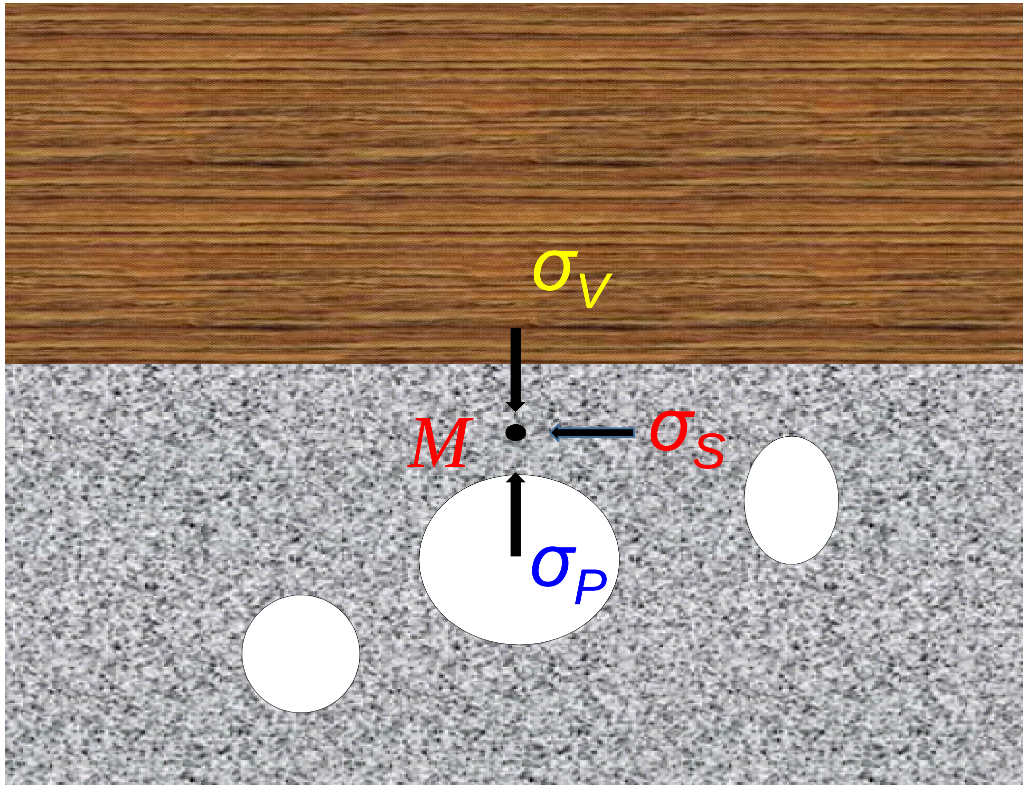
Outline

- Introduction
- Linearized reflection coefficient and elastic impedance (EI)
- Nonlinear inversion for fluid and stress parameters
- Numerical examples
- Conclusions

Effective stress



σ_V: Vertical stress

σ_s: Horizontal stress

σ_P: Pore pressure

$$\sigma_e = f\left(\sigma_V, \sigma_S, \sigma_P\right)$$

1. Vertical stress

$$\sigma_V = \int_0^z \rho(z)\, g dz$$

2. Pore pressure

$$\sigma_P = \sigma_V - \left[\frac{1}{a} \ln\left(\frac{\phi_0}{\phi}\right)\right]$$

(Zoback, 2007)

ϕ_0 is the initial porosity.

3. Horizontal stress

1) Isotropic media: $\sigma_S = \sigma_H - \sigma_h = 0$

2) Fractured media: $\sigma_S = \sigma_H - \sigma_h = f(\Delta_N, \Delta_T)$

(Gray, 2012)

Introduction

Assuming isotropic media (Fractures or cracks distribute randomly but equally in all directions. *Pointer et al., 2000*):

$$\sigma_e = \sigma_V - \sigma_P = \left[\frac{1}{a}\ln\left(\frac{\phi_0}{\phi}\right)\right]$$

Porosity and effective stress function (Smith, 1971)

$$\phi \approx \phi_0 P_{\mathrm{e}}$$

Stress- sensitive parameter

$$P_{\mathrm{e}} = \exp\left(-a\sigma_{\mathrm{e}}\right)$$

Effective stress

Empirical relationships among velocity, density, porosity and clay volume

$$V_P = 5.81 - 9.42\phi - 2.21V_{clay}$$

$$V_S = 3.89 - 7.07\phi - 2.04V_{clay}$$

(Castagna et al., 1985)

$$\rho = \left[V_{clay}\rho_{clay} + \left(1 - V_{clay}\right)\rho_{quartz}\right]\left(1 - \phi\right) + \phi\rho_f$$

(Mavko et al., 2009)

Gassmann's fluid substitution equation (Gassmann, 1951; Mavko et al., 2009)

$$K_{\mathrm{sat}} = K_{\mathrm{dry}} + \frac{(1-K_{\mathrm{dry}}/K_0)^2}{\phi/K_{\mathrm{f}}+(1-\phi)/K_0-K_{\mathrm{dry}}/K_0^2}$$

Saturated rock bulk modulus

Dry rock bulk modulus

Fluid bulk modulus

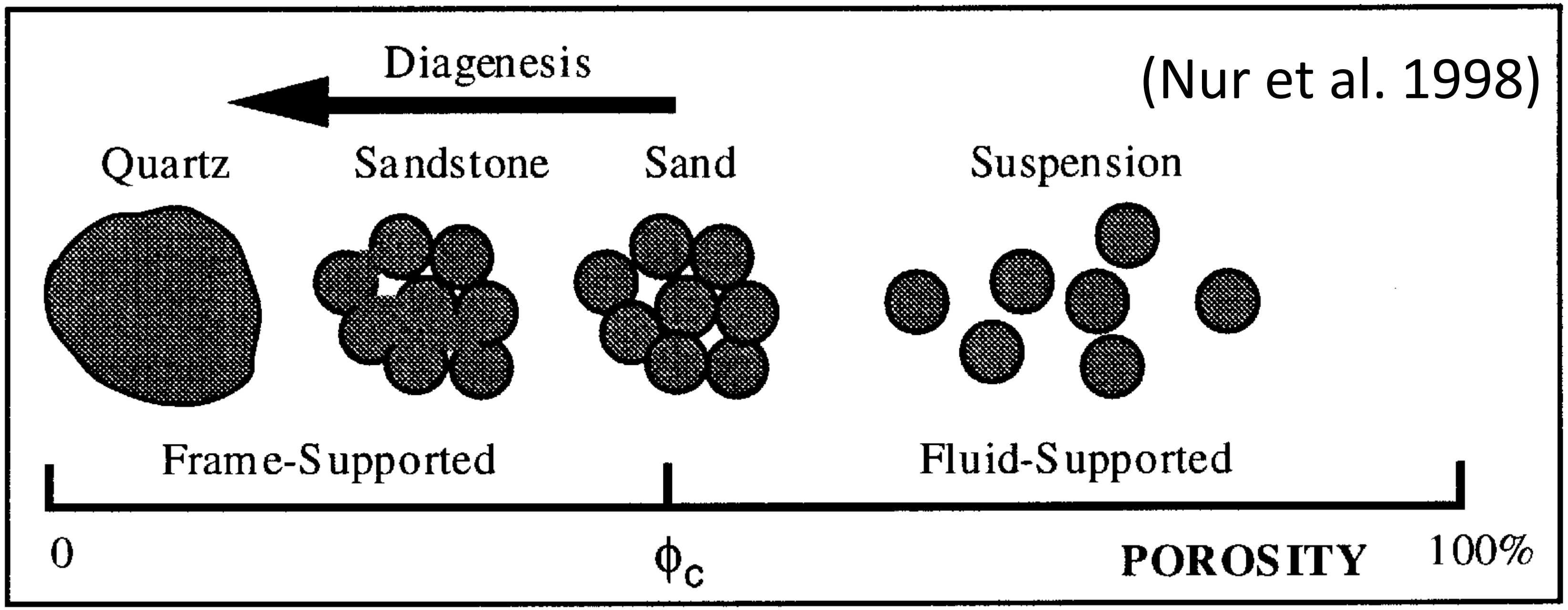
Porosity

Mineral bulk modulus

Relationship between dry rock bulk modulus and porosity

$$K_{\text{dry}} = K_0 \left(1 - \frac{\phi}{\phi_c}\right)$$

Critical porosity



Linearized reflection coefficient and EI

Assuming ϕ_0=ϕ_c, the approximate bulk modulus of saturated rock

$$K_{\mathrm{sat}} = K_{\mathrm{dry}} + \frac{(\phi/\phi_{\mathrm{c}})^2}{\phi/K_{\mathrm{f}}+(1-\phi)/K_0-\left(1-\frac{\phi}{\phi_{\mathrm{c}}}\right)/K_0}$$

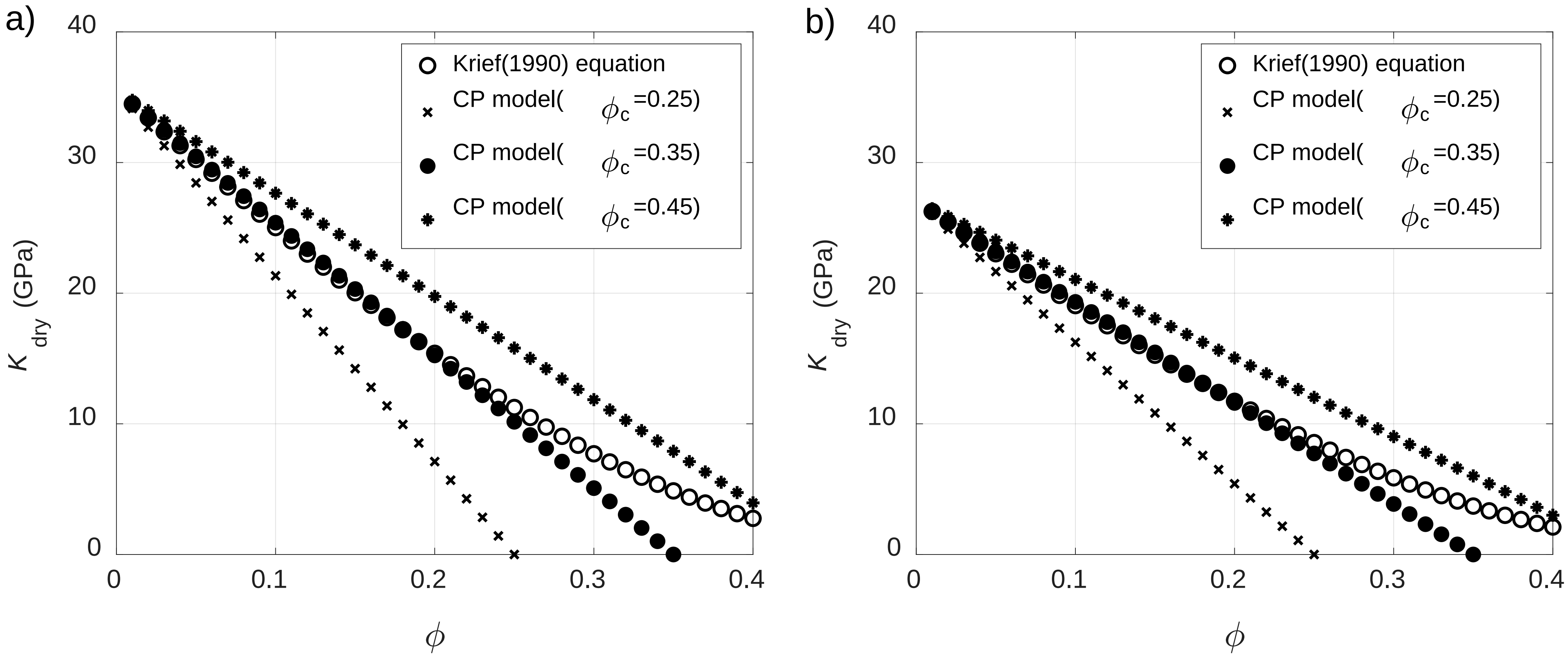
$$= K_{\mathrm{dry}} + \frac{K_0(\phi/\phi_{\mathrm{c}})^2}{\phi K_0/K_{\mathrm{f}}-\left(\phi-\frac{\phi}{\phi_{\mathrm{c}}}\right)}$$

$$K_{\mathrm{sat}} \approx K_{\mathrm{dry}} + \frac{\phi}{(\phi_{\mathrm{c}})^2} K_{\mathrm{f}} \approx \underline{K_{\mathrm{dry}} + \frac{K_{\mathrm{f}}}{\phi_{\mathrm{c}}} P_{\mathrm{e}}}$$

Fluid saturated rock bulk modulus related to effective stress

Linearized reflection coefficient and EI

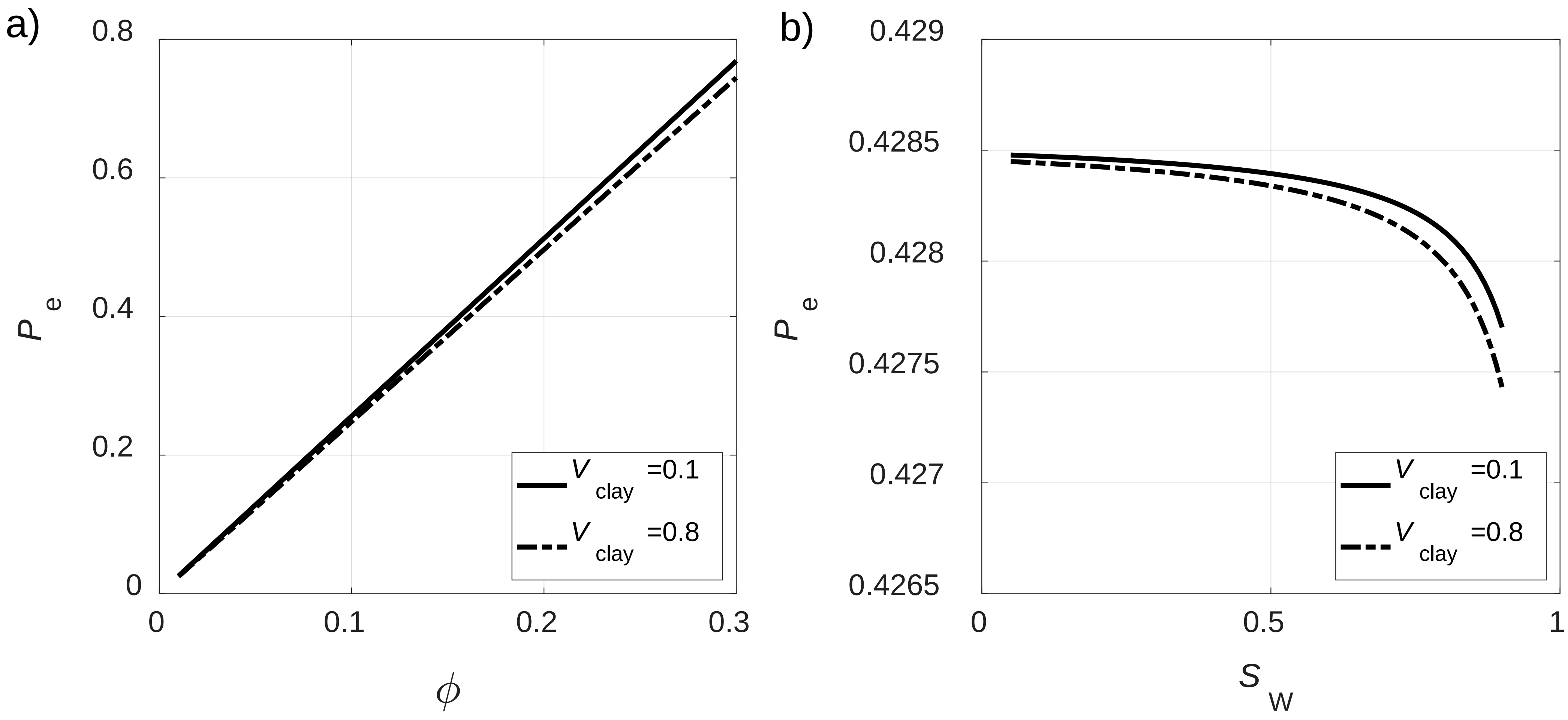
Clay volume 0.1



Clay volume 0.8

$$\phi_c = 0.35$$

Linearized reflection coefficient and EI



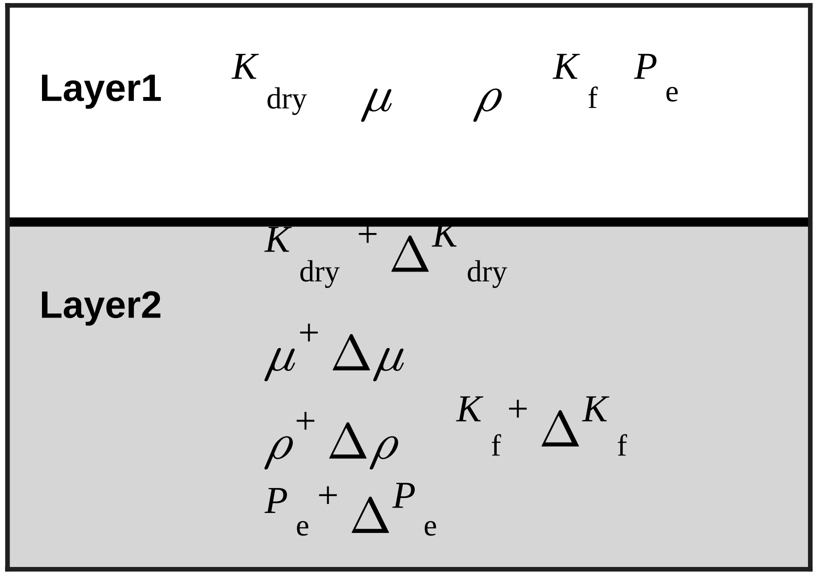
Porous and oil-bearing reservoirs exhibit high values of P_e.

Linearized reflection coefficient and EI

P-to-P reflection coefficient related to the scattering function (Shaw and Sen, 2006)

$$R_{\mathrm{PP}} = \frac{1}{4\rho\cos^2\theta} S,$$

$$S = \Delta\rho\cos 2\theta + \Delta C_{33}\left(\xi_a\right) + \Delta C_{55}(\xi_b),$$



$$\Delta C_{33} = \left(K_{\mathrm{sat}} + \tfrac{4}{3}\mu\right)_{layer2} - \left(K_{\mathrm{sat}} + \tfrac{4}{3}\mu\right)_{layer1}$$
$$\approx \Delta K_{\mathrm{dry}} + \frac{\Delta K_{\mathrm{f}}}{\phi_{\mathrm{c}}} P_{\mathrm{e}} + \frac{K_{\mathrm{f}}}{\phi_{\mathrm{c}}} \Delta P_{\mathrm{e}},$$

$$\Delta C_{55} = \Delta\mu$$

Linearized reflection coefficient and EI

$$R_{\mathrm{PP}} = \frac{1}{2\cos^2\theta}\left(\frac{\gamma_{\mathrm{sat}}}{\gamma_{\mathrm{dry}}} - \frac{4}{3}\gamma_{\mathrm{sat}}\right)R_{K_{\mathrm{dry}}} - 4\gamma_{\mathrm{sat}}\sin^2\theta R_\mu + \frac{\cos 2\theta}{2\cos^2\theta}R_F$$
$$+ \frac{1}{2\cos^2\theta}\left(2\sin^2\theta - \frac{\gamma_{\mathrm{sat}}}{\gamma_{\mathrm{dry}}}\right)R_{K_{\mathrm{f}}} + \frac{1}{2\cos^2\theta}\left(1 - \frac{\gamma_{\mathrm{sat}}}{\gamma_{\mathrm{dry}}}\right)R_{P_{\mathrm{e}}}$$

$$\mathrm{EI} = \mathrm{EI}_0\left(\frac{K_{\mathrm{dry}}}{K_{\mathrm{dry}0}}\right)^{a_{K_{\mathrm{dry}}}(\theta)}\left(\frac{\mu}{\mu_0}\right)^{a_\mu(\theta)}\left(\frac{F}{F_0}\right)^{a_F(\theta)}\left(\frac{K_{\mathrm{f}}}{K_{\mathrm{f}0}}\right)^{a_{K_{\mathrm{f}}}(\theta)}\left(\frac{P_{\mathrm{e}}}{P_{\mathrm{e}0}}\right)^{a_{P_{\mathrm{e}}}(\theta)}$$

$$R_{K_{\mathrm{dry}}} = \frac{\Delta K_{\mathrm{dry}}}{2K_{\mathrm{dry}}}, R_\mu = \frac{\Delta\mu}{2\mu}, R_F = \frac{\Delta F}{2F}, R_{K_{\mathrm{f}}} = \frac{\Delta K_{\mathrm{f}}}{2K_{\mathrm{f}}}, R_{P_{\mathrm{e}}} = \frac{\Delta P_{\mathrm{e}}}{2P_{\mathrm{e}}};$$
$$F = \rho K_{\mathrm{f}}$$

dry rock properties

Fluid-sensitive factors

Stress-sensitive parameter

Procedure of seismic inversion

Step1: Using seismic data to estimate EI

Least-squares algorithm is employed to solve the inversion problem (Chen and Innanen, 2018).

$$\mathbf{S}\left(\theta_i\right) = \mathbf{WDL}\underline{\mathbf{EI}\left(\theta_i\right)}\ ,$$

Seismic data vector

Wavelet vector

Difference vector

EI vector

Step2: Using the estimated EI to implement a nonlinear inversion

$$\mathbf{d} = \mathbf{G}(\mathbf{m})$$

Estimated EI vector

Parameter vector

Model vector

$$\mathbf{m} = \left[\mathbf{K}_{\mathrm{dry}} \quad \boldsymbol{\mu} \quad \mathbf{F} \quad \mathbf{K}_{\mathrm{f}} \quad \mathbf{P}_{\mathrm{e}}\right]^T$$

Nonlinear inversion

Residual energy: $\mathbf{E} = \|\mathbf{d}_{\mathrm{mod}} - \mathbf{d}_{\mathrm{obs}}\|_2 = \frac{1}{2}(\delta\mathbf{d})^T(\delta\mathbf{d})$

Iteration process: $\mathbf{m} = \mathbf{m}_{\mathrm{i}} + \delta\mathbf{m}$

How to obtain $\delta\mathbf{m}$?

Residual energy Taylor expansion:

$$\mathbf{E}(\mathbf{m}_{\mathrm{i}} + \delta\mathbf{m}) \approx \mathbf{E}(\mathbf{m}_{\mathrm{i}}) + \left(\frac{\partial\mathbf{E}}{\partial\mathbf{m}}\right)\delta\mathbf{m} + \frac{1}{2}\left(\frac{\partial^2\mathbf{E}}{\partial^2\mathbf{m}}\right)(\delta\mathbf{m})^2$$

Derivative of E with respect to δm: $\frac{\partial\mathbf{E}(\mathbf{m}_{\mathrm{i}}+\delta\mathbf{m})}{\delta\mathbf{m}} = \left(\frac{\partial\mathbf{E}}{\partial\mathbf{m}}\right) + \left(\frac{\partial^2\mathbf{E}}{\partial^2\mathbf{m}}\right)(\delta\mathbf{m})$

Result: $\delta\mathbf{m} = -\mathbf{H}^{-1}\mathbf{g}$

Hessian matrix

Transferring the derivative of energy residual to the derivative of EI：

$$\frac{\partial \mathbf{E}}{\partial \mathbf{m}} = \frac{\partial\left[\frac{1}{2}(\mathbf{d}_{\mathrm{mod}}-\mathbf{d}_{\mathrm{obs}})^T(\mathbf{d}_{\mathrm{mod}}-\mathbf{d}_{\mathrm{obs}})\right]}{\partial \mathbf{m}} = \frac{\partial \mathbf{d}_{\mathrm{mod}}(\mathbf{m})}{\partial \mathbf{m}}(\delta \mathbf{d})$$

Derivative of EI with respect to model vector m：

$$\frac{\partial \mathbf{d}_{\mathrm{mod}}(\mathbf{m})}{\partial \mathbf{m}} = \begin{bmatrix} \frac{\partial(\mathbf{EI})}{\partial(\mathbf{K}_{\mathrm{dry}})} & \frac{\partial(\mathbf{EI})}{\partial(\boldsymbol{\mu})} & \frac{\partial(\mathbf{EI})}{\partial(\mathbf{F})} & \frac{\partial(\mathbf{EI})}{\partial(\mathbf{K}_{\mathrm{f}})} & \frac{\partial(\mathbf{EI})}{\partial(\mathbf{P}_{\mathrm{e}})} \end{bmatrix}^T$$

Nonlinear inversion

Details of inversion (A four-step inversion)

1) Using $EI(\theta_1)$ to estimate stress-sensitive parameter $\boldsymbol{P_e}$;

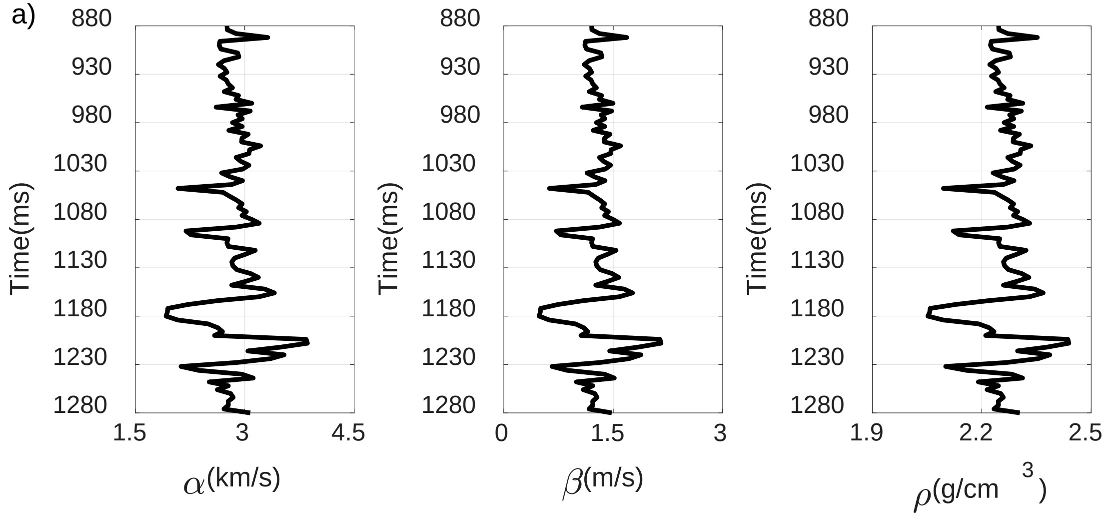
2) Using $EI(\theta_2)$ to estimate fluid bulk modulus $\boldsymbol{K_f}$;

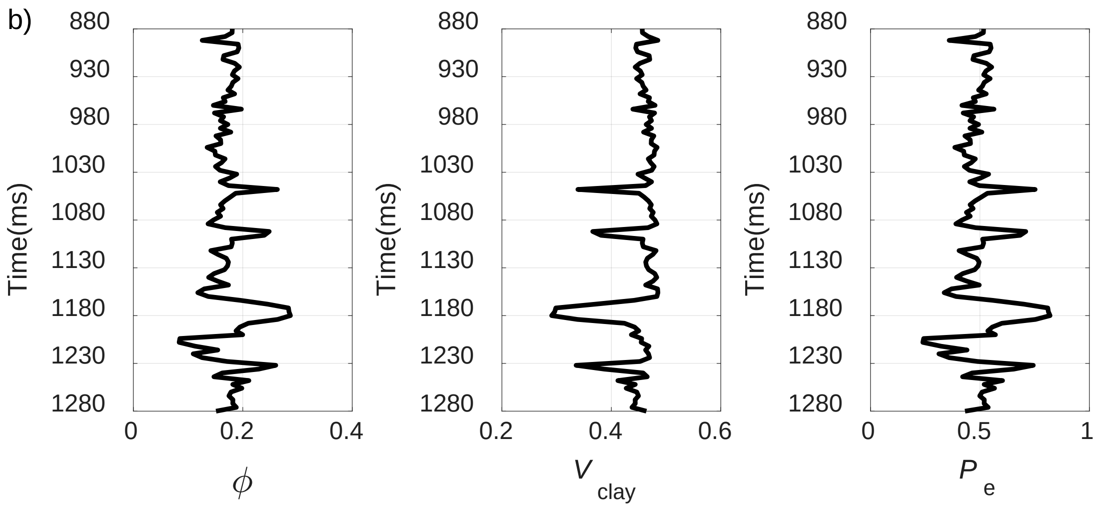
3) Using $EI(\theta_3)$ to estimate shear modulus $\boldsymbol{\mu}$;

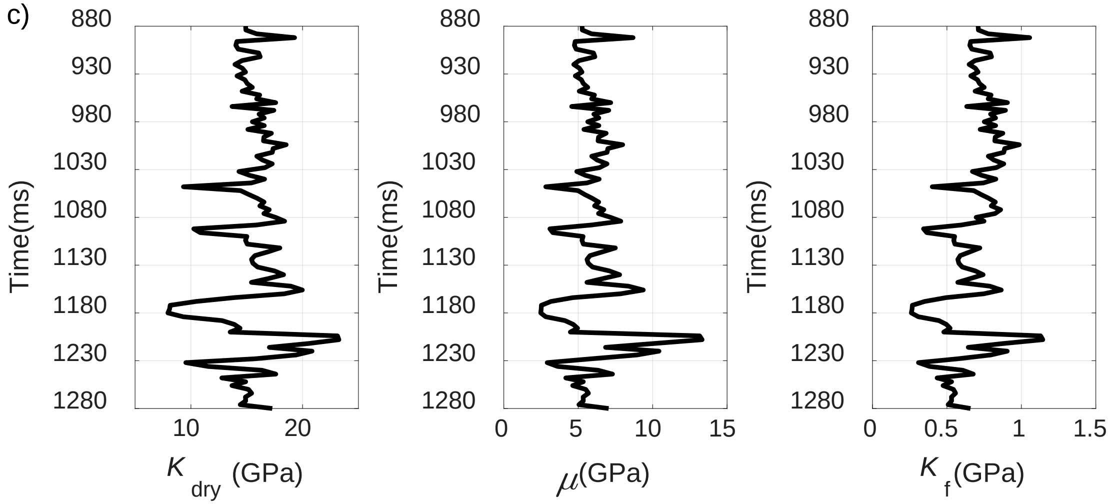
4) Using $EI(\theta_4)$ and $EI(\theta_5)$ to obtain inversion results of dry rock bulk modulus $\boldsymbol{K_{dry}}$ and fluid factor $\boldsymbol{F}$.

Numerical examples

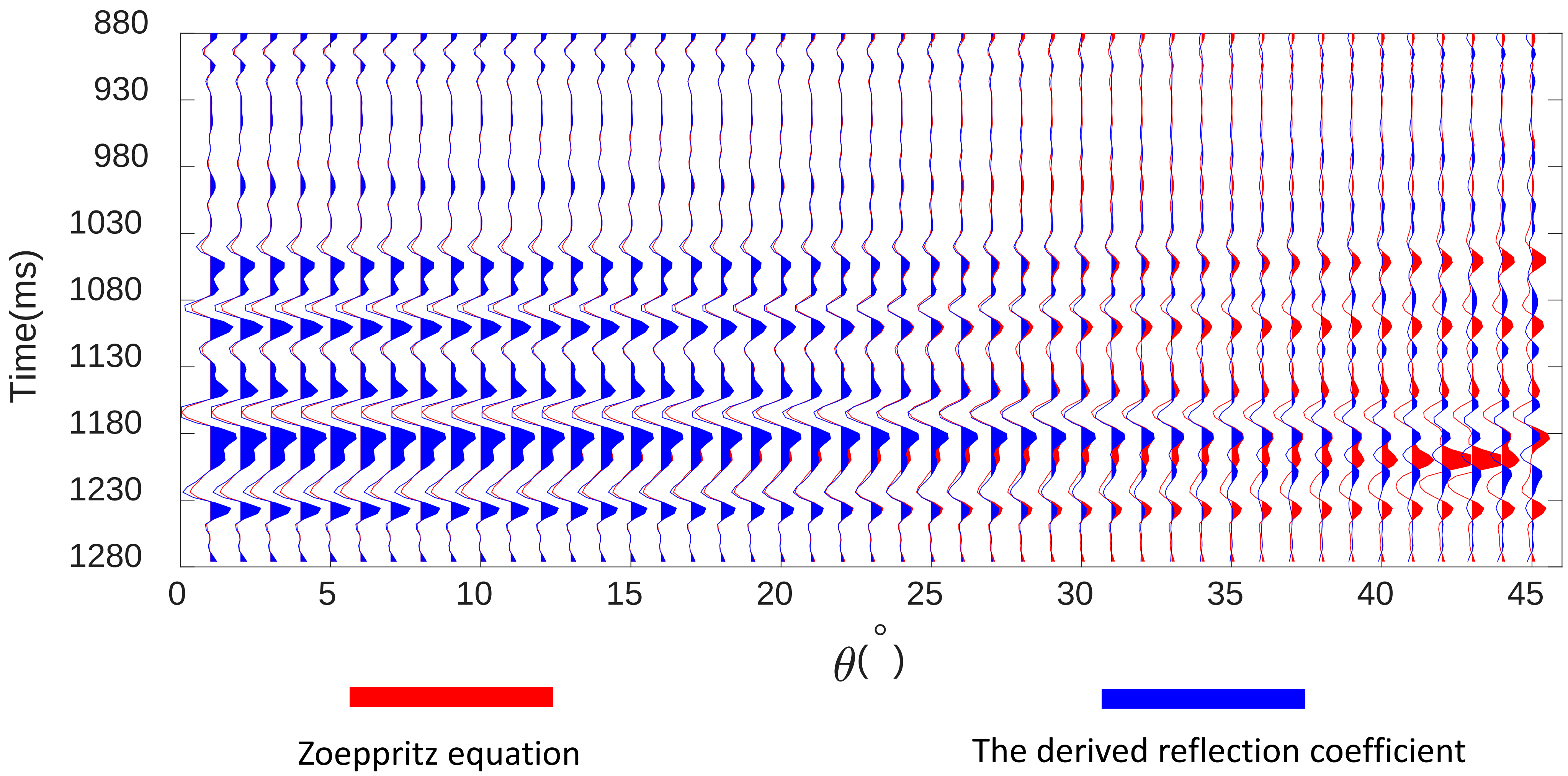
Well log model



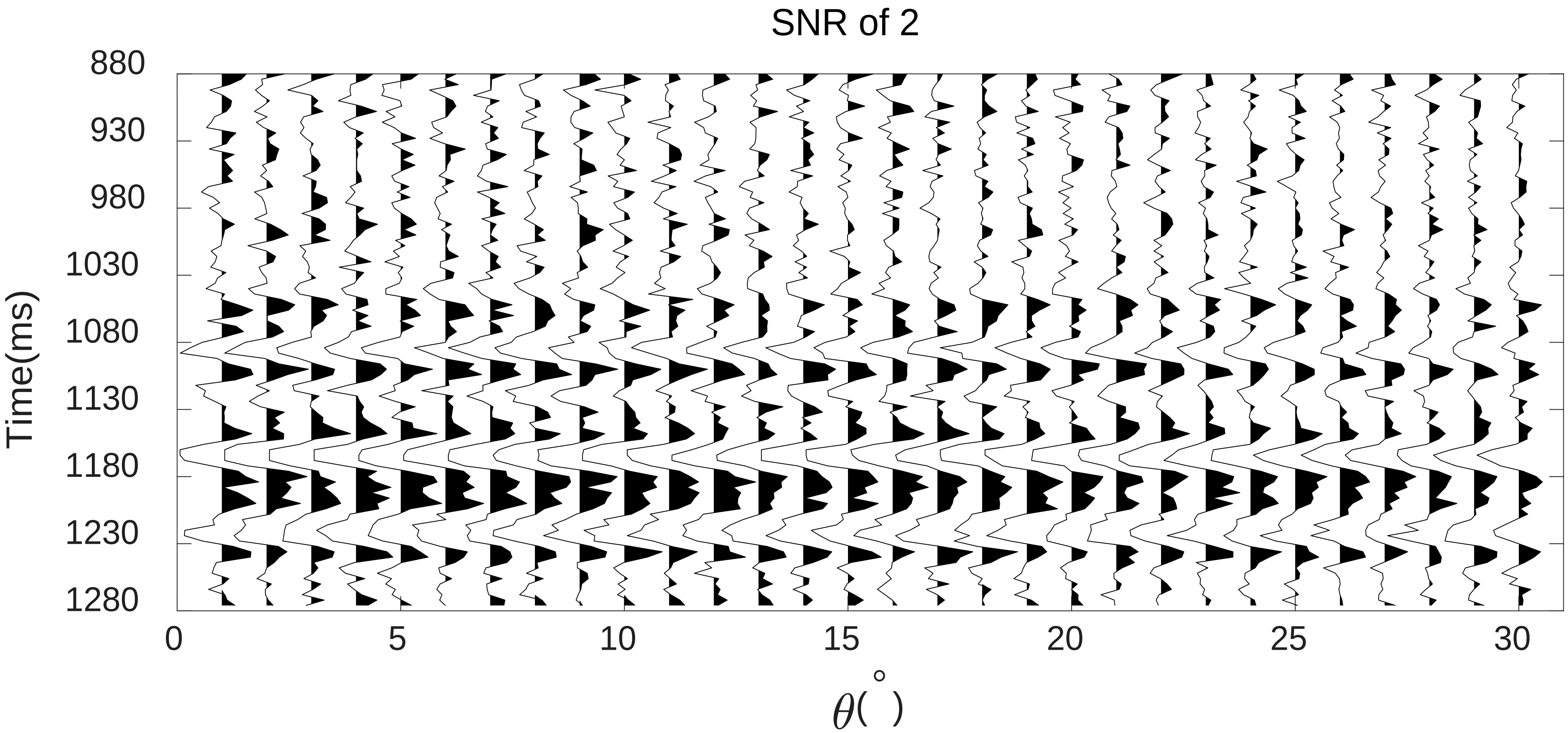




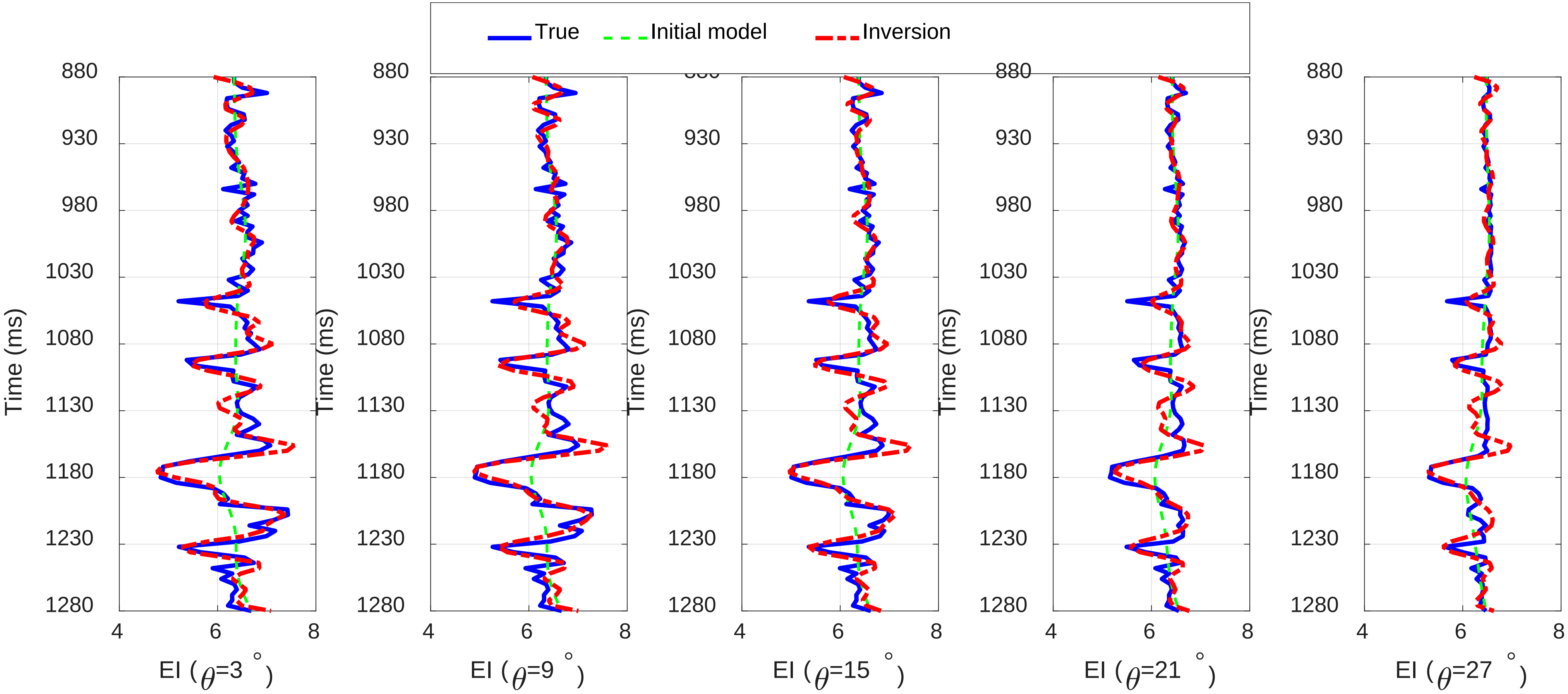
Verification of accuracy of reflection coefficient



Noisy synthetic data



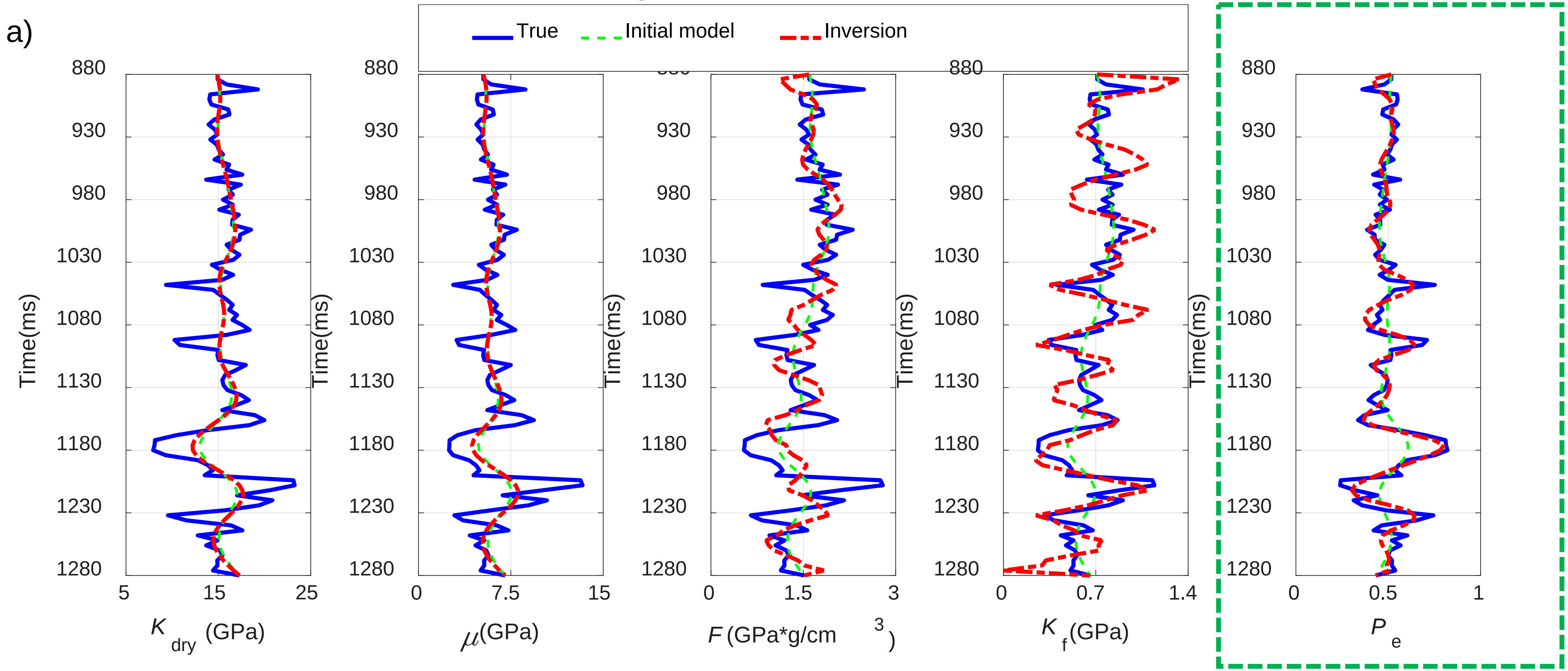
Step 1: Inversion of EI (Least-squares algorithm)



SNR=2

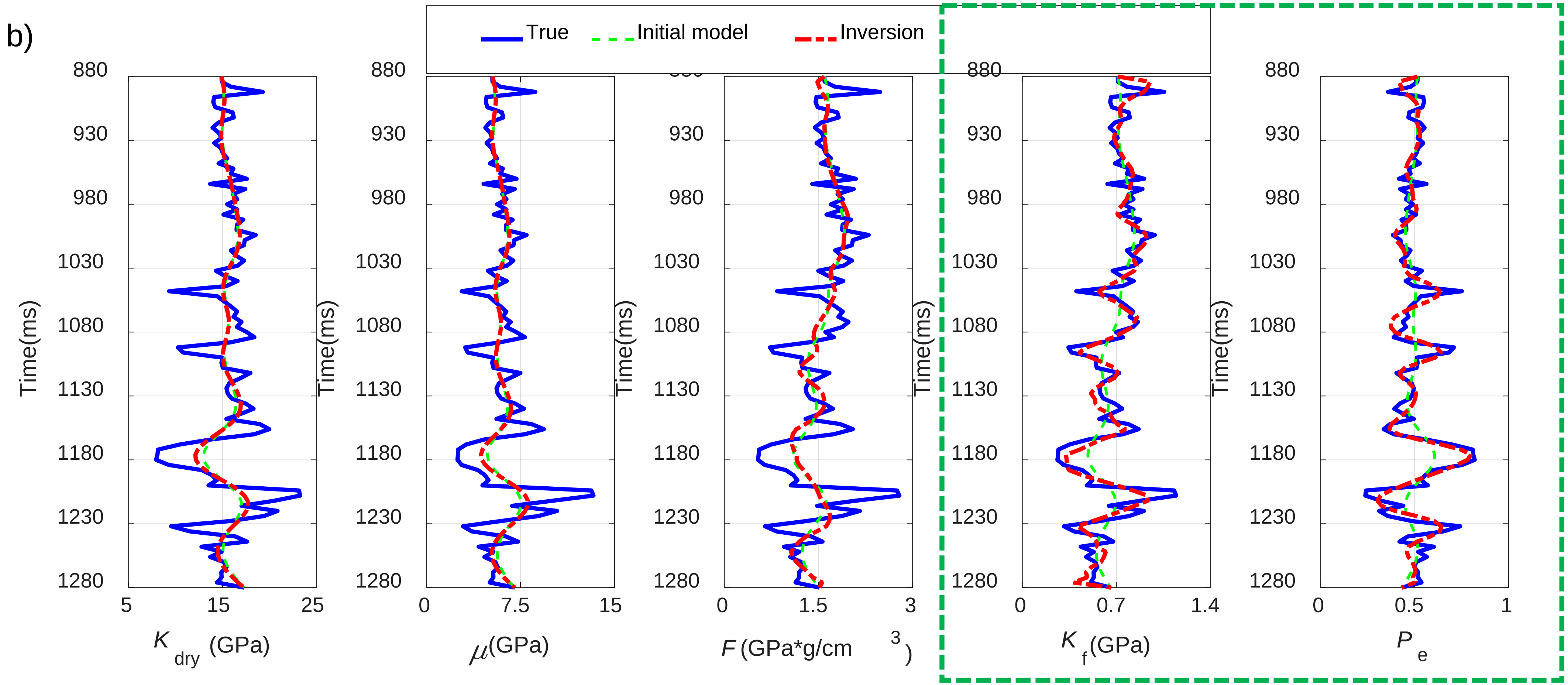
Step2: Nonlinear inversion for unknown parameters one by one

a) Using EI(θ=3°) to estimate P_e

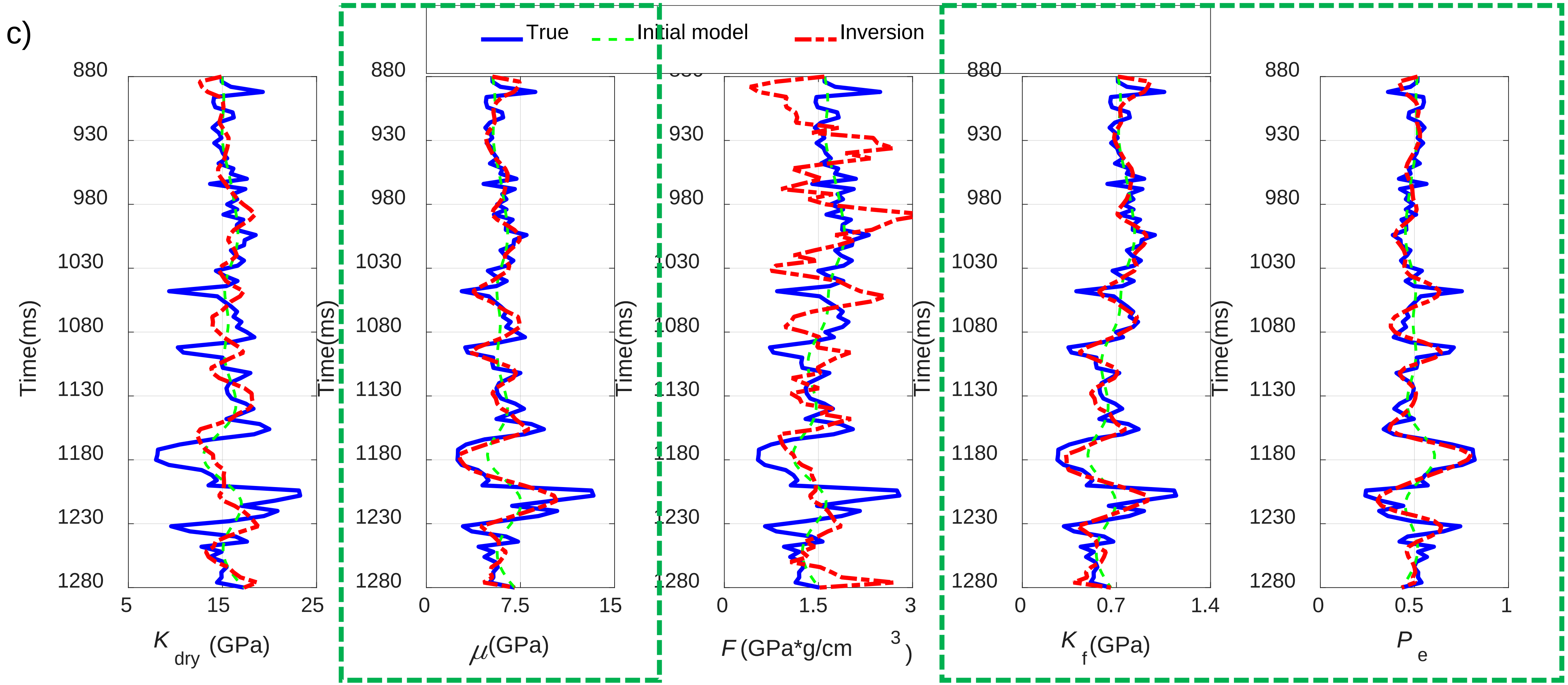


Numerical examples

b) Using EI(θ=9^o) to estimate K_f

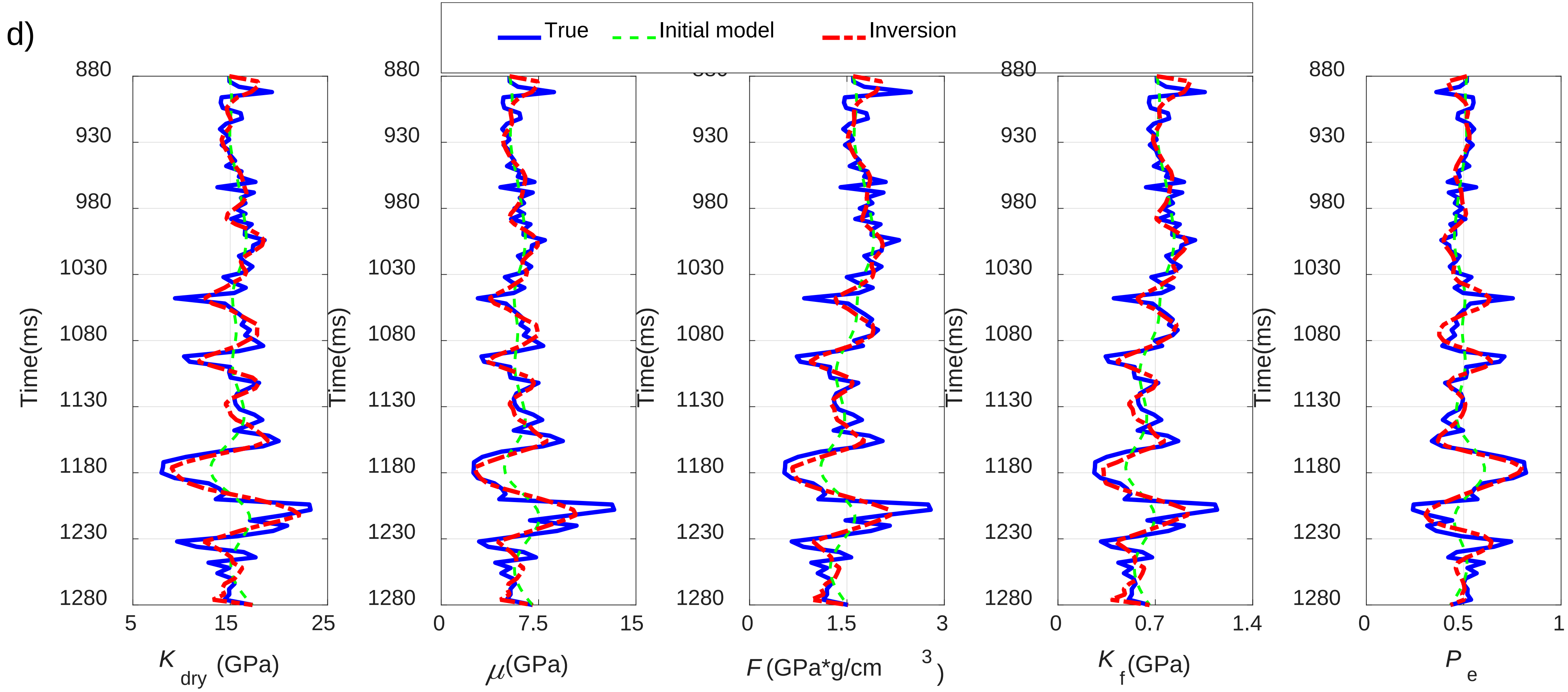


c) Using EI(θ=15^o) to estimate μ

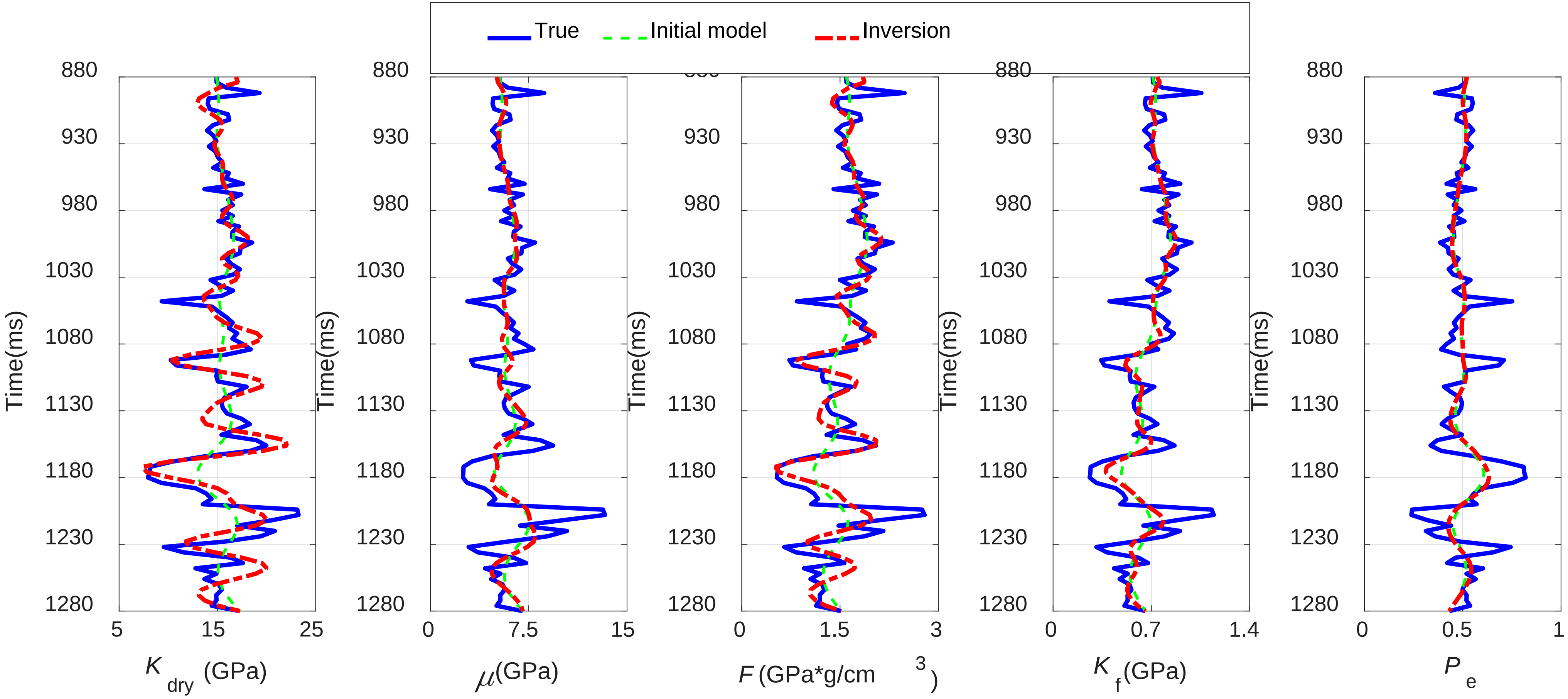


Numerical examples

d) Using EI(θ=21°) and EI(θ=27°) to estimate K_{dry} and F



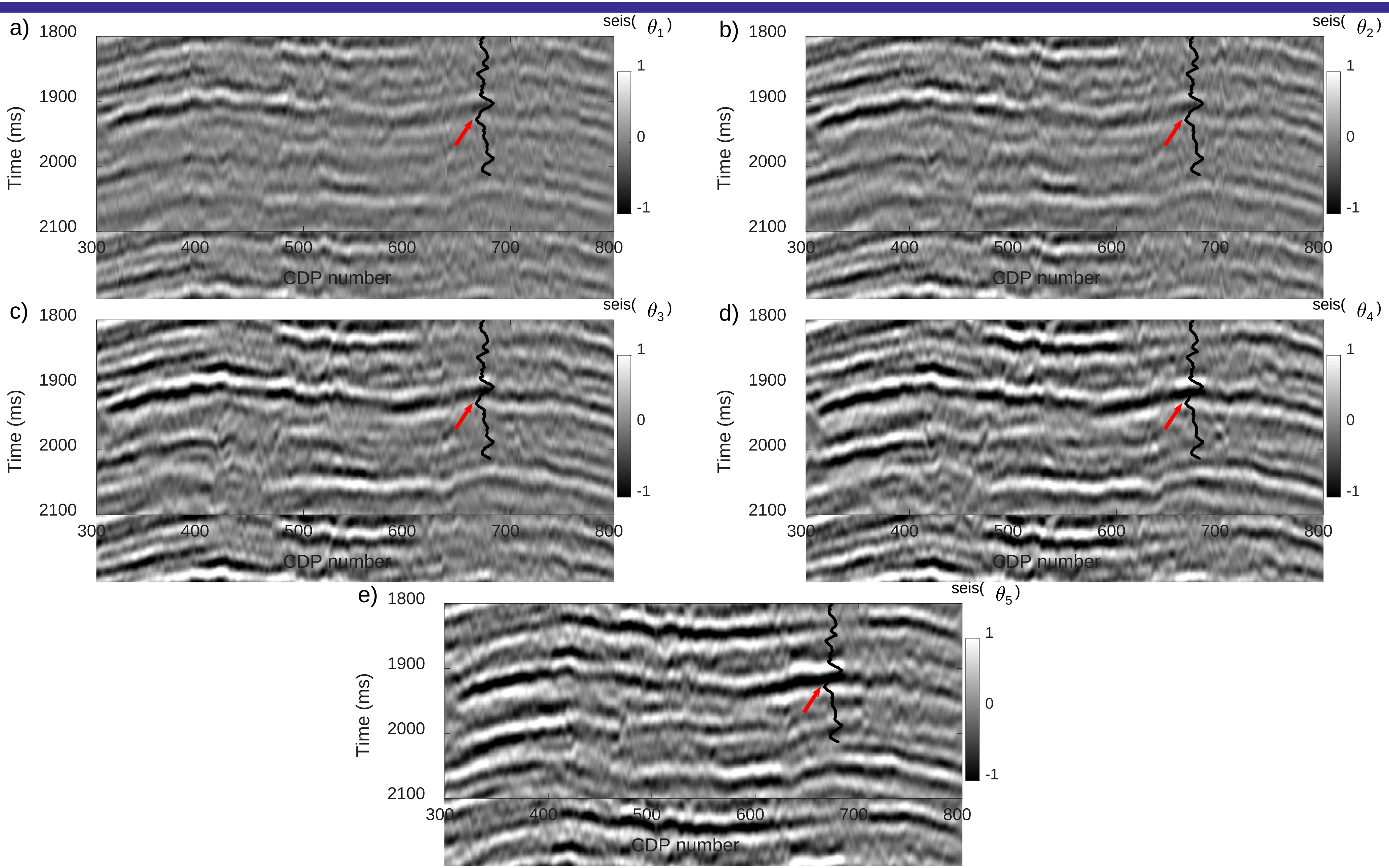
AVO simultaneous inversion results



Numerical examples

Real data

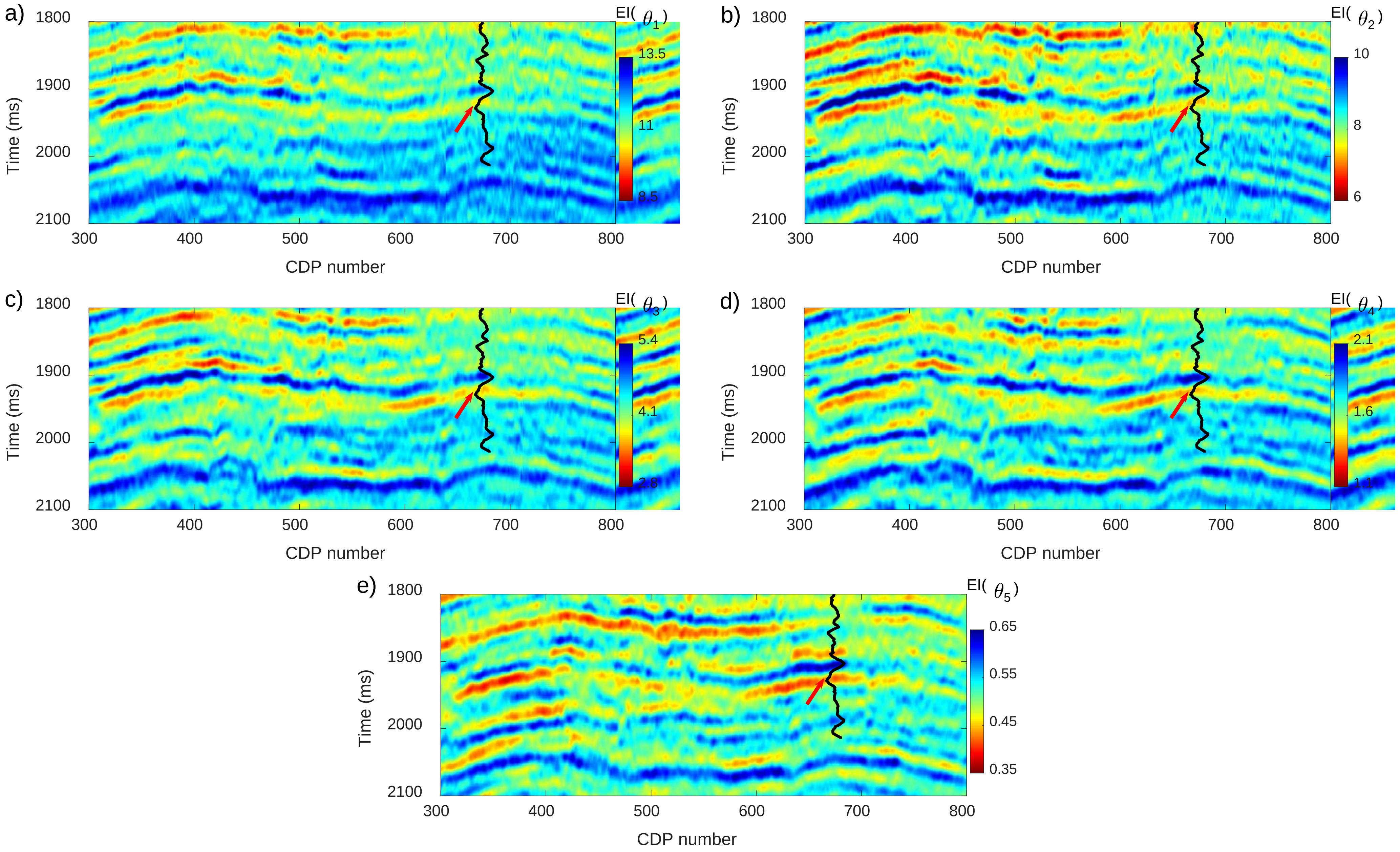
Stacked Seismic Data



Numerical examples

Real data

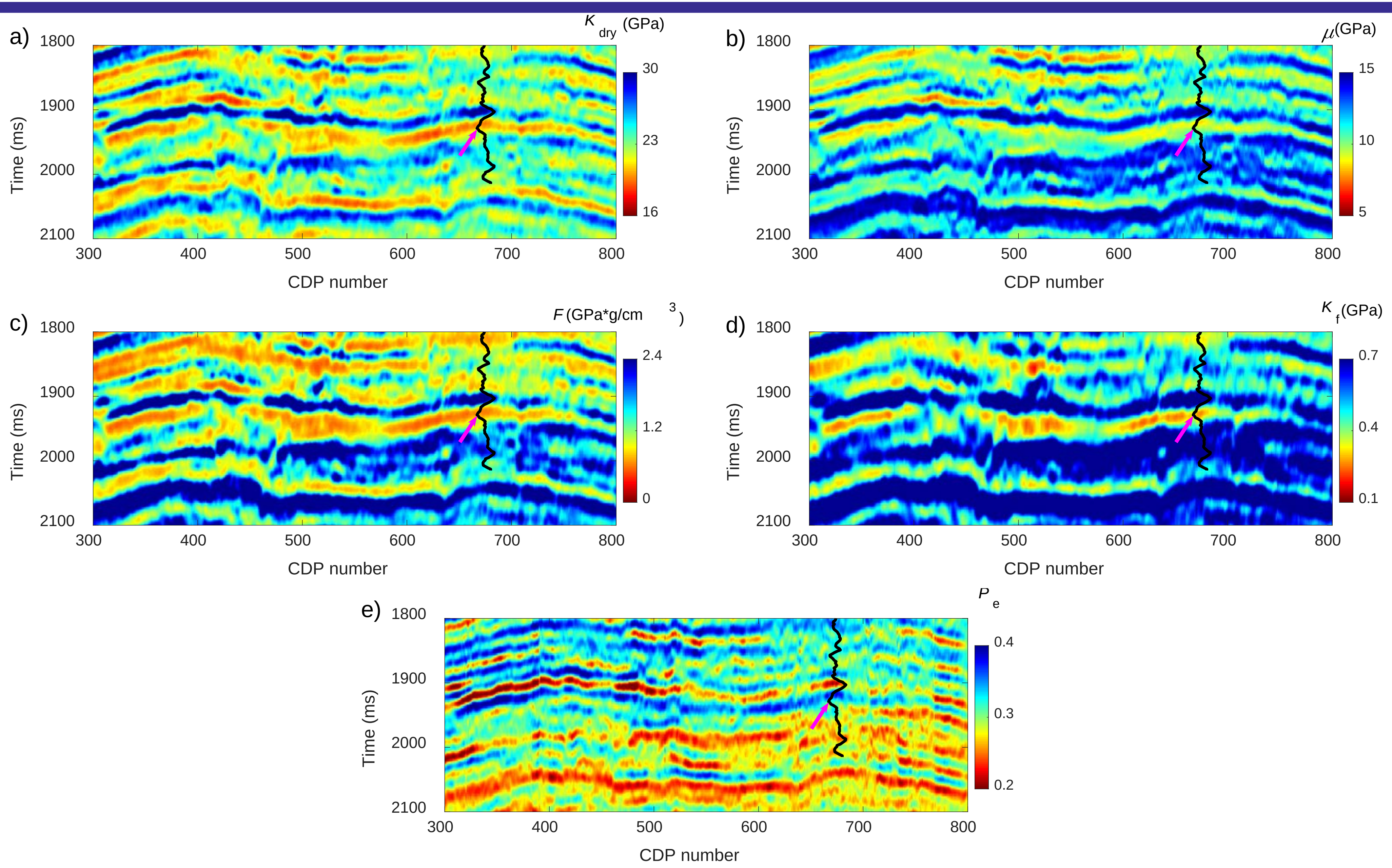
Inversion Results of EI



Numerical examples

Real data

Inversion results of unknown parameters



Conclusions

- Using the simplified Gassmann's equation, we derived a linearized reflection coefficient and the corresponding EI as a function of effective stress-sensitive parameter;
- We proposed a two-step approach of employing seismic data to estimate fluid factors and stress-sensitive parameter based on the derived reflection coefficient an EI;
- Tests on noisy synthetic data and real data implied that the proposed approach is stable and it can generate meaningful results for fluid identification and pressure estimation.
- Future work: Stress estimation in HTI, VTI, and OA media.

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