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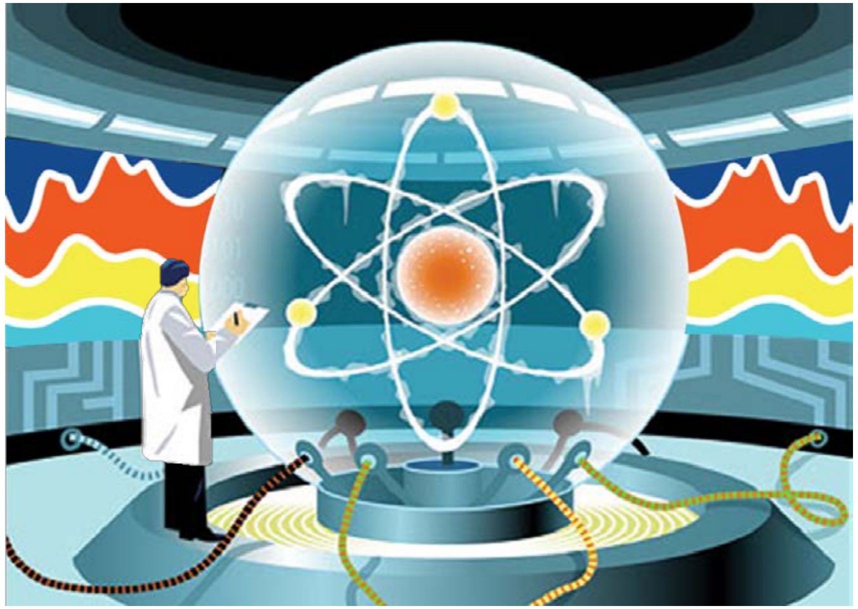
Quantum computing for seismic problems

Shahpoor Moradi, Daniel Trad and Kris Innanen

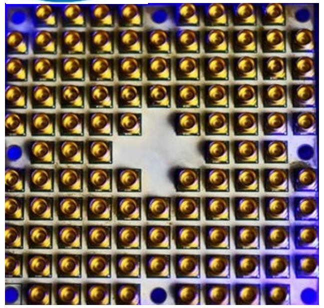


November 30th , 2018

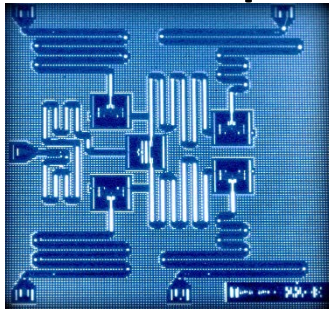
NSERC CRSNG CMC Containment & Monitoring Institute UNIVERSITY OF CALGARY FACULTY OF SCIENCE Department of Geoscience

Latest news in developing quantum computers (2018)

intel 49-qubit



IBM 50-qubit



rigetti 17-qubit

Google 72-qubit

- Any of these quantum computers can solve the real world problems?
  Too small for classical problems such as code breaking or seismic modeling
  Probably large enough to emulate the quantum chemistry
- What sorts of problems are quantum computers good for in general?
  Quantum chemistry, prime factorization, search and probably optimization
- What about seismic problems? modeling? inversion? imaging?
  We are working on this....

Outline

- Quantum computation
- Computational challenges in exploration seismology
  Classical and quantum resources
- Quantum algorithms with applications in seismic problems
- Future work and summary

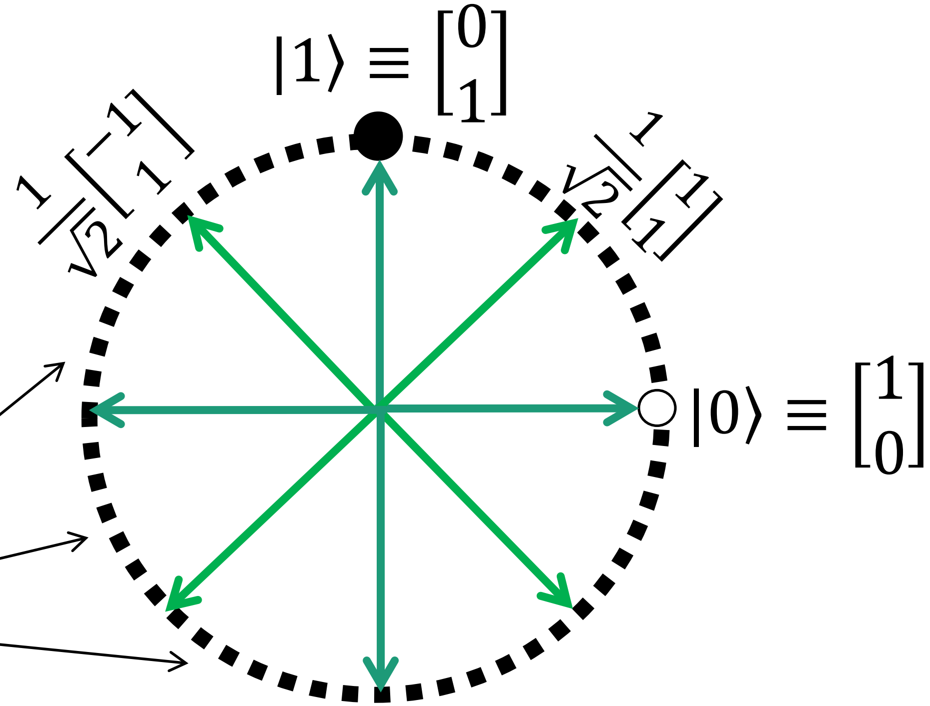
Quantum bit

Smallest unit of information in a quantum computer

A **classical bit** is either ‘0’ or ‘1’

A **quantum bit** can be an arbitrary superposition of ‘0’ and ‘1’.

A **quantum bit** state can be represented by any point on the unite circle.



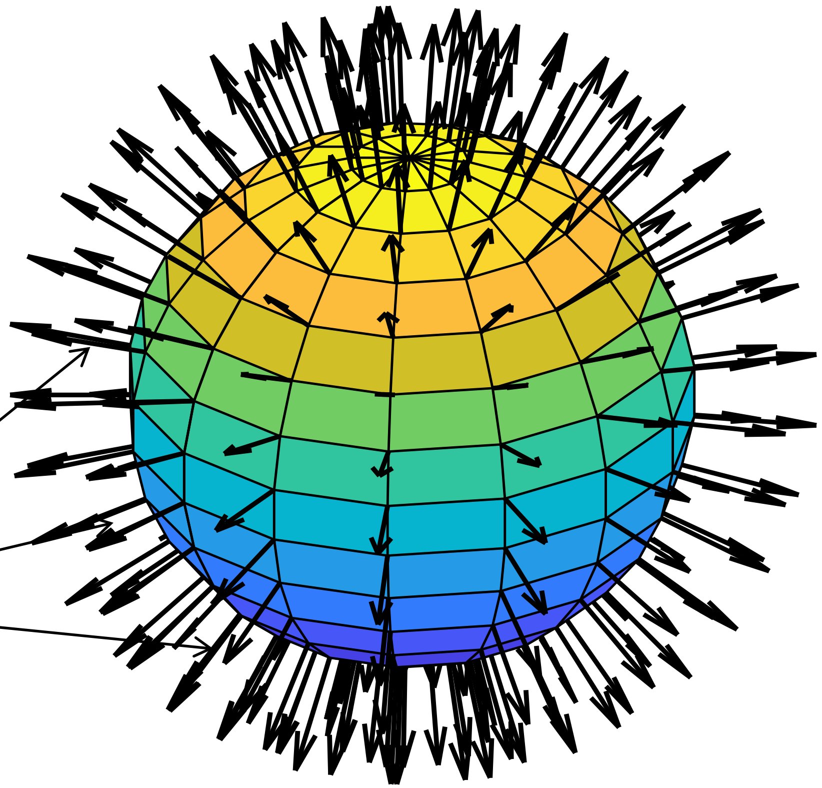
Quantum bit

Smallest unit of information in a quantum computer

A **classical bit** is either ‘0’ or ‘1’

A **quantum bit** can be an arbitrary superposition of ‘0’ and ‘1’.

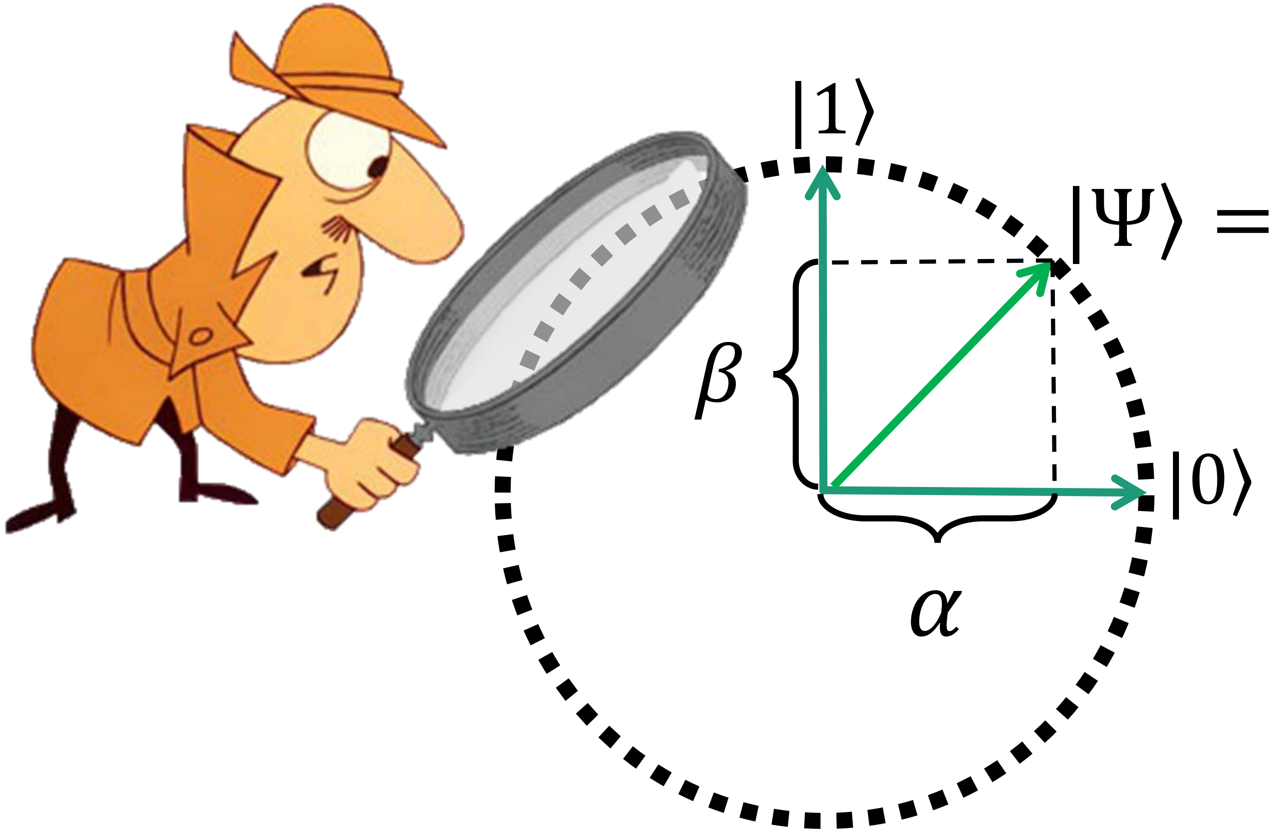
A **quantum bit** state can be represented by any point on the sphere.



Measurement

Has anybody ever seen the superposition?

Measurement collapses the quantum bit to one of the two classical states

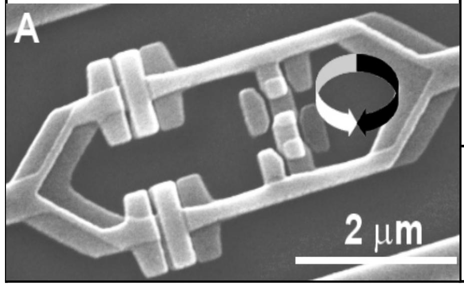


$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$|\alpha|^2$ Probability that qubit collapses to $|0\rangle$

$|\beta|^2$ Probability that qubit collapses to $|1\rangle$

Physical realization of qubit

Physical support	Name	Information support	$\|0\rangle$	$\|1\rangle$
Superconducting qubit	Charge qubit	Charge	Uncharged island	Charged island
	Flux qubit	Current	Clockwise current	Counterclockwise current
	Phase qubit	Energy	Ground state	First excited state

https://en.wikipedia.org/wiki/Qubit

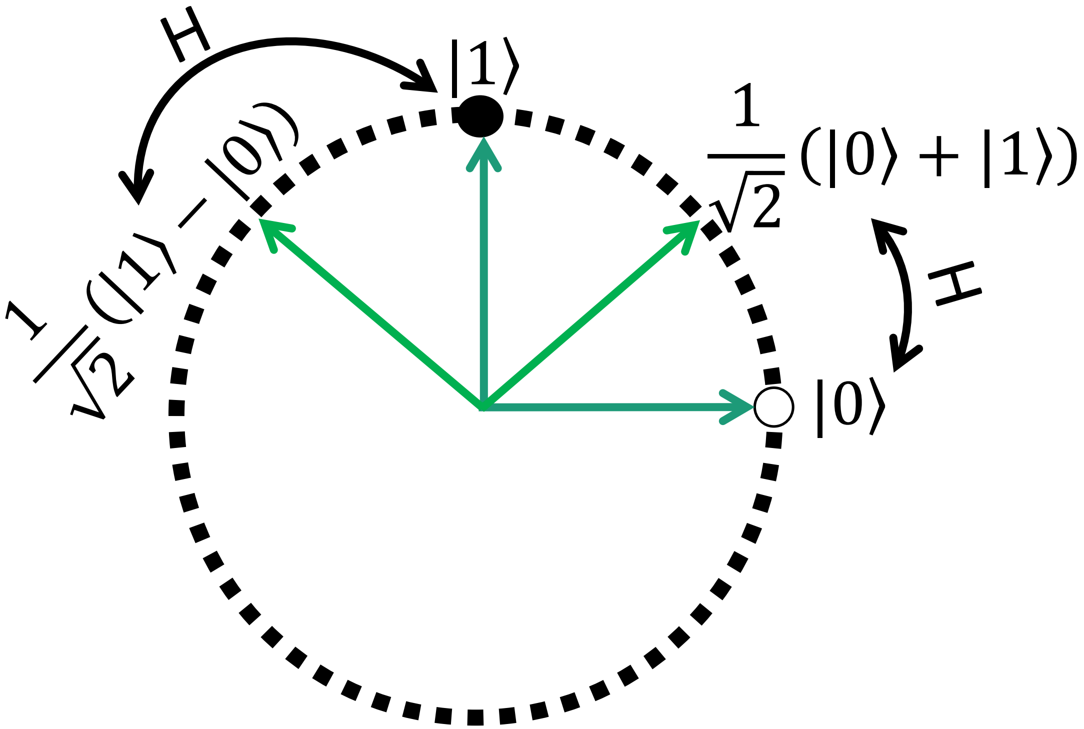
Quantum gates

In classical circuit theory, only two single logic gates are possible, Identity gate and NOT gate.
In quantum circuit theory, any 2×2 unitary matrix ($U^\dagger U = I$) is a valid single qubit gate.

Single qubit gate

$$U|\Psi\rangle = U(\alpha|0\rangle + \beta|1\rangle) = \alpha U|0\rangle + \beta U|1\rangle$$

Simultaneous computation on two different states!



Quantum parallelism: power of quantum computation

- 1 qubit quantum computer can register ‘0’ and ‘1’ at the same time and perform two computations simultaneously
- 2 qubit quantum computer can register four states at the same time and perform four computations simultaneously

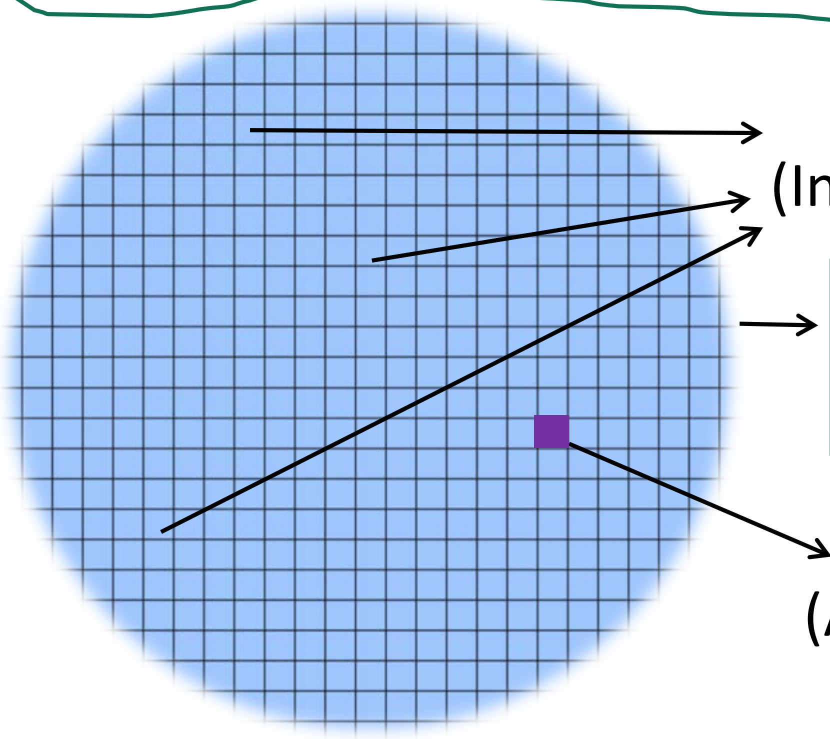
⋮

- N qubit quantum computer can register 2^N states at the same time and perform 2^N computations simultaneously

$$2^{50}=1,125,899,906,842,624 \sim 10^{15}$$

Output of quantum computer is not fully accessible!

Quantum computer process considerable amounts of information at a rate that cannot be matched in real time by any classical means, yet at the same time, most of this processed information is kept hidden from us!



Quantum information
(Inaccessible information)

2^{N} cells each contains N bits of information

Classical information
(Accessible information)

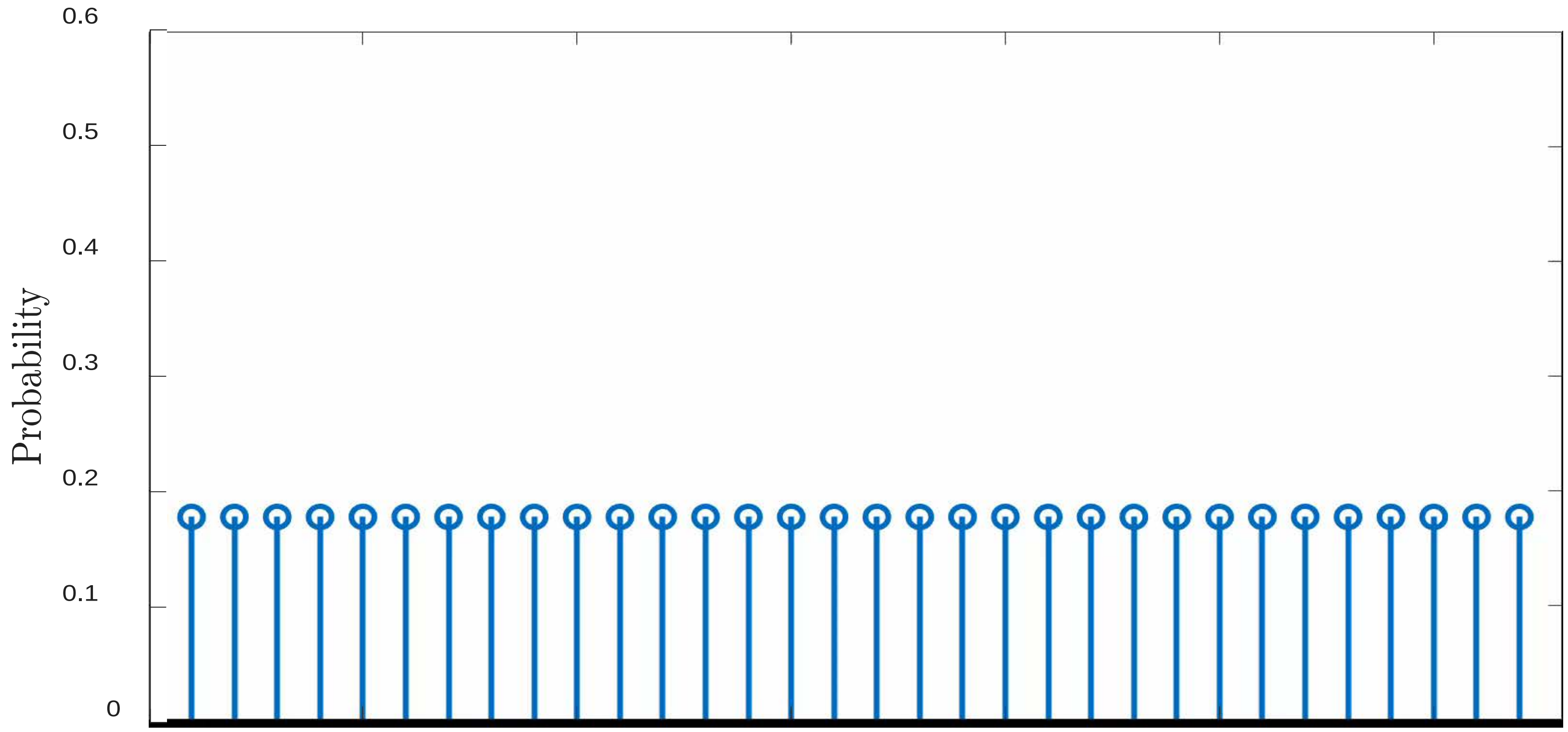
Holevo's theorem

A quantum state with 2^{N} information content: at most $\mathrm{N} = \log 2^{\mathrm{N}}$ classical bits of information can be extracted

Quantum algorithm: 5 qubit test

Probabilities for observing each of the 32 basis states are equal $(0.177)^2$=0.0312

$$|\psi\rangle = 0.177|00000\rangle + 0.177|00001\rangle + 0.177|00010\rangle \ldots + 0.177|11111\rangle$$



Classical resources

Computational complexity of seismic modeling

N: Num. of grids~1000
N_f: Num. of frequencies
N_{it}: Num. of iterations
N_s: Num. of sources

$$O(N_f N_{it} N_s N^3), \quad N_f \to N, \quad N_{it} \to N, \quad N_s \to N^2$$

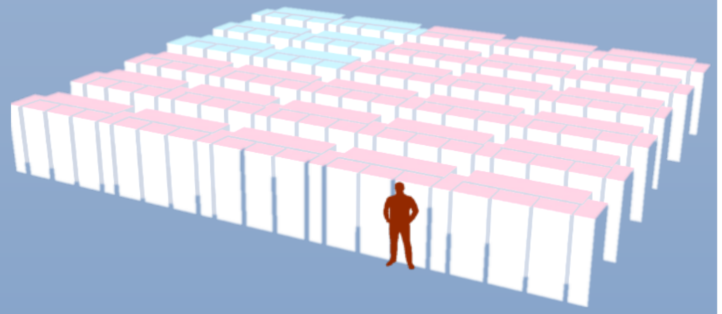
$$\mathrm{N}^7 \sim (1000)^7 = 10^{21} \text{ operations}$$

Full Waveform Inversion (FWI)

$$\textbf{3D FWI} \xrightarrow[\text{Computational complexity}]{} \mathrm{N}^8 \xrightarrow[\mathrm{N}=1000]{} 10^{24} \text{ operations}$$

Current supercomputers

1 Peta-Flops
=1,000,000,000,000,000
calculations per second



3D Modeling~ 12 days
3D FWI ~ 32 years !!

Quantum resources

Seismic modeling

70 qubits can handle 2^{70} or $(2^{10})^7$ or $(10^3)^7 \sim 10^{21}$ calculations simultaneously….

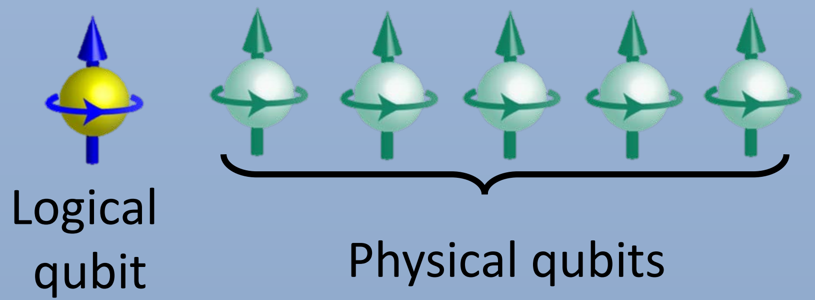
$70 \times 6 = 420$ **qubits**

Full Waveform Inversion (FWI)

80 qubits can handle 2^{80} or $(2^{10})^8$ or $(10^3)^8 \sim 10^{24}$ calculations simultaneously….

$80 \times 6 = 480$ **qubits**

Quantum error correction



For every one qubit, you have an additional five copies of that qubit, so, if the occasional one decoheres, you can still make a majority decision over the remaining five. Quantum superposition can thus be protected (Vedral, 2010)

Developed quantum algorithms

- **Factorization of large numbers**: much of modern day cryptography is based on the difficulty of factoring large prime numbers (exponential speed-up)

- **Searching a large database**: search for the correct answer amongst several (or a few million) incorrect answers (quadratic speed-up)

- **Quantum algorithm for linear systems of equations:** to accelerate machine learning tasks (QML) such as support vector machines (SVM), which are used for data classification and regression (exponential speed-up)

  P Rebentrost ea al., PRL, 2014

Harrow, A.W., Hassidim, A. and Lloyd, S., 2009. Quantum algorithm for linear systems of equations PRL.

Quantum solution for

$$A\,x = b$$

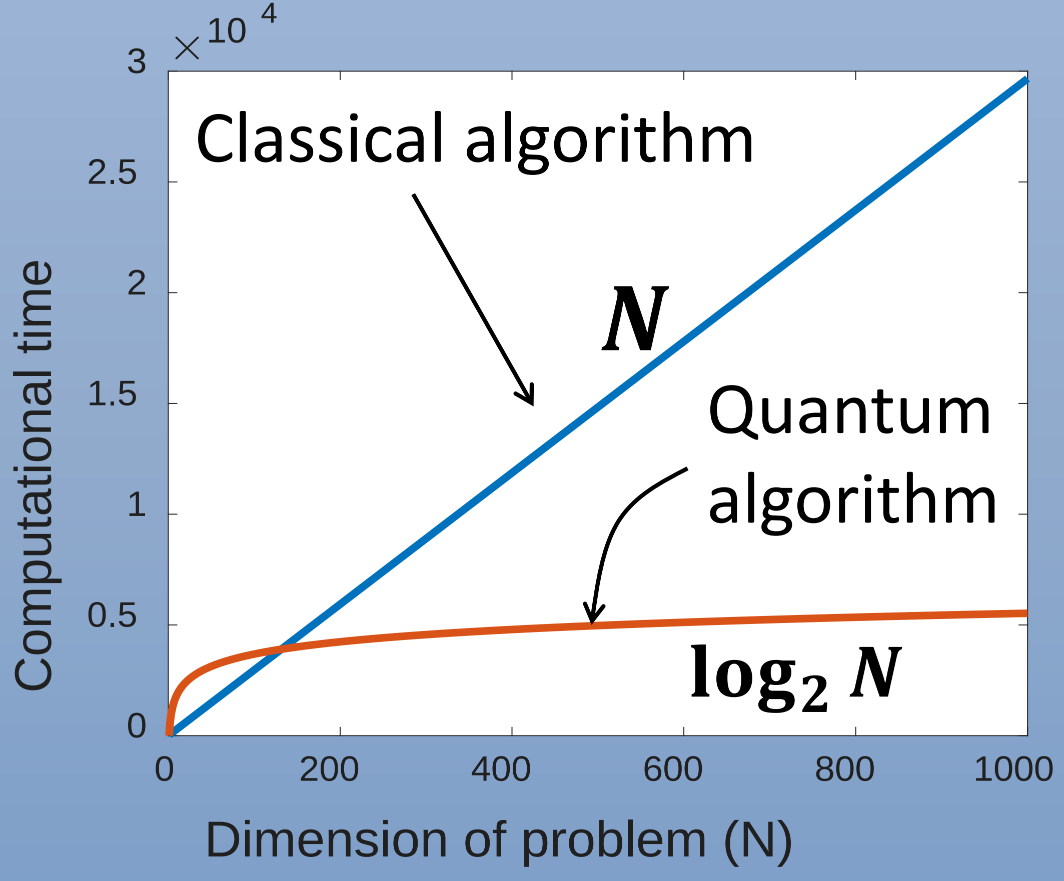
- A $N \times N$ Matrix
- $\vec{x}$ Unknown vector
- $\vec{b}$ Unit vector

$$|x\rangle = A^{-1} \boxed{|b\rangle}$$

log *N* qubits

$$\sum_{i=1}^{\log N} b_i |i\rangle$$

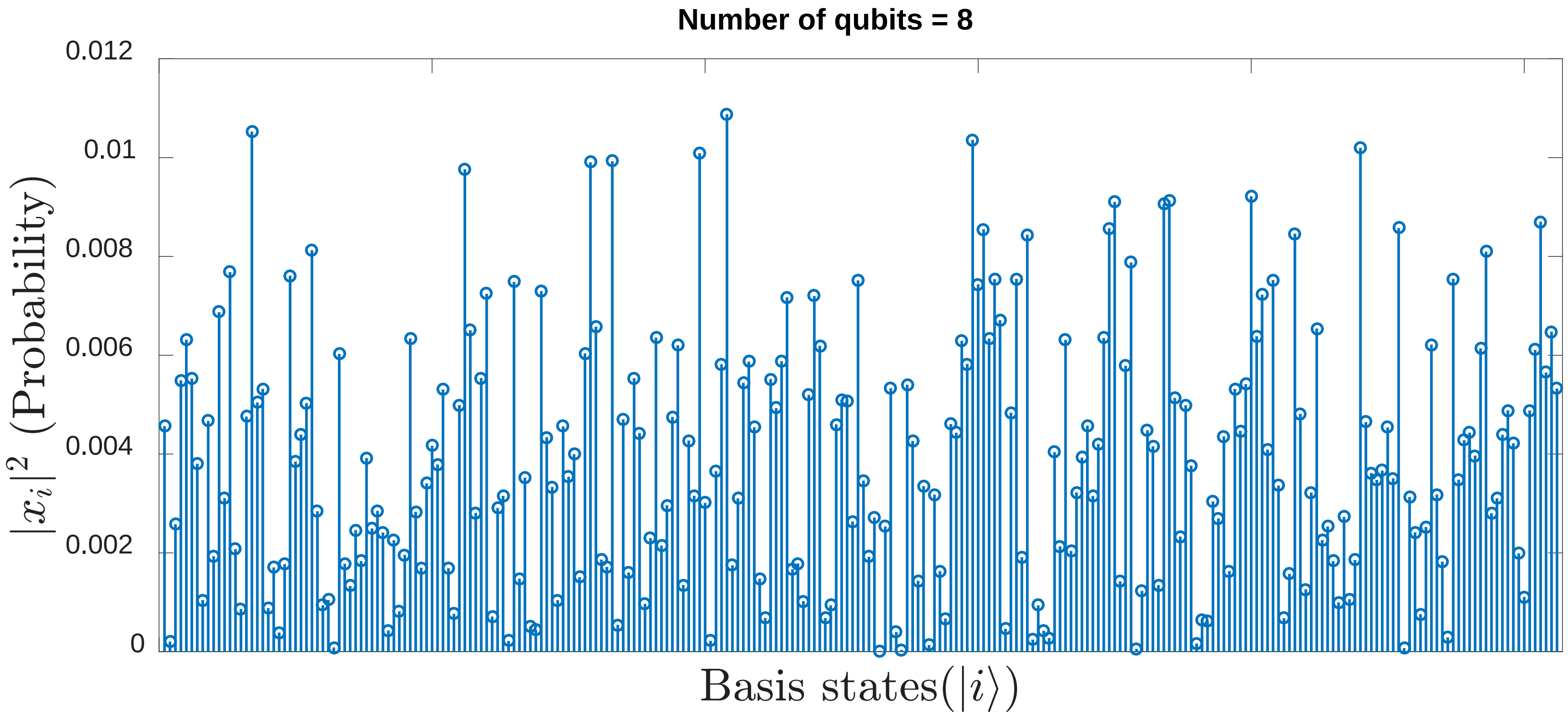
Quantum state
With ***N*** components



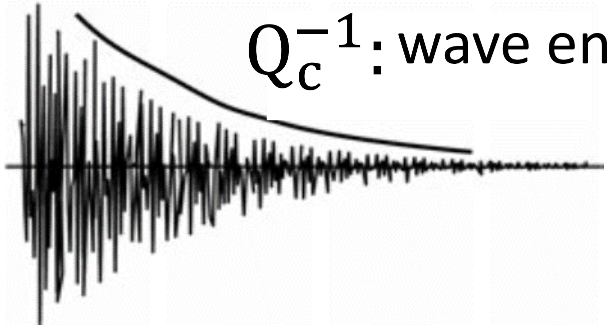
Applications in seismic problems

The output is the quantum state: $|x\rangle = x_0|0\rangle + x_1|2\rangle + \cdots + x_{N-1}|N-1\rangle$
with amplitudes x_i which encode the components of the vector $\boldsymbol{x} = (x_0, .., x_{N-1})$.

We can take advantages of the solution if we are interested in some statistical features of the solution. Classify $|x\rangle$ into one of the two clusters $|z\rangle$ and $|y\rangle$ by estimation of $\langle x|z\rangle$ and $\langle x|y\rangle$.

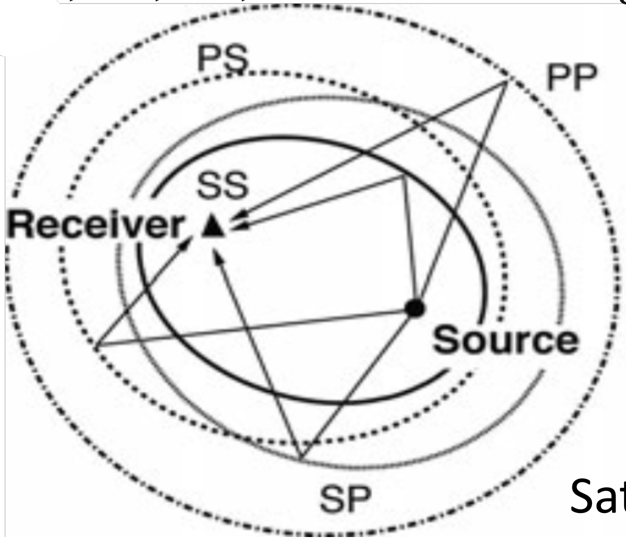


Seismic wave attenuation estimation



Q_c^{-1}: wave energy decreases with increasing travel distance due to scattering

PP, SS, PS, and SP Scattering modes



Sato et al., 2012

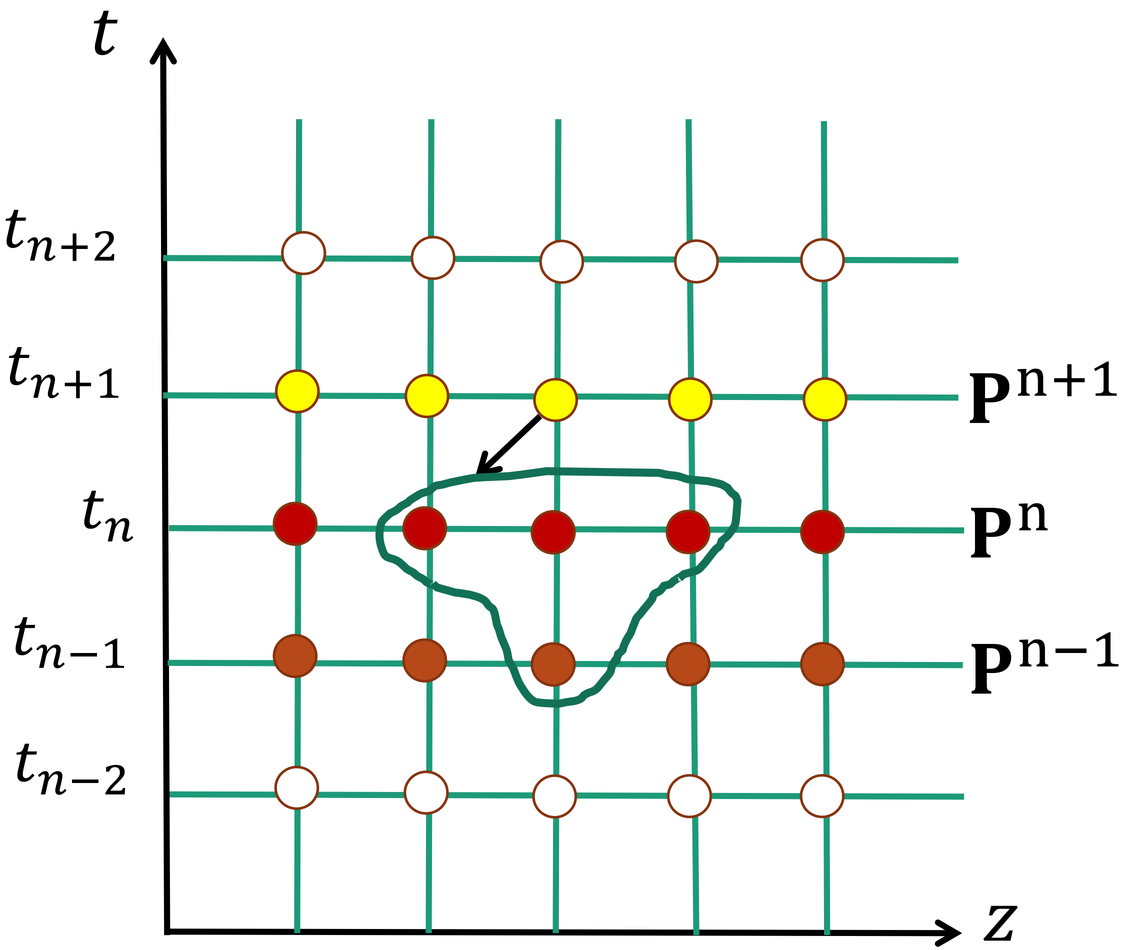
Scattering amplitude

$$\text{Scattered wave} \sim F(\theta, \varphi)\frac{e^{-ikr}}{r}$$

$$Q_P^{-1} \sim \langle F^{PP}|R|F^{PP}\rangle + \langle F^{PS}|R|F^{PS}\rangle$$

$$Q_S^{-1} \sim \langle F^{SP}|R|F^{SP}\rangle + \langle F^{SS}|R|F^{SS}\rangle$$

Wave modeling in time domain



1D wave equation

$$\frac{\partial^2 P}{\partial t^2} = V^2(z)\frac{\partial^2 P}{\partial z^2}$$

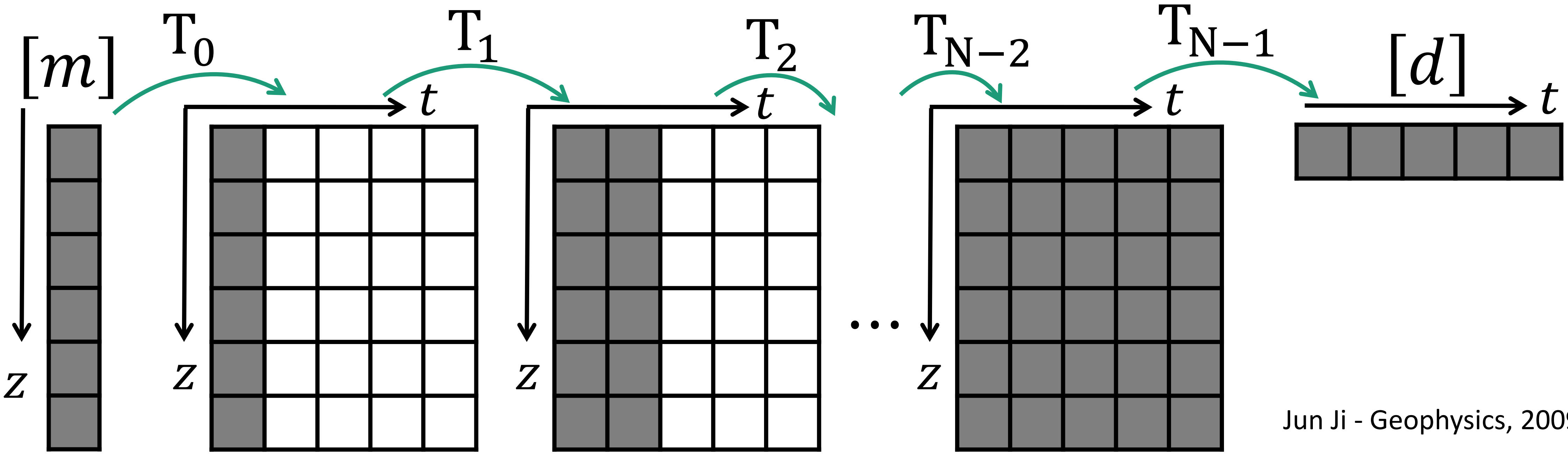
forward-time extrapolation equation

$$[\mathbf{P}^{n+1}]=[-\mathbf{I} \quad \mathbf{T}]\begin{bmatrix}\mathbf{P}^{n-1}\\ \mathbf{P}^{n}\end{bmatrix}$$

Jun Ji - Geophysics, 2009

Future work

Wave modeling in time domain



Jun Ji - Geophysics, 2009

Forward modeling $[d] = T_{N-1} \times T_{N-2} \ \cdots \ T_1 \times T_0 \times [m]$

Reverse time migration $[\tilde{m}] = T_0^* \times T_1^* \ \cdots \ T_{N-2}^* \times T_{N-1}^* \times [d]$

Summary

- Quantum computing offers the exponential speedup for seismic wave modeling (frequency domain)
- The output of the quantum computer as a superposition of many states is not fully accessible. We can take advantages of the solution if we are interested in some statistical features of the solution.

Future work

- Design an algorithm for time domain finite difference modeling and RTM
- Applications of quantum machine learning in seismic problems

Acknowledgement

- CREWES sponsors and NSERC
- CREWES staff
- Dr. Sam Gray

Thank you

When will we see working quantum computers solving real-world problems?

Timeline of digital technology

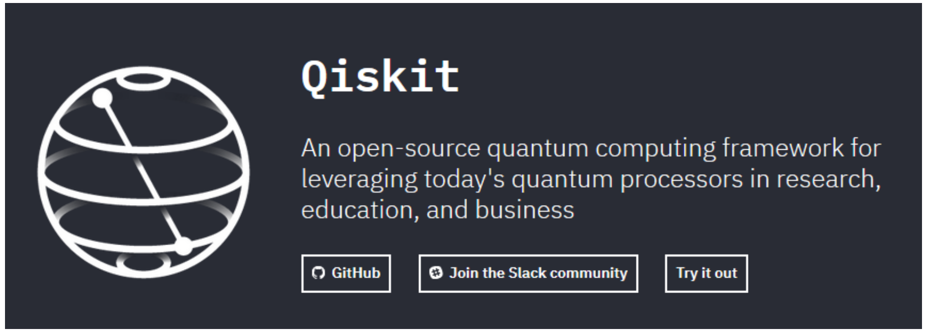
1947 The first transistor was introduced in.
1971 Intel developed the first microprocessor with 2300 transistors
1980s Home computers became common

Timeline of quantum computing

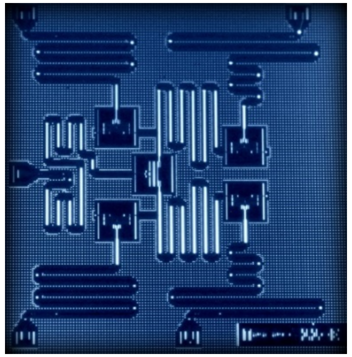
1998 First experimental demonstration of a quantum algorithm.
2-qubit NMR quantum computer, Oxford University
2018 Google announces the creation of a 72-qubit quantum chip
2020 Rigetti Computing plans to build a 128-qubit quantum computer

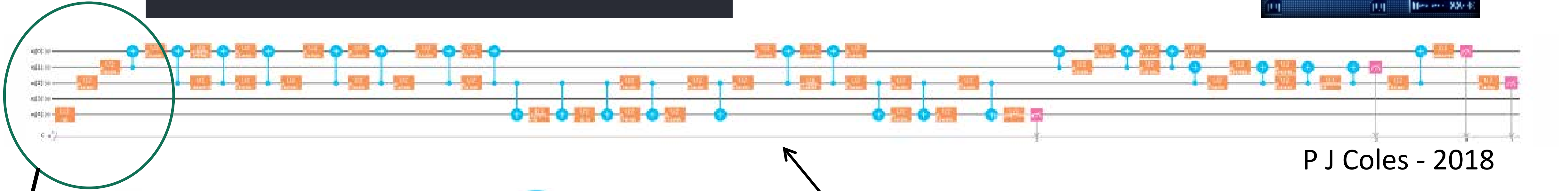
Quantum computation

Test your algorithms!

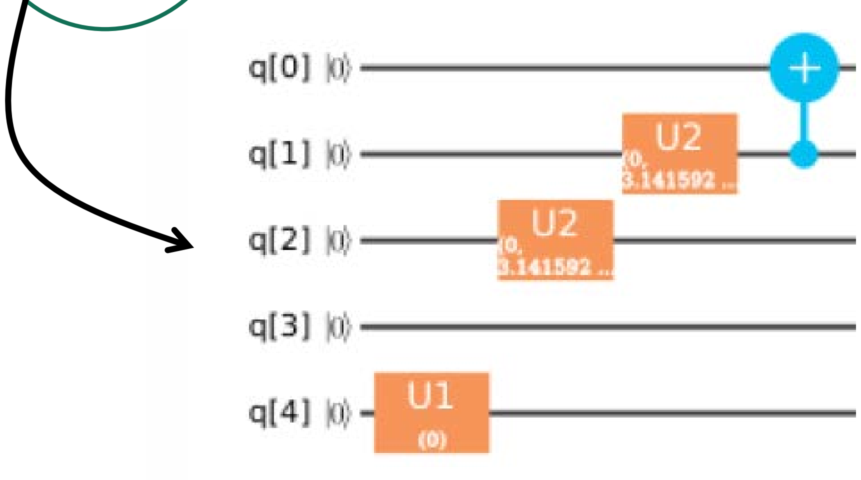


IBM Q experience
(5 qubits)





P J Coles - 2018



Quantum algorithm for linear system of equations

$$A\mathbf{x} = \mathbf{b}$$

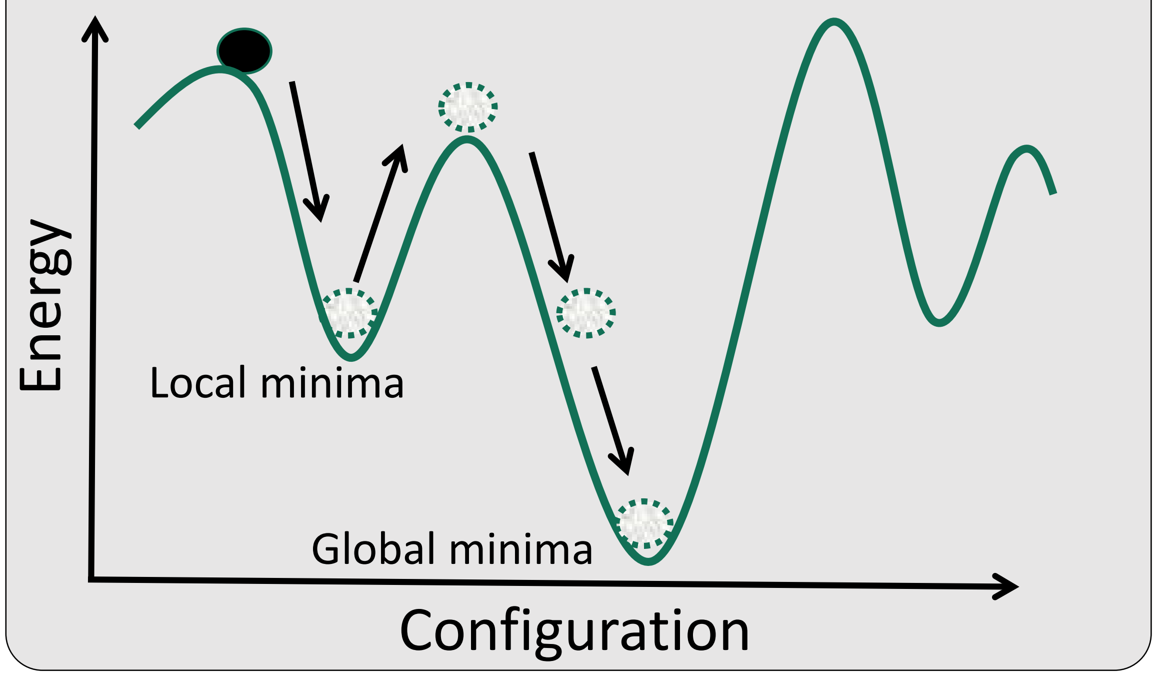
$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

D-wave Quantum annealer

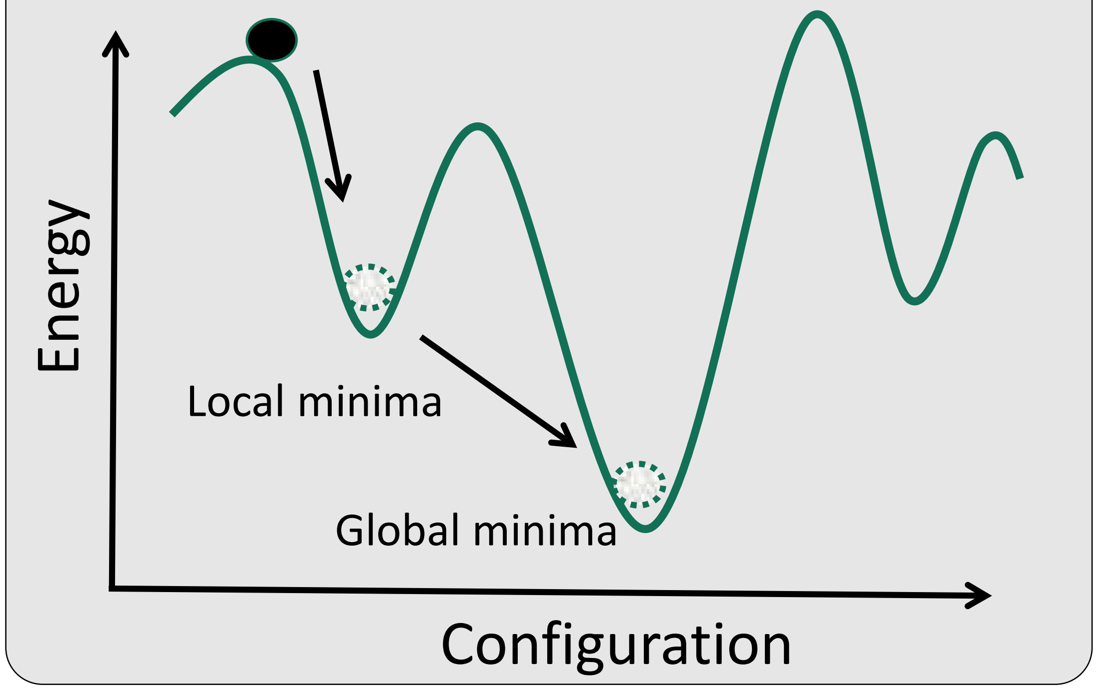
How effectively quantum tunneling processes possibly lead to the global minimum?

Pass through a barrier by quantum annealing rather than having to jump over it by thermal excitations.

Classical annealing can walk
Over the land scape only
(must climb over energy barriers)



Quantum annealing can
tunnel through the landscape
(penetrate the barriers)



Hamiltonian simulation

- Process of matrix inversion through the time evolution operator
- Hamiltonian is energy operator corresponds to the matrix A

$$x = A^{-1} b$$

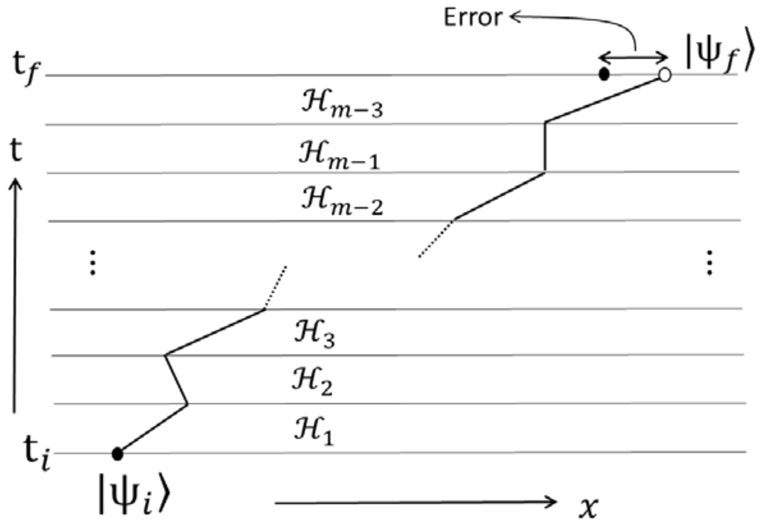
$$|\psi_f\rangle = e^{-i\mathcal{H}(t_f - t_i)} |\psi_i\rangle$$

Final state of quantum computer at time t_f

Time evolution operator

Initial state of quantum computer at time t_i

Hamiltonian $\mathcal{H} = \sum_{k=1}^{m} \mathcal{H}_k$



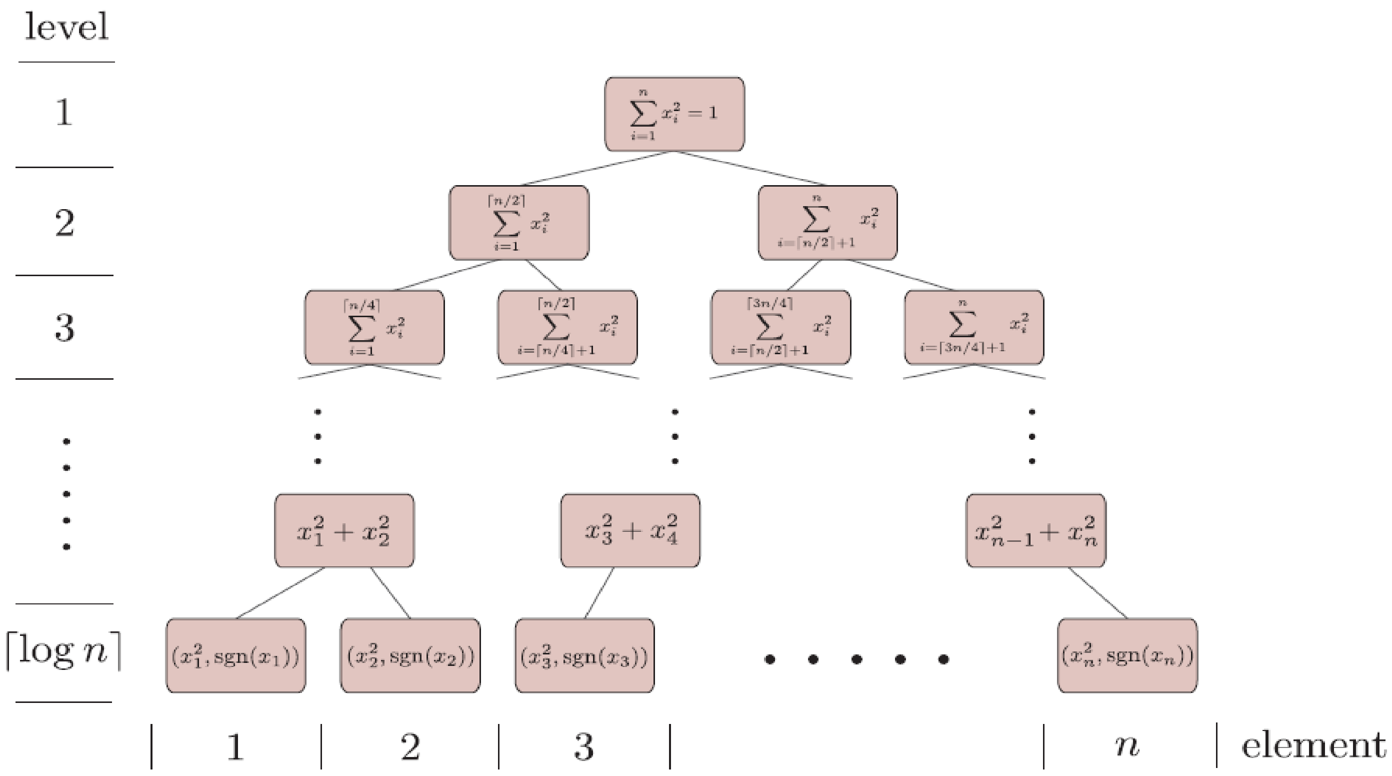


Figure 4: An illustration of a classical data structure $B_{\mathbf{x}}$ that, when equipped with quantum access, constitutes a qRAM, storing a vector $\mathbf{x} \in \mathbb{R}^n$, $\|\mathbf{x}\|_2 = 1$. The vector $\mathbf{x}$ is stored in the binary tree shown. To each element of $\mathbf{x}$, x_i, there is a leaf of the tree. Each leaf contains the squared amplitude of the element x_i and its sign $\mathrm{sgn}(x_i)$. Every other node contains the sum of its child nodes (ignoring the $\mathrm{sgn}(x_i)$ terms for the $(\lceil \log n \rceil - 1)^{\mathrm{th}}$ level). To load the vector, we move through the tree from the root node, appending relevant qubits to the computational register where necessary and rotating conditioned on the values stored in the corresponding nodes. This procedure is detailed in Algorithm 1.