Review of tomographic methods

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Outline

- Classical reflection tomography
- Prestack depth migration (PSDM) tomography
- Stereotomography
- Compare synthetic test results from classical stereotomography and adjoint stereotomography
- Conclusions

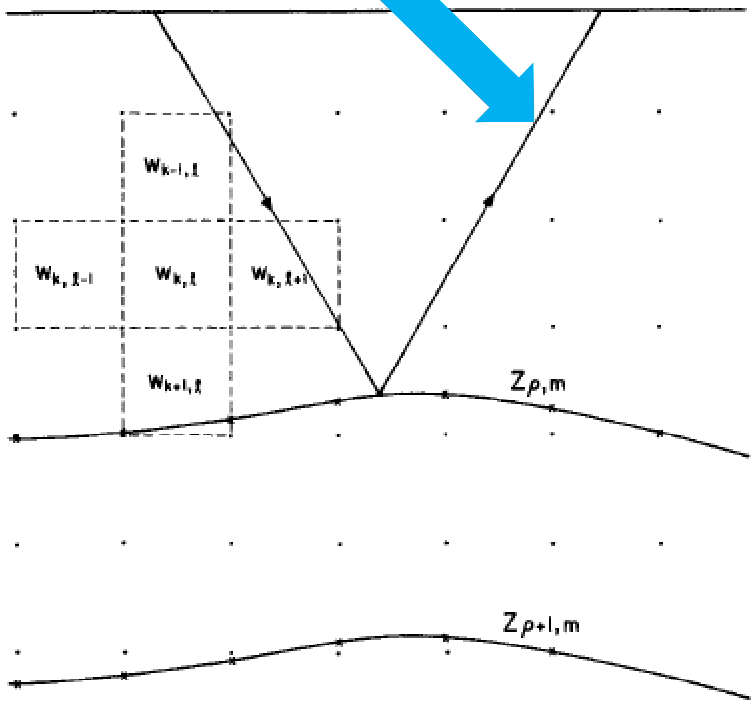
Classical Reflection Tomography

$$t_{raypath} = \int_{raypath} s(x,z)\,dl$$

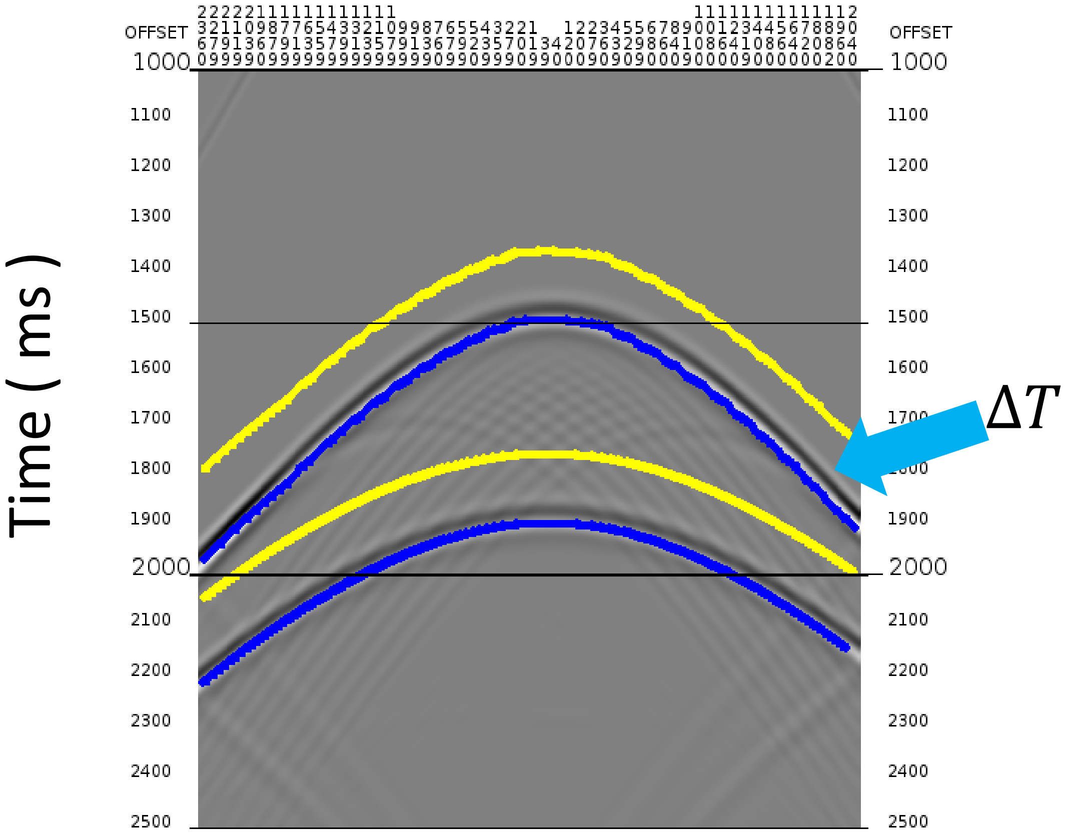
$$T = L\,S$$

$$\Delta T = L\,\Delta S$$

$$L = \begin{bmatrix} L_{1,1} & L_{1,2} & L_{1,3} & L_{1,4} & .. & L_{1,m} \\ L_{2,1} & L_{2,2} & L_{2,3} & L_{2,4} & .. & L_{2,m} \\ L_{3,1} & L_{3,2} & L_{3,3} & L_{3,4} & .. & L_{3,m} \\ .. & .. & .. & .. & .. & .. \\ l_{n,1} & l_{n,2} & l_{n,3} & l_{n,4} & .. & l_{n,m} \end{bmatrix} \qquad \Delta T = \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ .. \\ \Delta t_n \end{bmatrix} \qquad \Delta S = \begin{bmatrix} \Delta s_1 \\ \Delta s_2 \\ \Delta s_3 \\ .. \\ \Delta s_m \end{bmatrix}$$



Bishop (1985)



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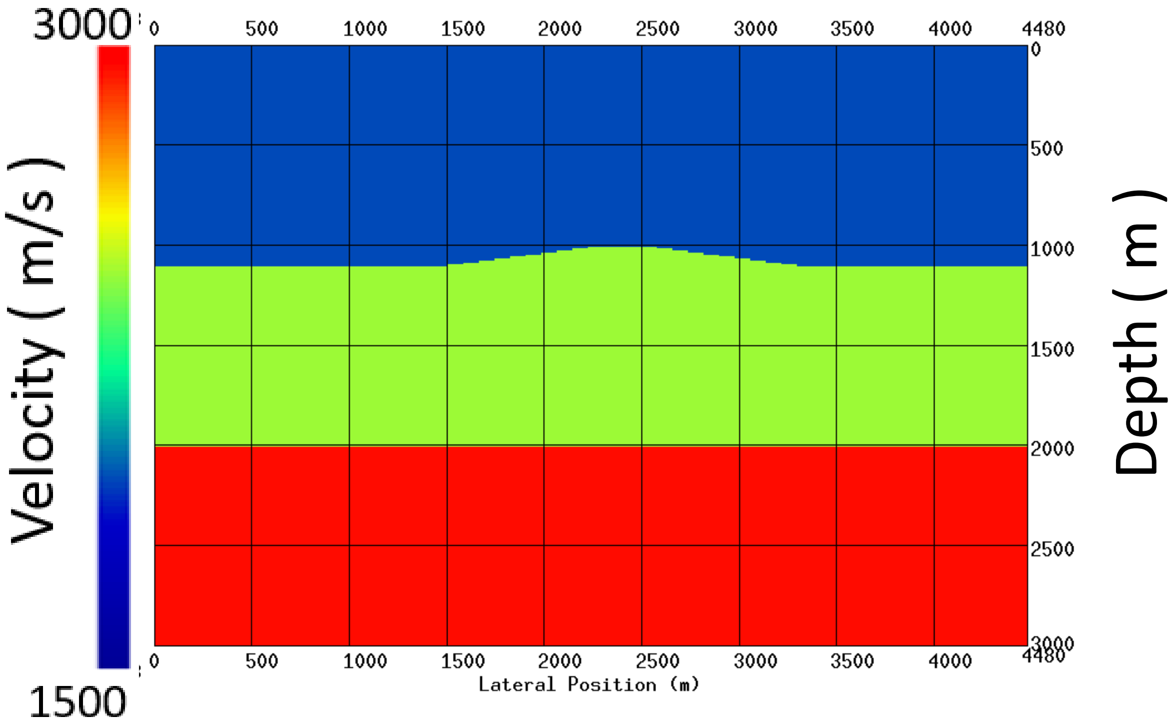
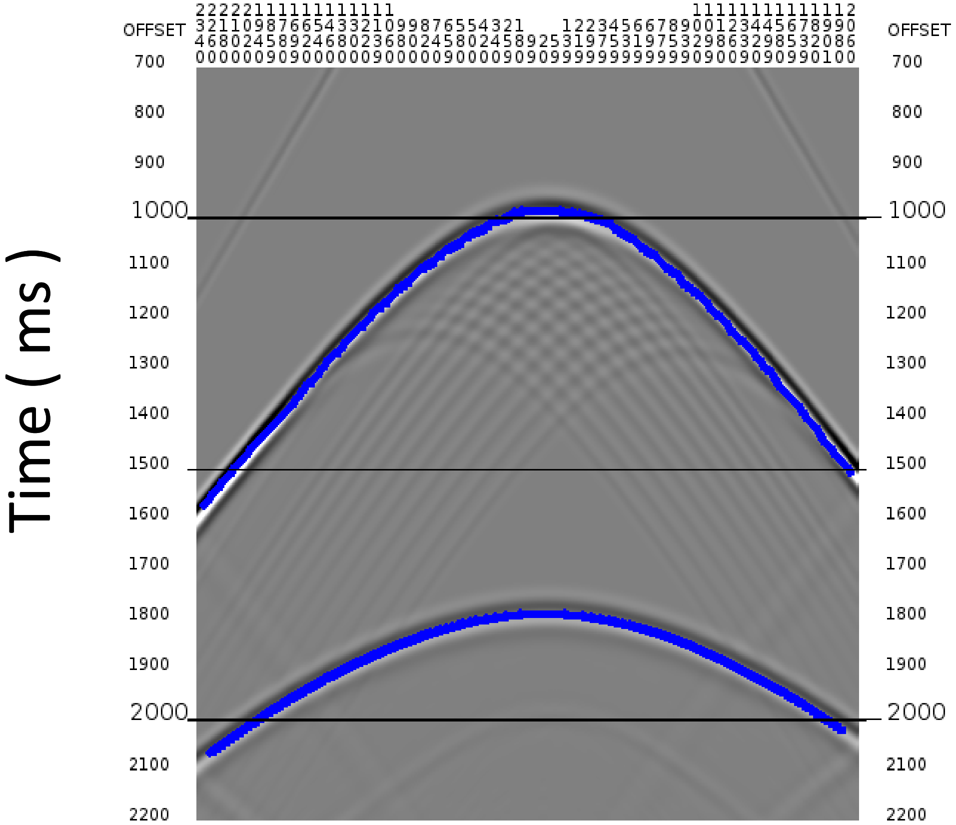


True Model



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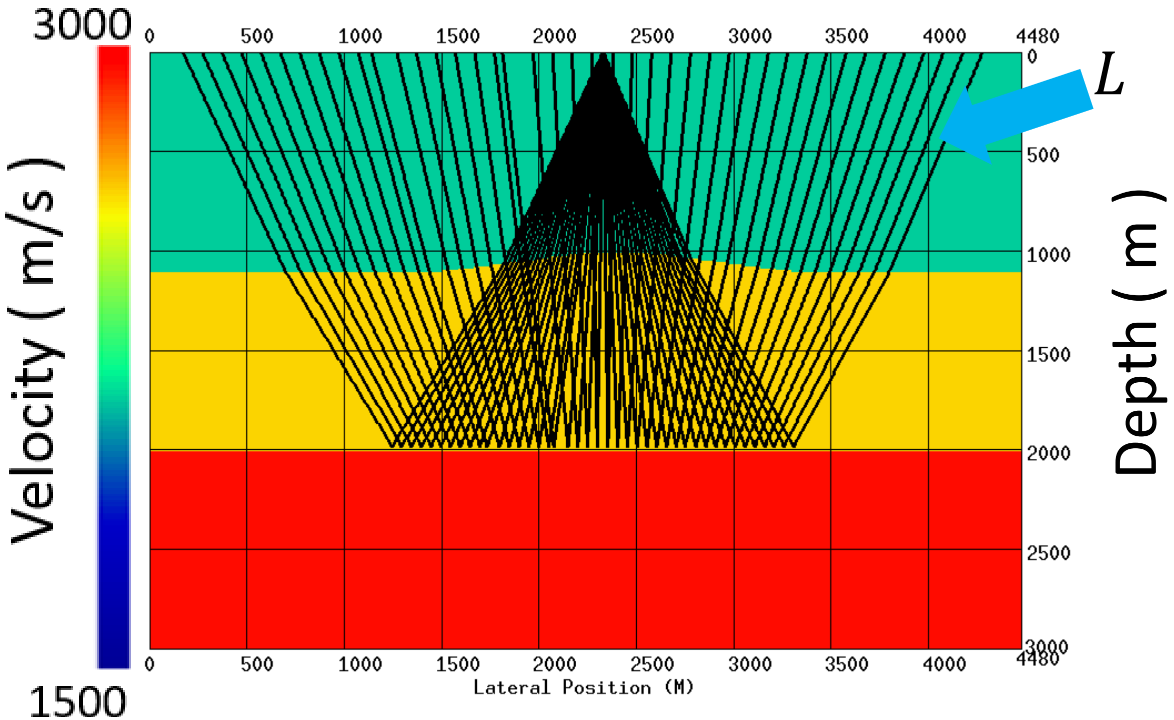
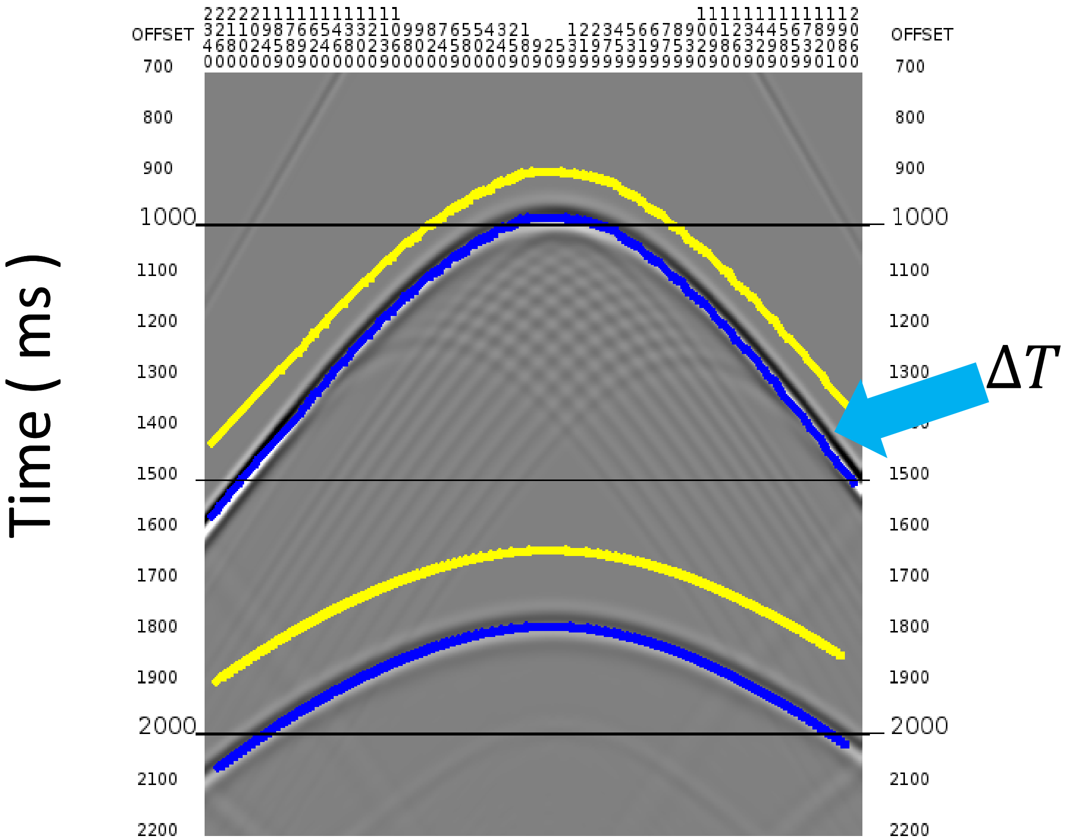


Initial velocity model



Classical Reflection Tomography

$$t_{raypath} = \int_{raypath} s(x,z)dl$$

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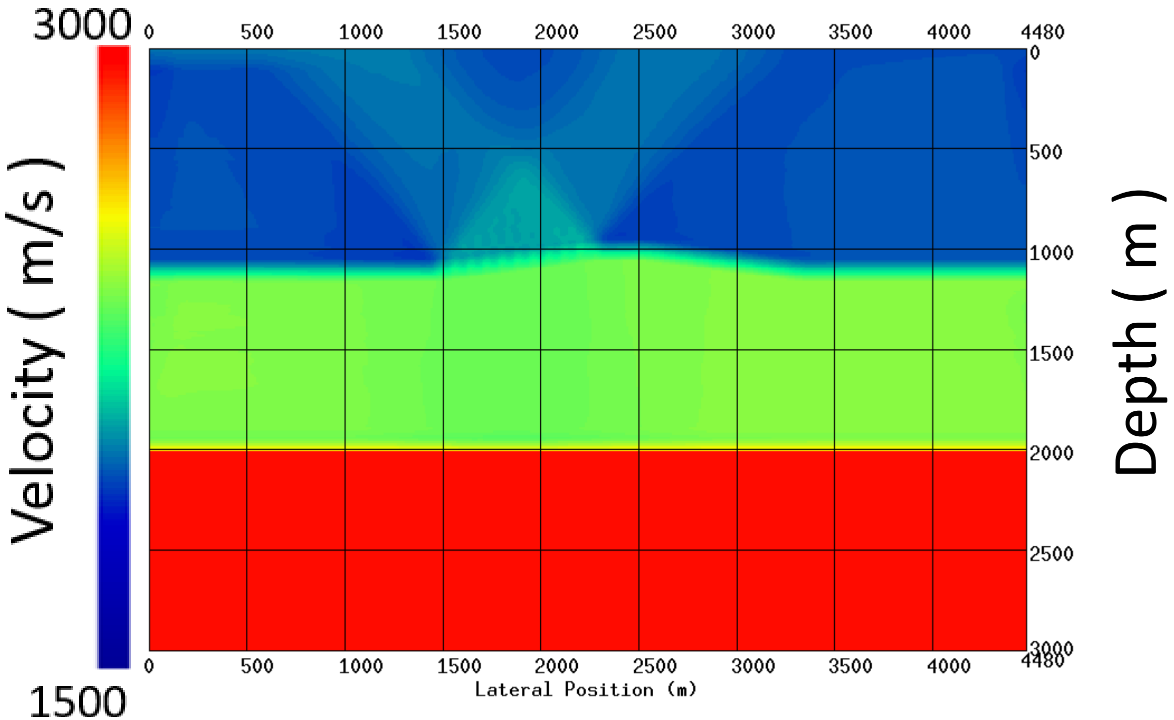

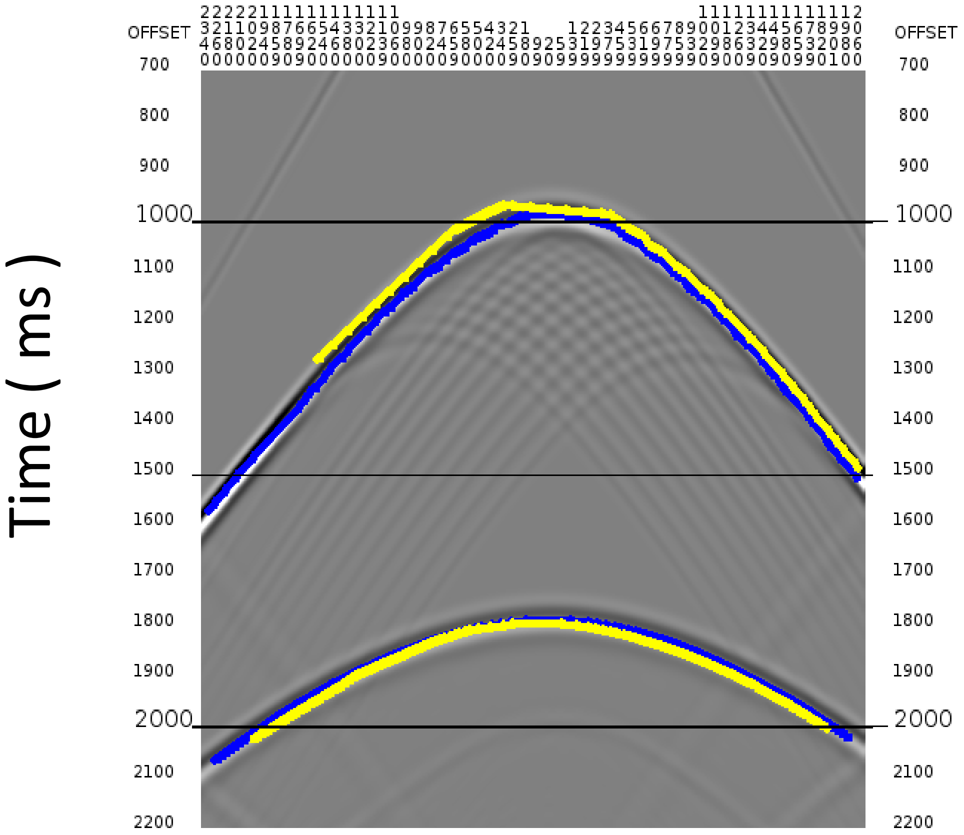

Velocity model after 40 internal iterations

Classical Reflection Tomography

$$t_{raypath} = \int_{raypath} s(x,z)dl$$

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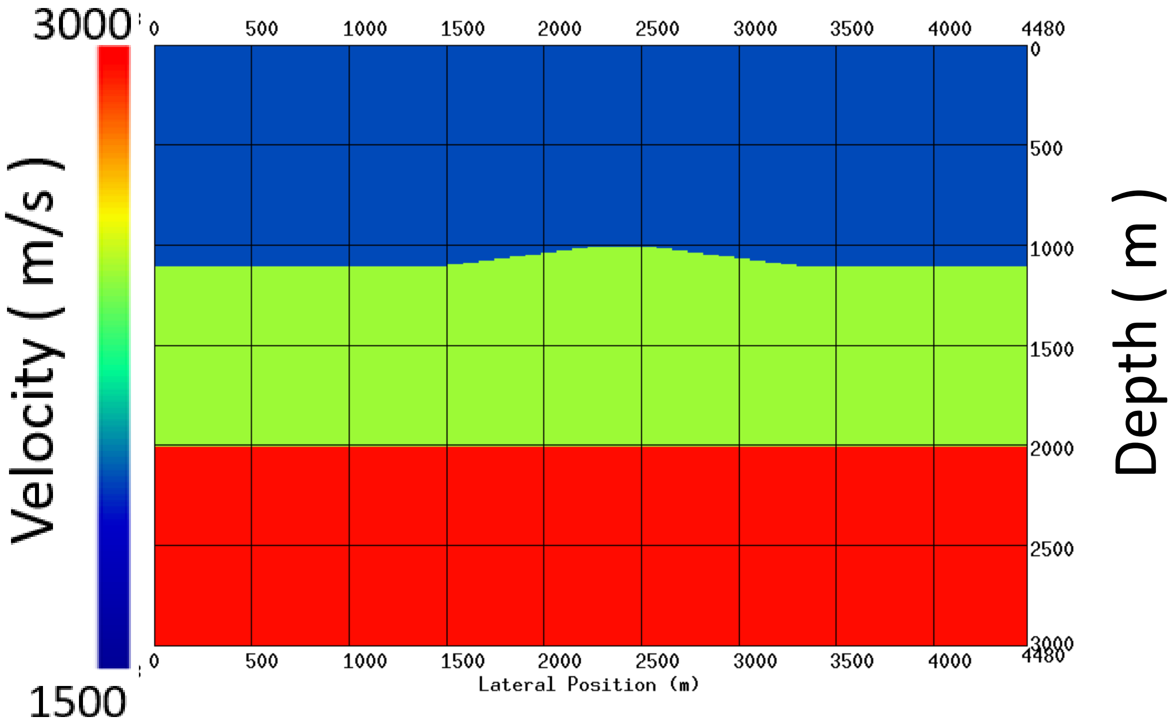
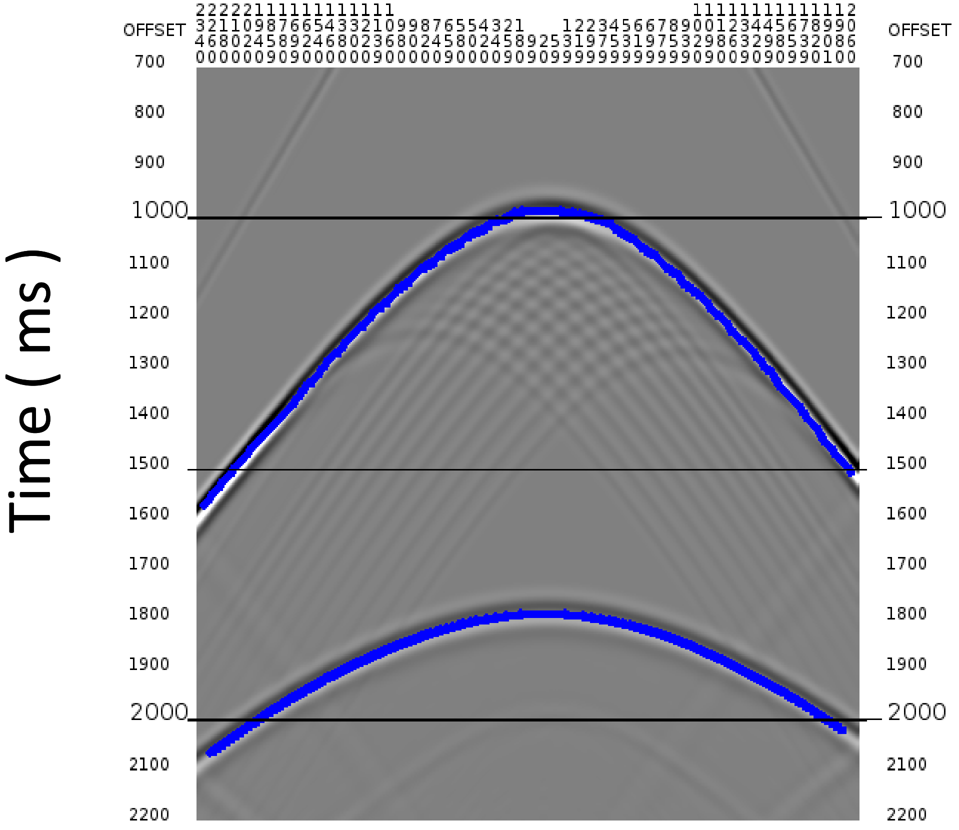
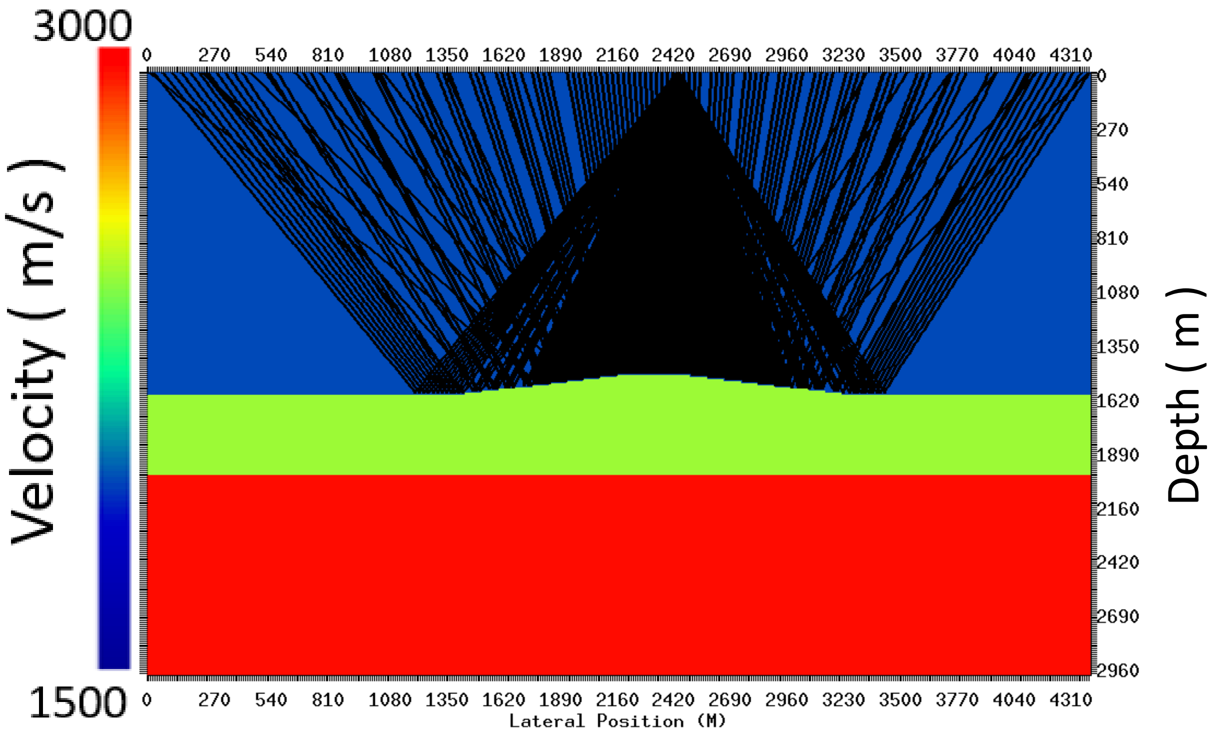


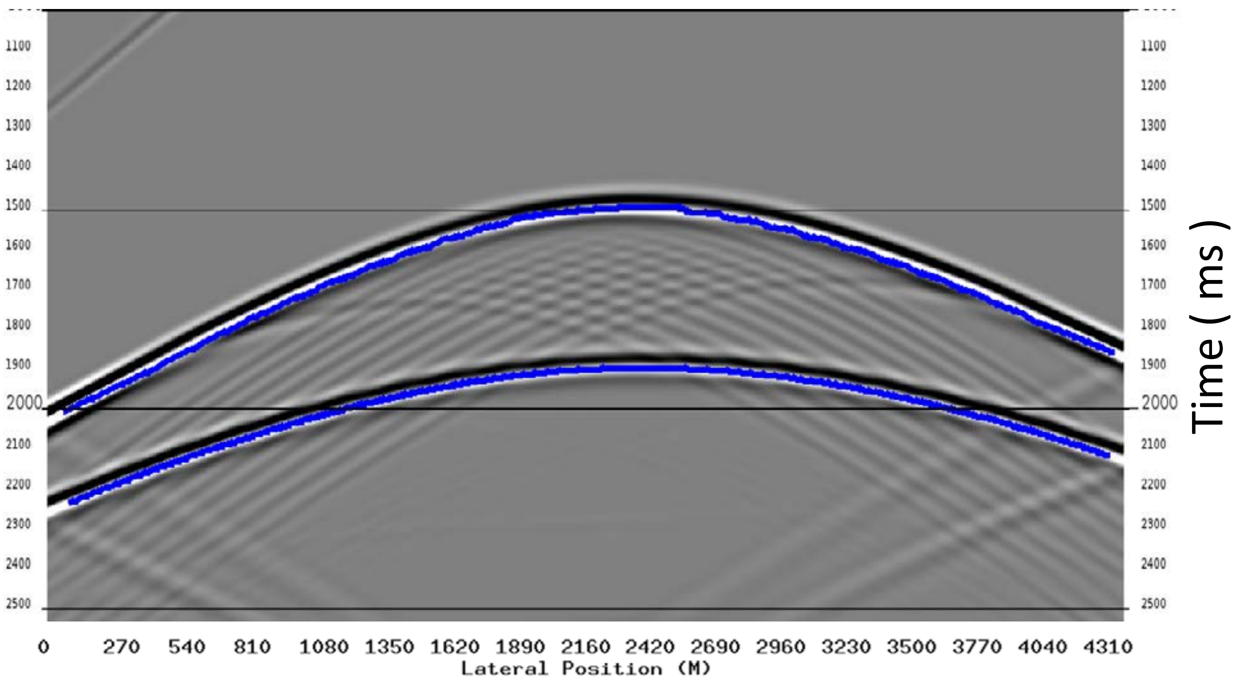
True Model



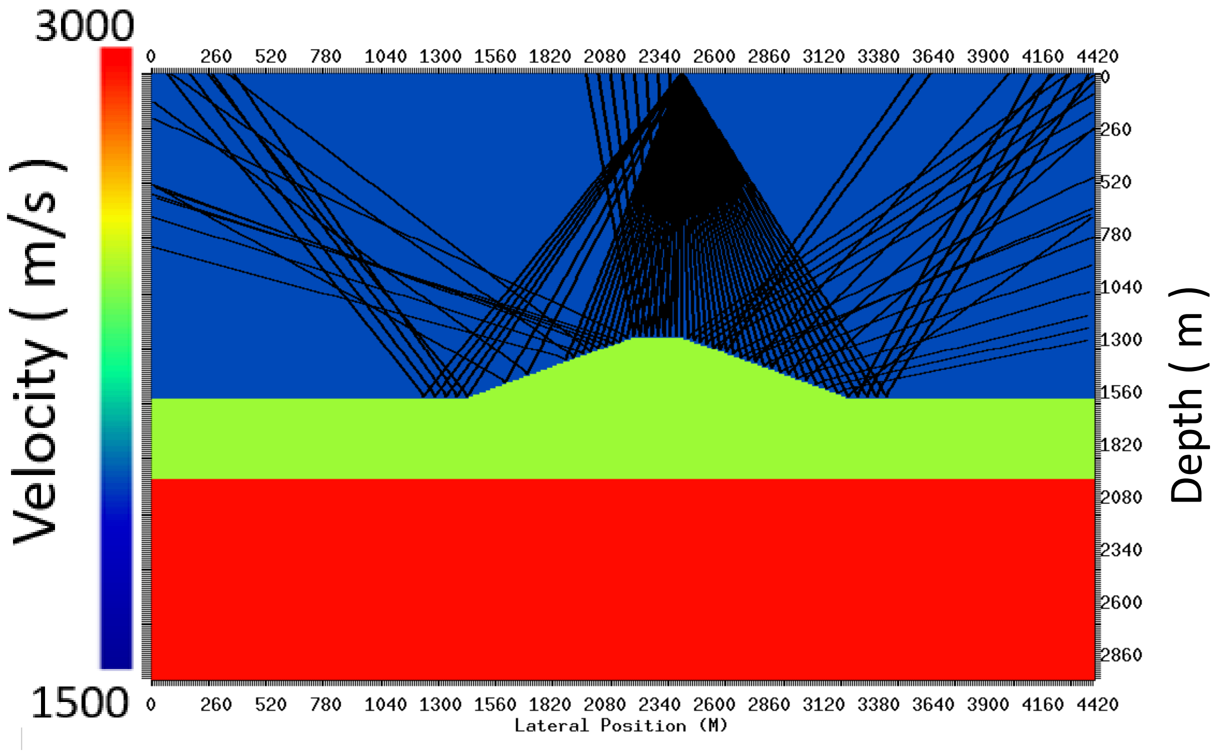
Traveltime picking difficulties



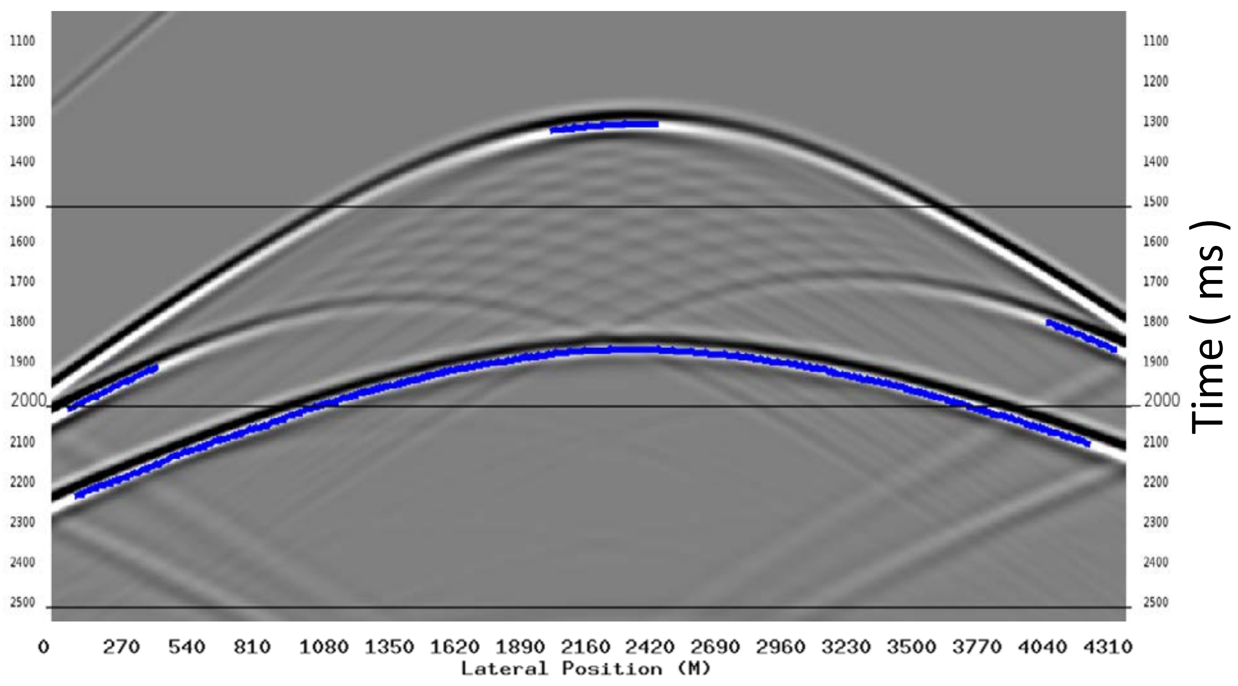
Common Shot Gather



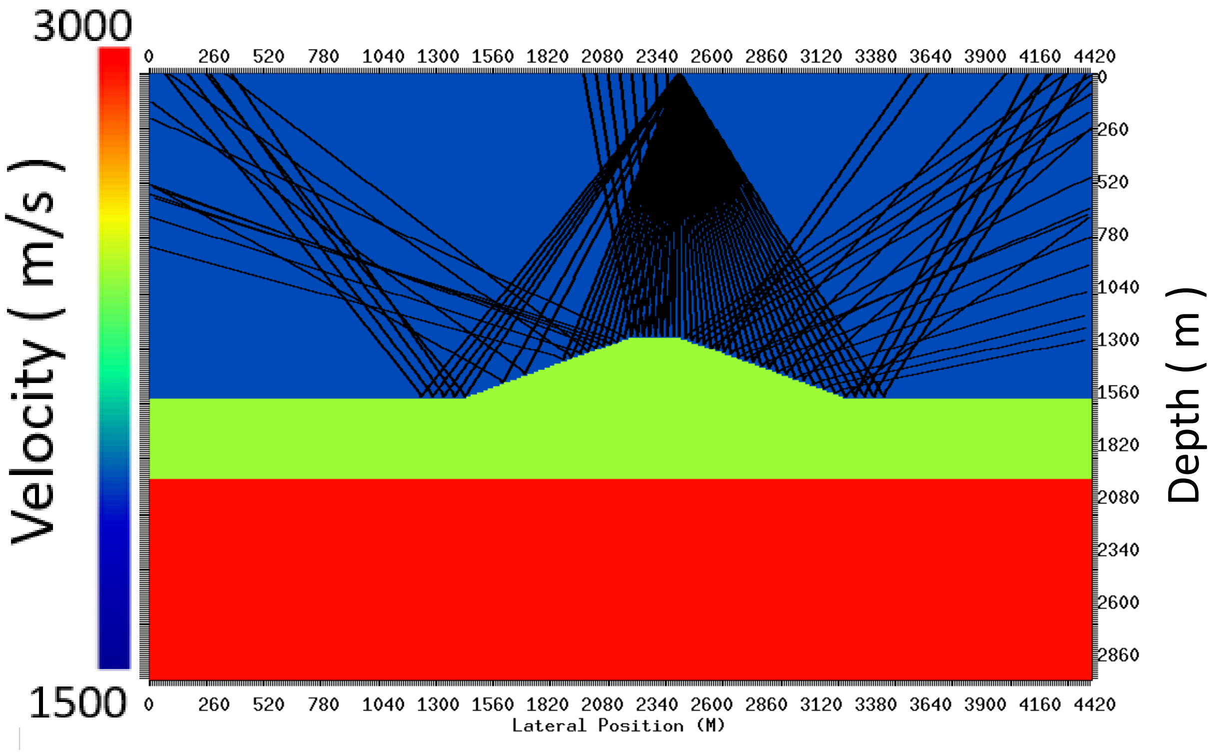
Traveltime picking difficulties



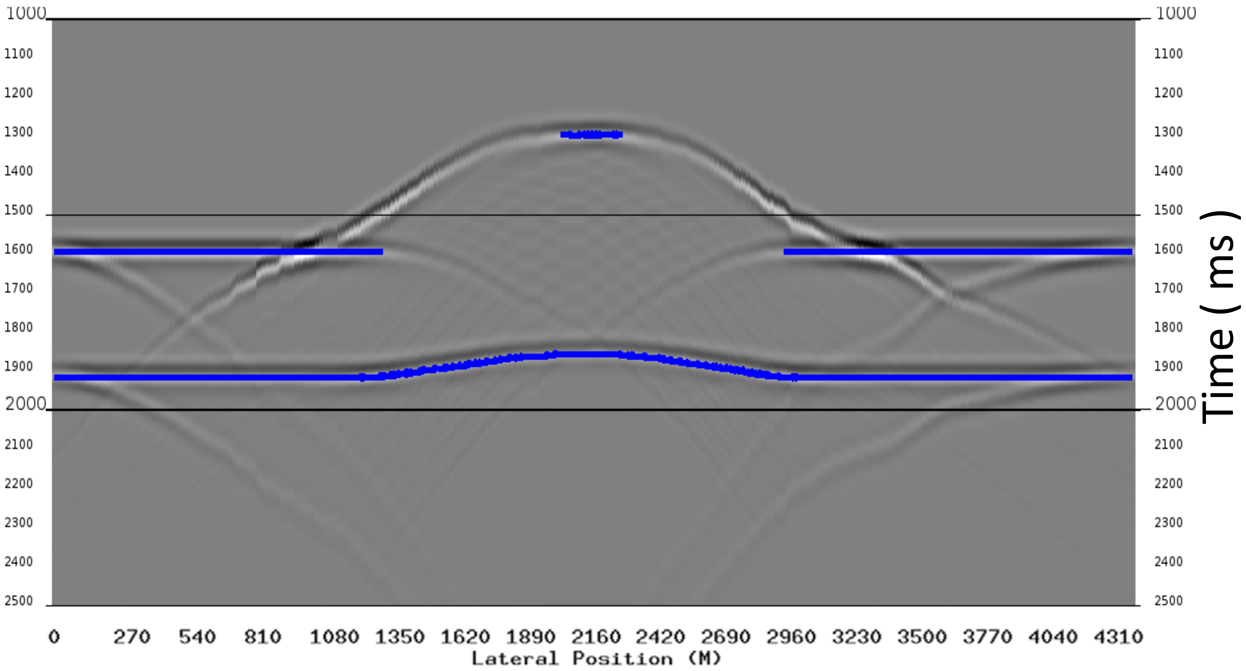
Common Shot Gather



Traveltime picking difficulties

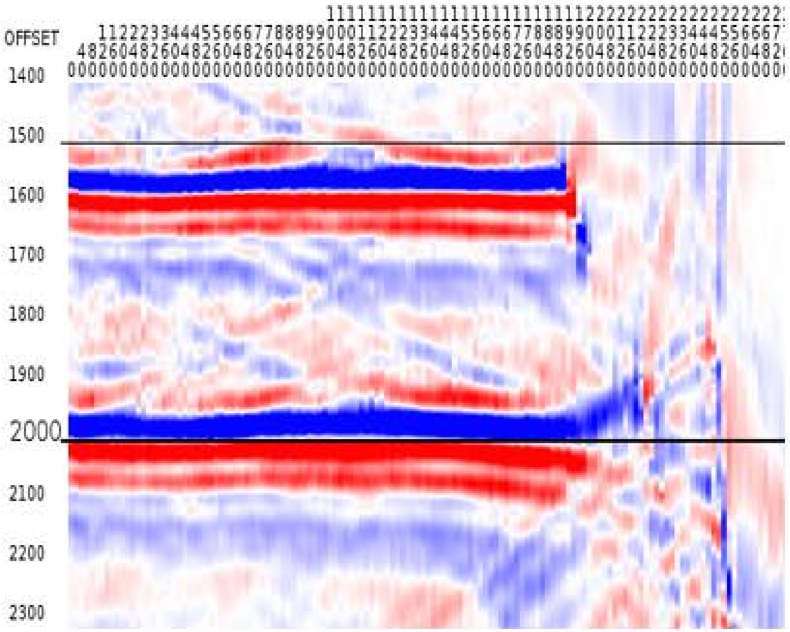


Common Offset Gather

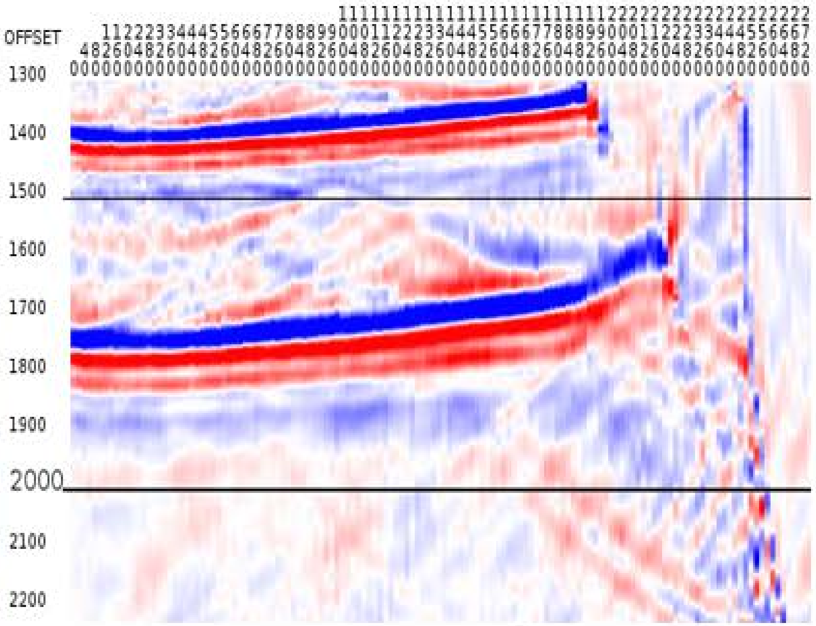


Residual moveout within a Common Image Gather (CIG)

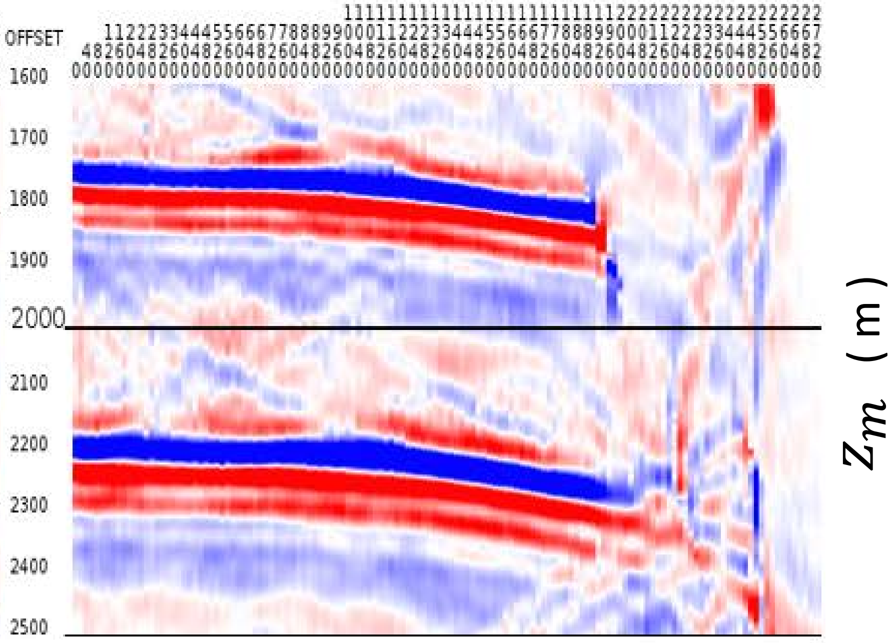
True Velocity



Slow Velocity



Fast Velocity



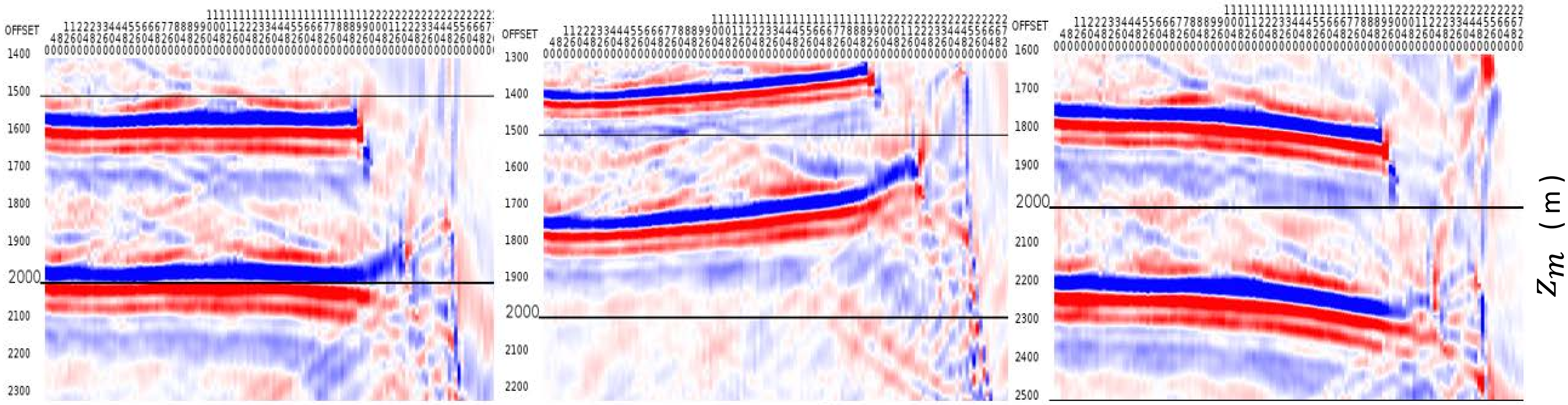
PSDM tomography

Residual moveout is scanned automatically

$$z_m = \sqrt{\gamma^2 z^2 + (\gamma^2 - 1)x^2} \qquad \gamma = \tilde{V}_m / \tilde{V}$$

Al-Yahya (1989)

True Velocity | Slow Velocity | Fast Velocity



$\gamma = 1$ | $\gamma < 1$ | $\gamma > 1$

PSDM tomography

Residual moveout is scanned automatically

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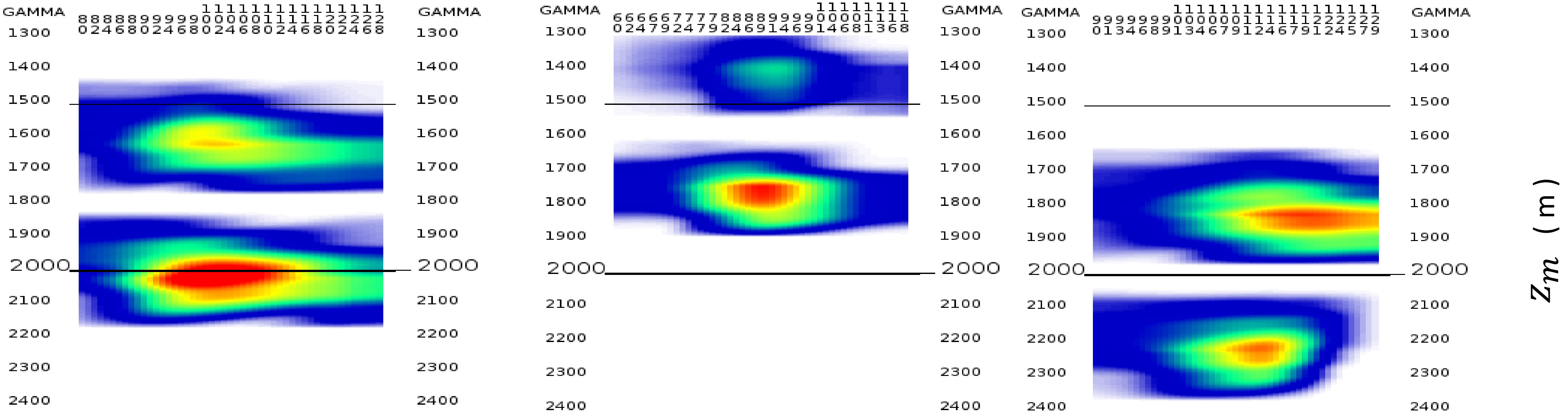
True Velocity | Slow Velocity | Fast Velocity



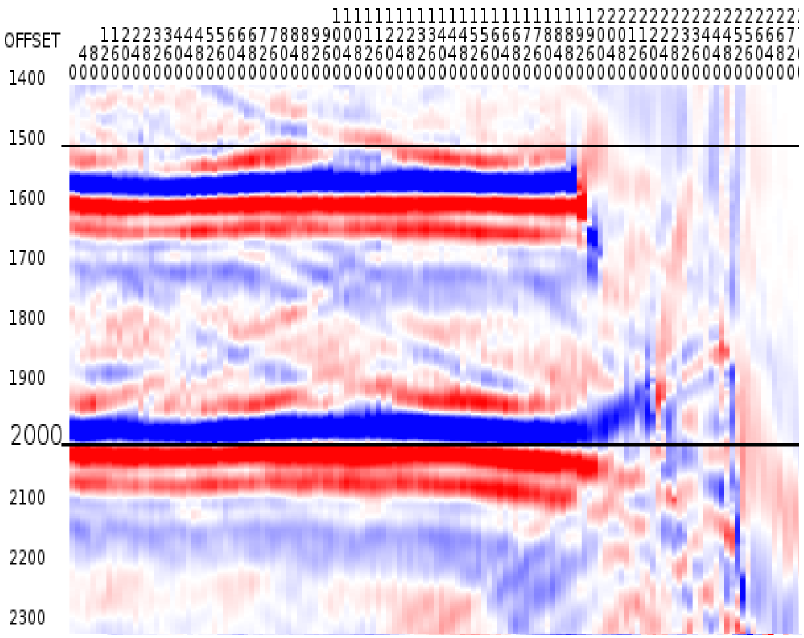
$\gamma = 1$ | $\gamma = 0.89$ | $\gamma = 1.13$

Residual moveout is scanned automatically

$$z_m = \sqrt{\gamma^2 z^2 + (\gamma^2 - 1)x^2} \qquad \gamma = \tilde{V}_m / \tilde{V}$$

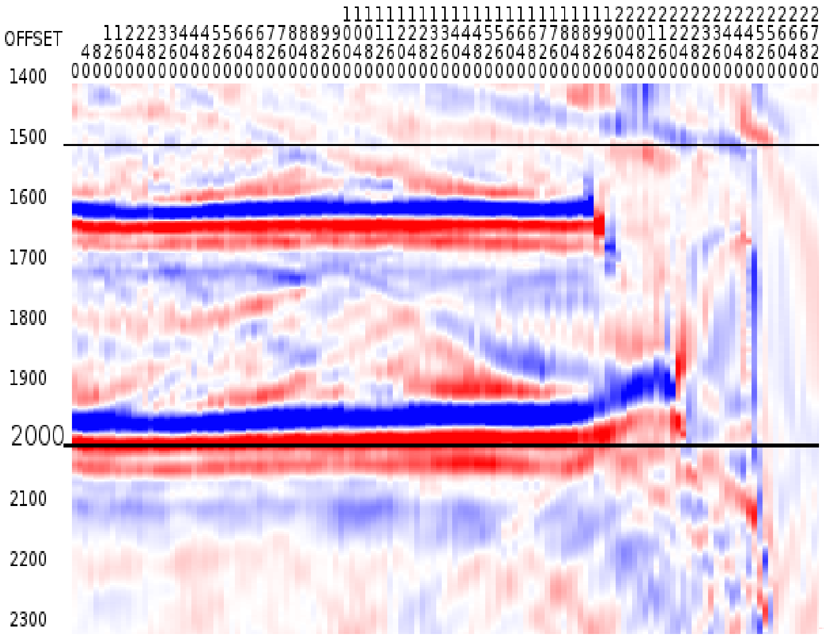
Al-Yahya (1989)

True Velocity



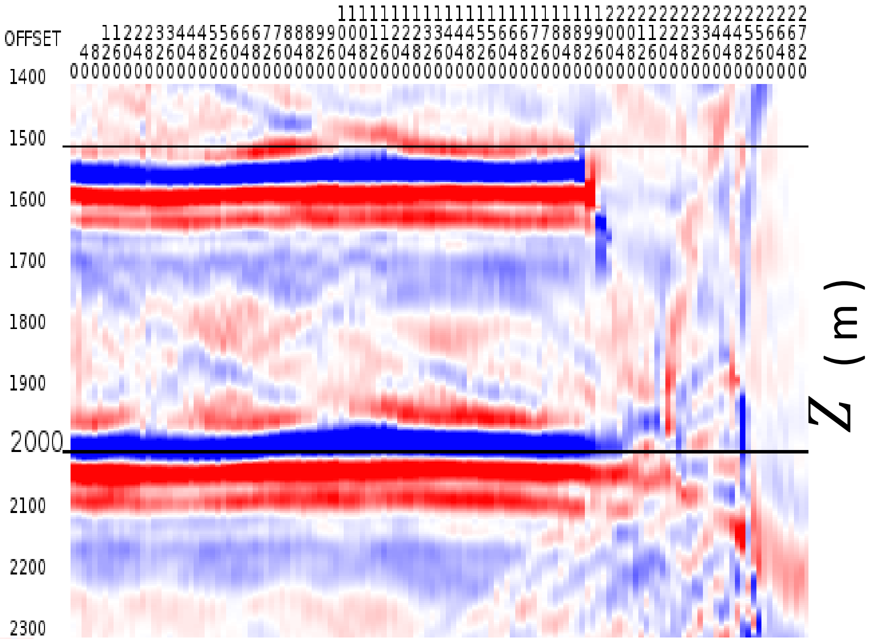
$\gamma = 1$

Slow Velocity



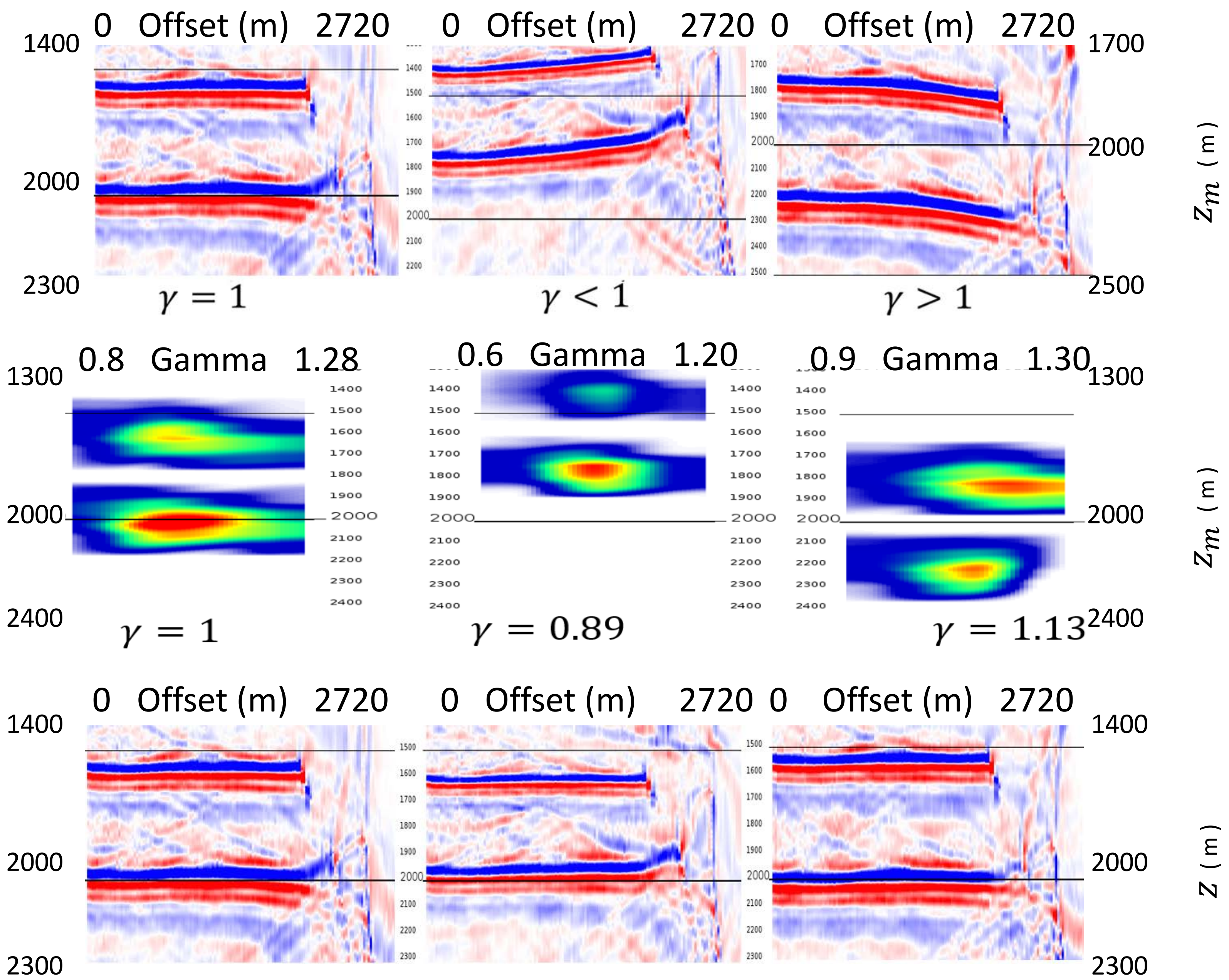
$\gamma = 0.89$

Fast Velocity



$\gamma = 1.13$

PSDM tomography



1. Create migrated common image gathers.

   Pick analysis windows and locations

   Adjust residual moveout to reflect velocity update of upper layers.

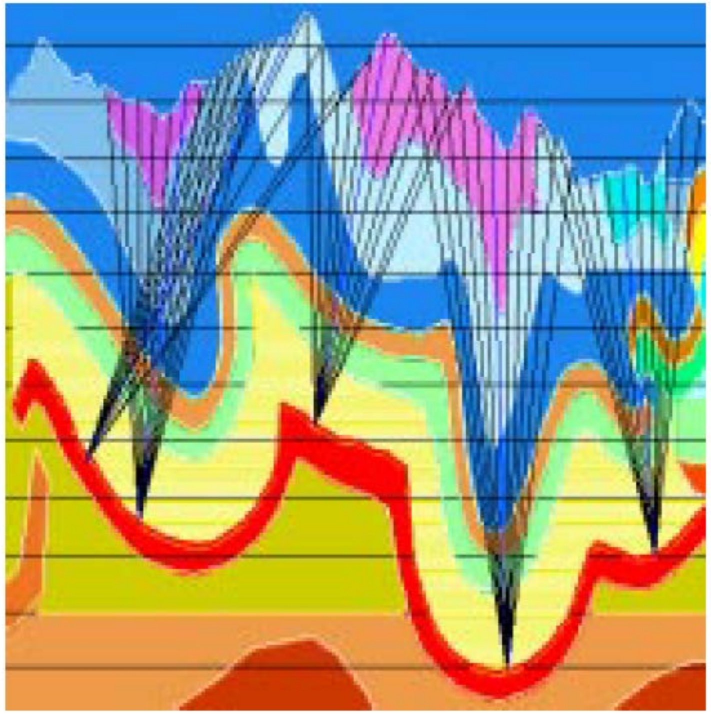
2. Create and pick gamma scans

3. Apply gamma picks to CIG gathers

PSDM tomography

Update velocity using γ

$$\gamma = \tilde{V}_m / \tilde{V}$$



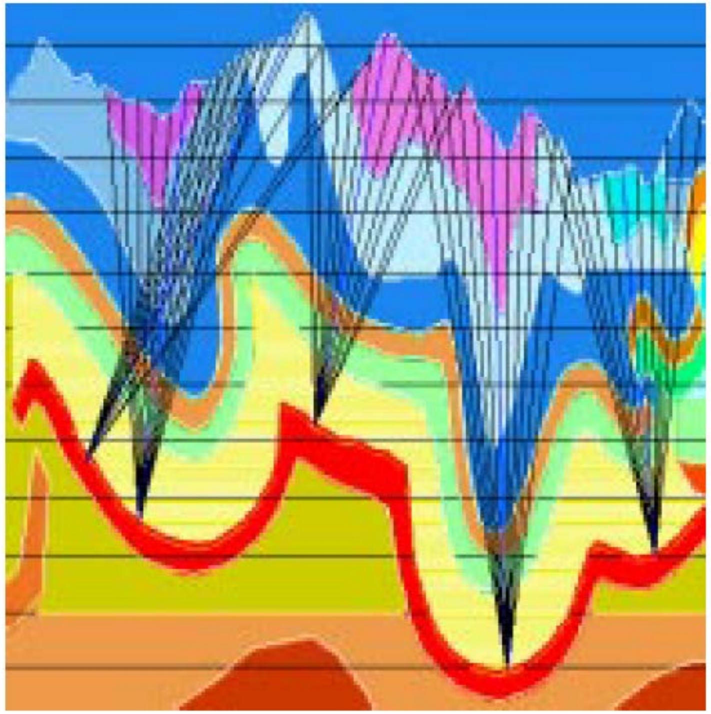
Gray (2000)

PSDM tomography

Update velocity using γ or Δz

$$\gamma = \tilde{V}_m / \tilde{V}$$

$$z_m = \sqrt{\gamma^2 z^2 + (\gamma^2 - 1)x^2}$$



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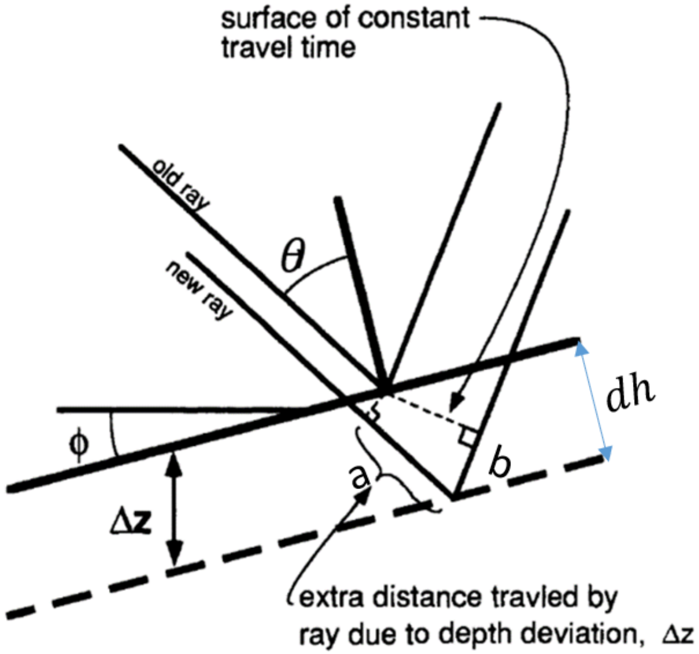

Stork (1991,1992)

$$\Delta t = (a + b) \cdot s$$

$$\Delta t = 2 \cdot s \cdot dh \cdot \cos(\theta)$$

$$\Delta t = 2 \cdot s \cdot \Delta z \cdot cos(\phi) \cdot cos(\theta)$$

$$L\,\Delta s = \Delta t$$

$$\boxed{L\,\Delta s = 2 \cdot s \cdot \Delta z \cdot \cos(\phi) \cdot \cos(\theta)}$$

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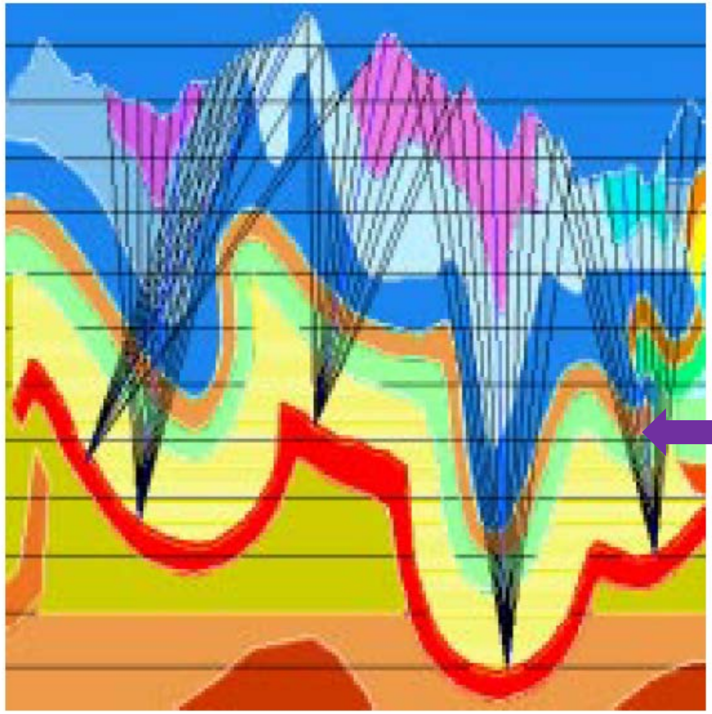

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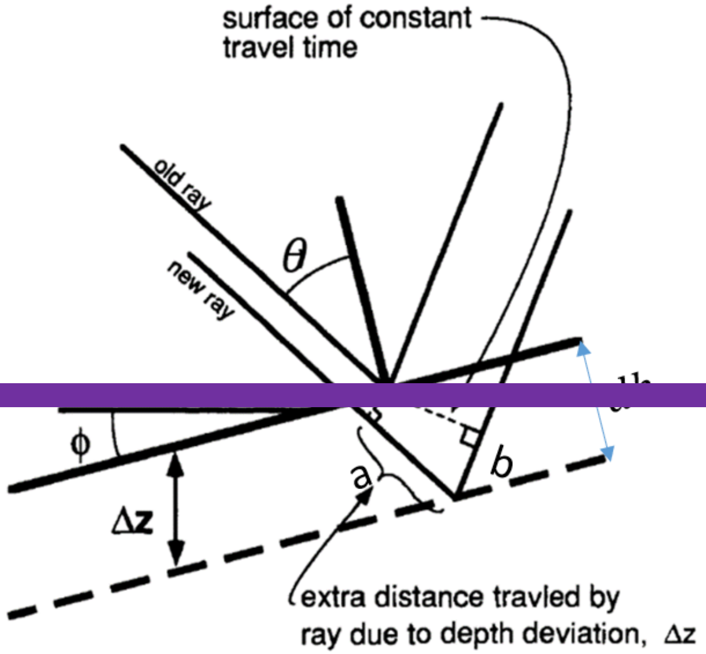

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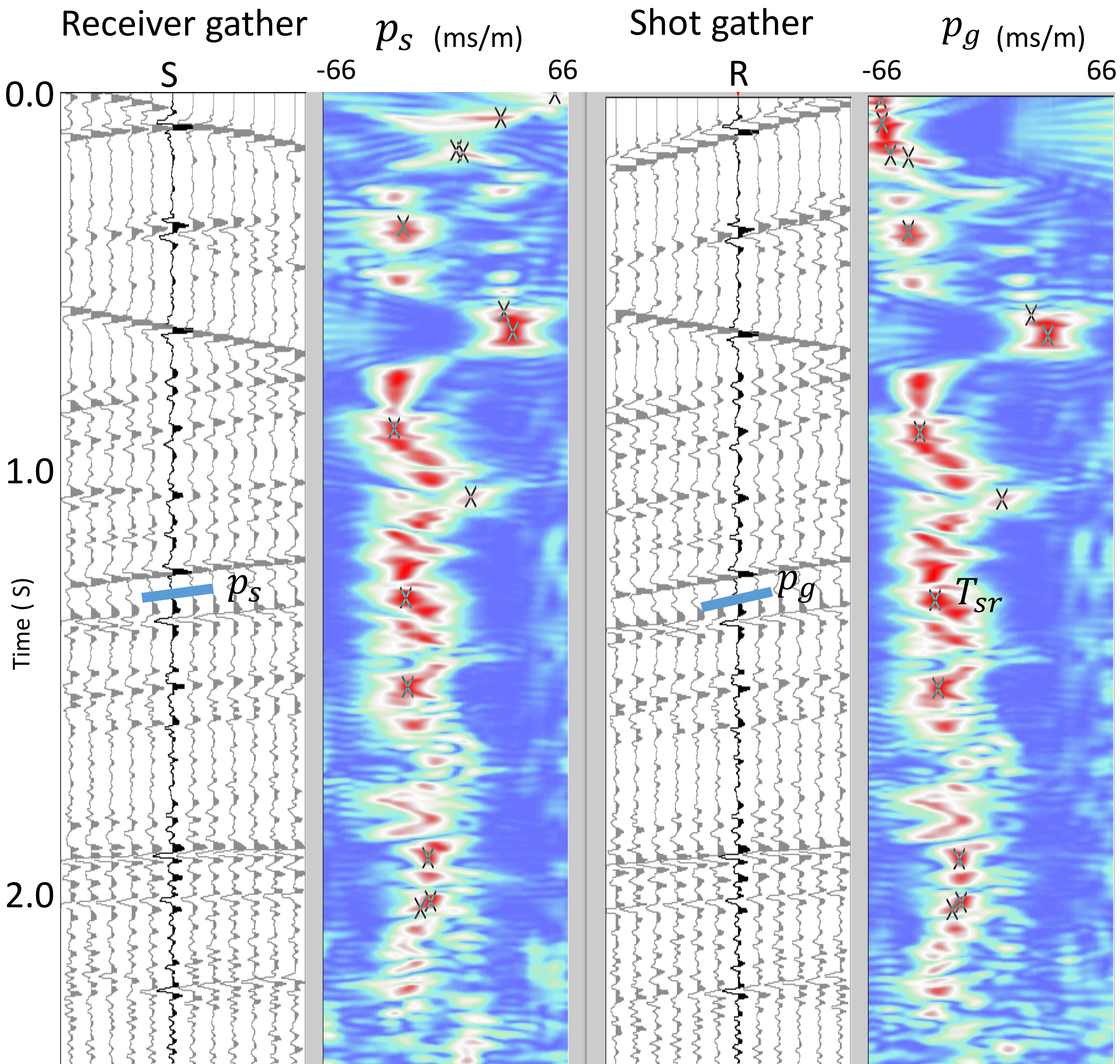
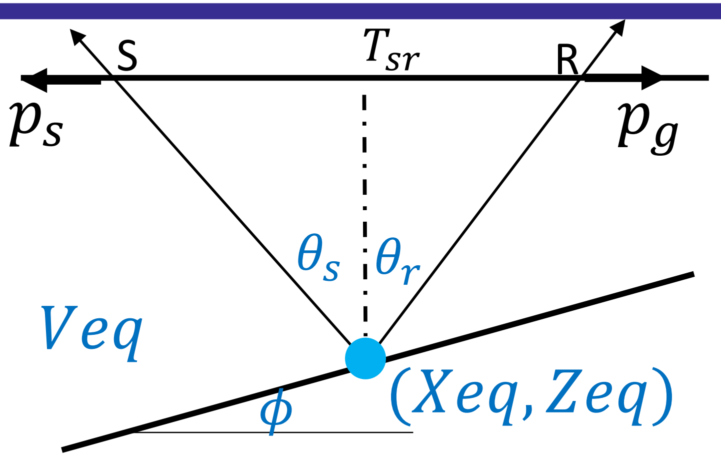
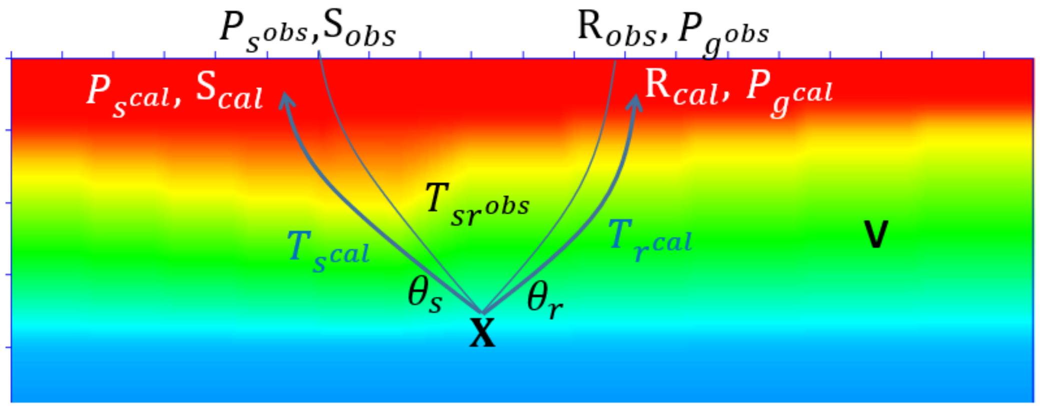
Stereotomography





Sword 1987
Billette et al. 1998,2003

Stereotomography and Adjoint stereotomography



Model space: $\mathbf{m} = [(X, \Theta_s, \Theta_r, T_s, T_r)_{i1=1,N}], [V]_{i2=1,M}]$

Data space : $\mathbf{d} = \left[S, R, P_s, P_g, T_{sr} \right]_{j=1,N}$

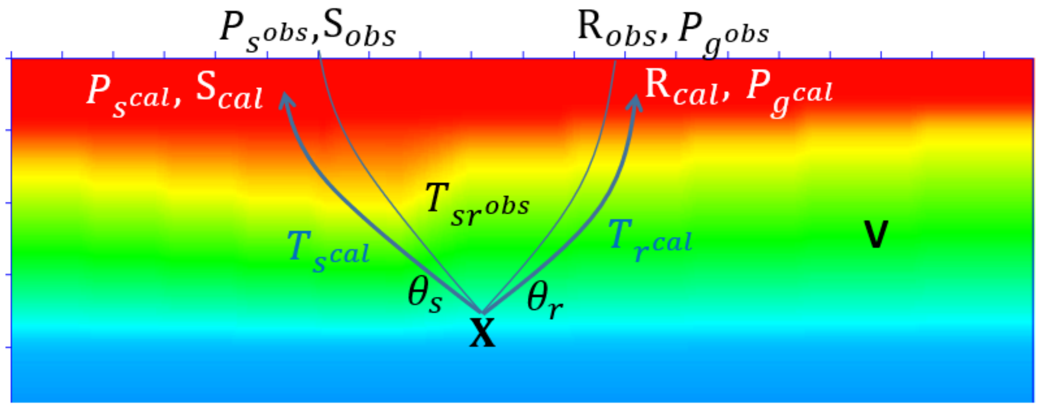
Fréchet derivative: $A_{ij} = \frac{\partial(S,R,P_s,P_r,T_{sr})}{\partial(X,\Theta_s,\Theta_r,T_s,T_r,V)}$

Inversion: $\mathbf{A}\,\mathbf{\Delta m} = \mathbf{\Delta d}$

A[6N,6N+M]

Stereotomography Billette and Lambaré 1998

Stereotomography and Adjoint stereotomography



Stereotomography Billette and Lambaré 1998

Model space: $\mathbf{m} = [(X, \cancel{\Theta_s, \Theta_r, T_s, T_r})_{i1=1,N}], [V]_{i2=1,M}]$

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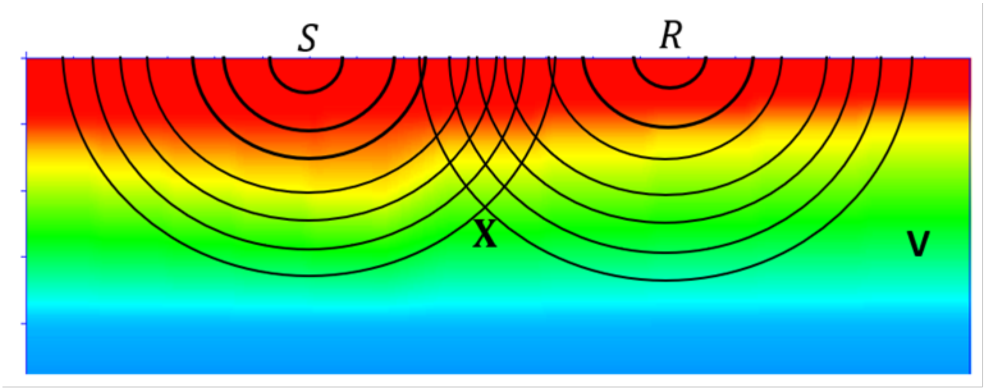
A[6N,6N+M]



Adjoint stereotomography Tavakoli 2017

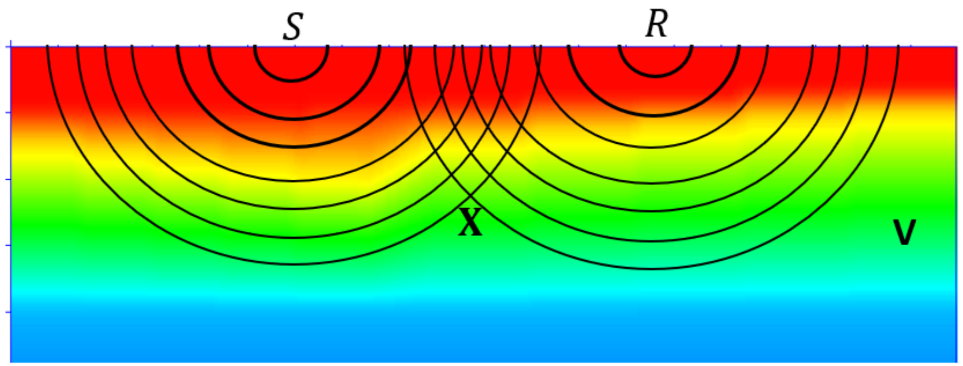
Model space : $[X_{j=1,N}], [V]_{i=1,M}]$

Data space : $\left[T_{sr}, P_s, P_g\right]_{j=1,N}$

Inversion : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial \boldsymbol{J}}{\partial \mathbf{m}}$

$\frac{\partial \boldsymbol{J}}{\partial \mathbf{m}}$ [2N+M]

Adjoint stereotomography



Adjoint Stereotomography Tavakoli 2017

Model space : $[X_{j=1,N}],[V]_{i=1,M}]$

Data space : $\left[T_{sr}, P_s, P_g\right]_{j=1,N}$

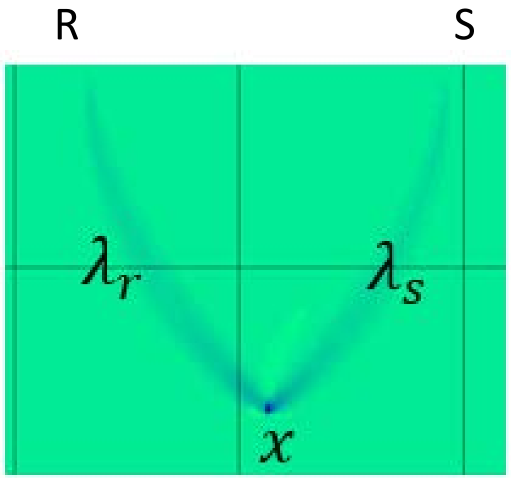
Inversion : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial J}{\partial \mathbf{m}}$

$$\frac{\partial J}{\partial v(x)} = -\frac{1}{v(x)^3} \sum (\lambda_r + \lambda_s)$$

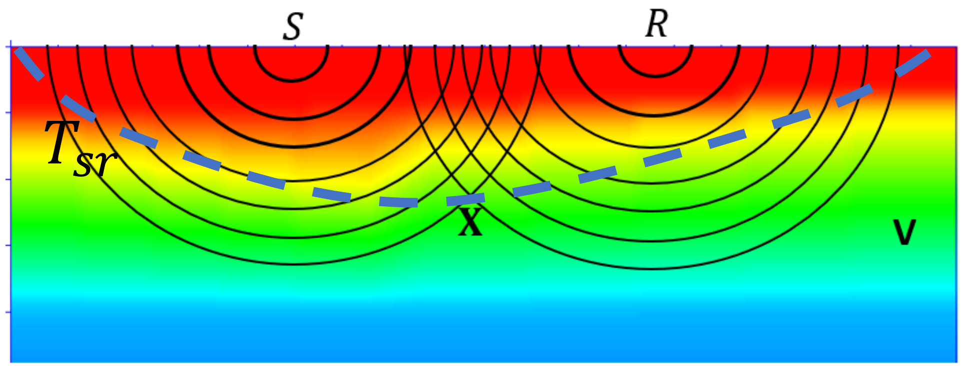
λ_s, λ_r = adjoint state variables correspond to traveltime equations for T_s and T_r

$$(\partial/\partial x, \partial/\partial z) \cdot (\lambda_s \cdot \nabla T_s) = S(\Delta T_{sr}, \Delta P_s)$$

$$(\partial/\partial x, \partial/\partial z) \cdot (\lambda_r \cdot \nabla T_r) = S(\Delta T_{sr}, \Delta P_g)$$



Adjoint stereotomography



Adjoint Stereotomography Tavakoli 2017

Model space : $[X_{j=1,N}],[V]_{i=1,M}$]

Data space : $\left[T_{sr}, P_s, P_g\right]_{j=1,N}$

Inversion : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial \boldsymbol{J}}{\partial \mathbf{m}}$

$$\frac{\partial J}{\partial X} = \Delta T_{sr} \nabla (T_s + T_r) + \frac{\Delta p_s}{2\Delta s} \nabla (T_{s+1} - T_{s-1}) + \frac{\Delta p_g}{2\Delta r} \nabla (T_{r+1} - T_{r-1})$$

Normal to the wavefront T_{sr}

Move x normal to T_{sr}

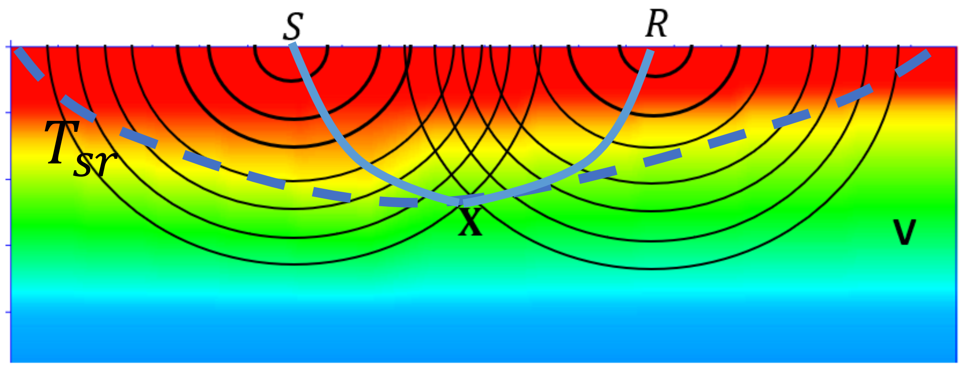
Normal to source-x ray path

Move x normal to source-x ray path

Normal to receiver-x ray path

Move x normal to receiver-x ray path

Adjoint stereotomography



Adjoint Stereotomography   Tavakoli 2017

Model space : $[X_{j=1,N}],[V]_{i=1,M}]$

Data space : $\left[T_{sr}, P_s, P_g\right]_{j=1,N}$

Inversion : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial J}{\partial \mathbf{m}}$

$$\frac{\partial J}{\partial X} = \Delta T_{sr} \nabla (T_s + T_r) + \frac{\Delta p_s}{2\Delta s} \nabla (T_{s+1} - T_{s-1}) + \frac{\Delta p_g}{2\Delta r} \nabla (T_{r+1} - T_{r-1})$$

Normal to the wavefront T_{sr}

Move x normal to T_{sr}

Normal to source-x ray path

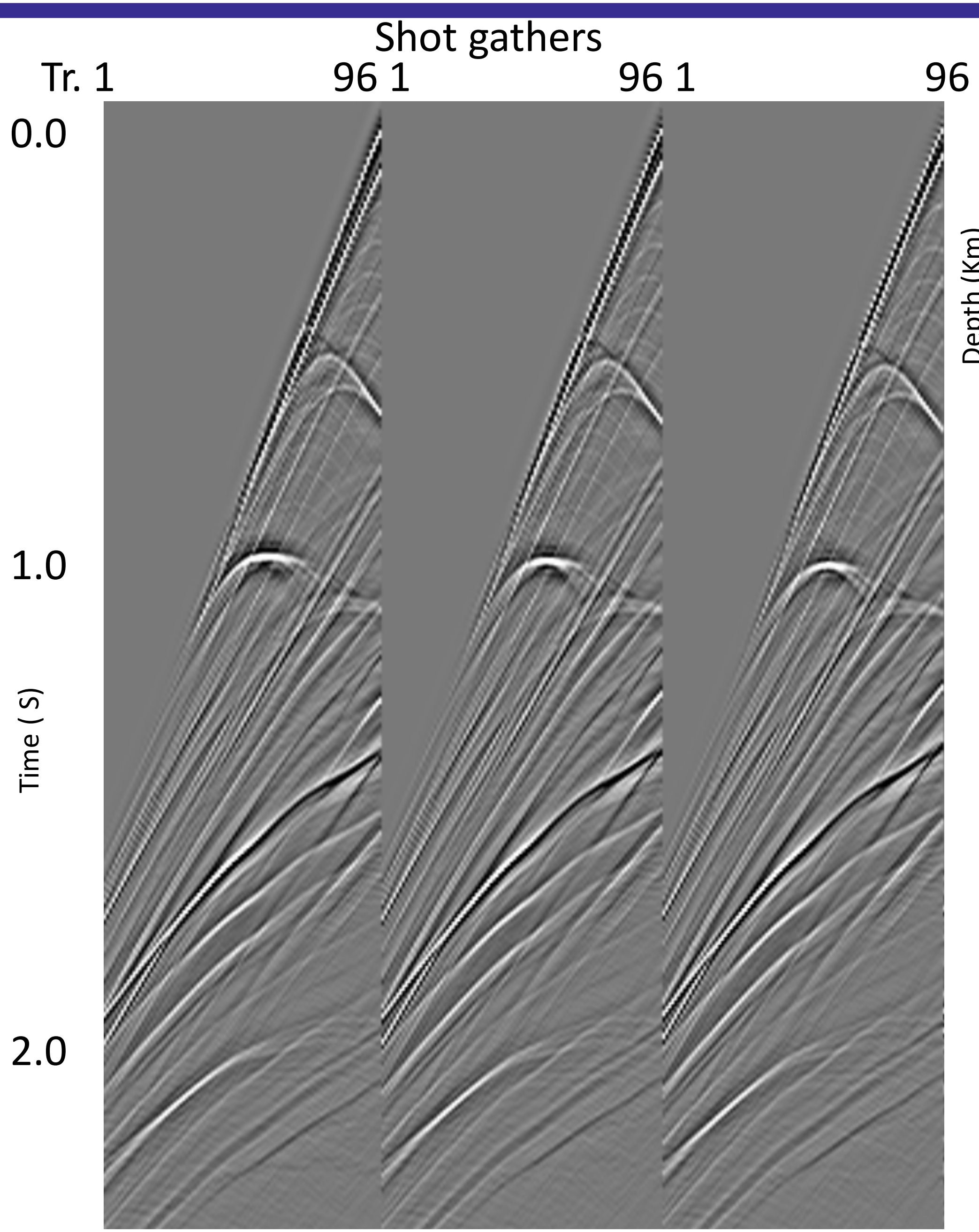
Move x normal to source-x ray path

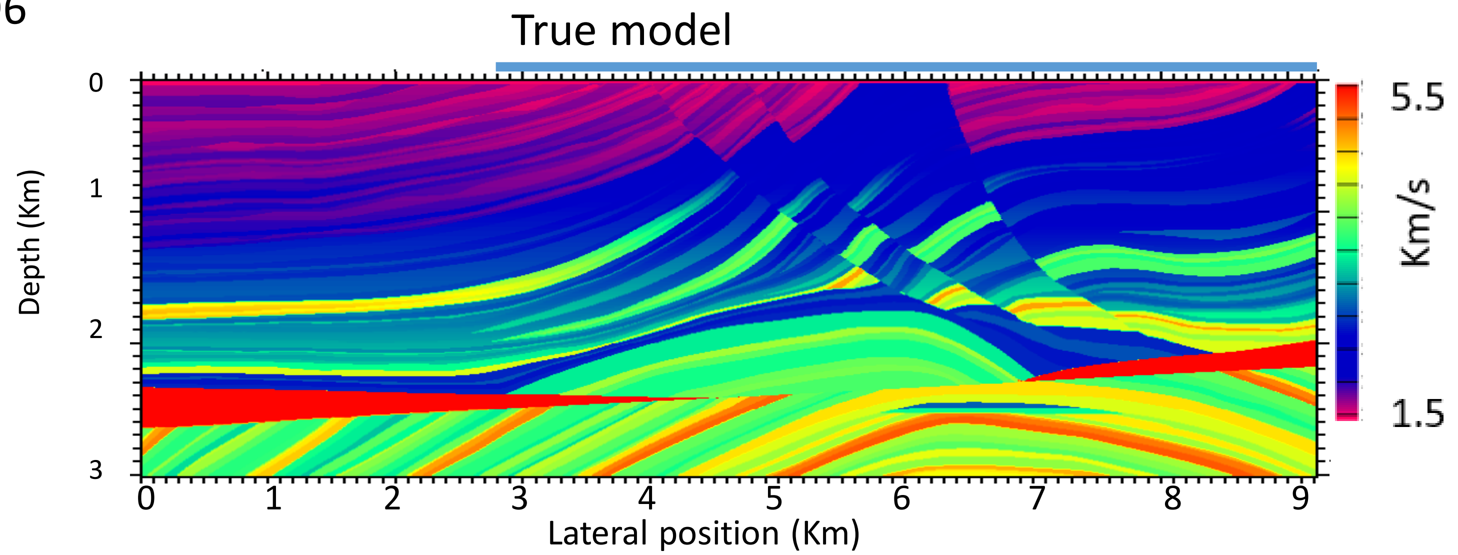
Normal to receiver-x ray path

Move x normal to receiver-x ray path

$$\Delta x = \alpha_x \frac{\partial J}{\partial x}$$

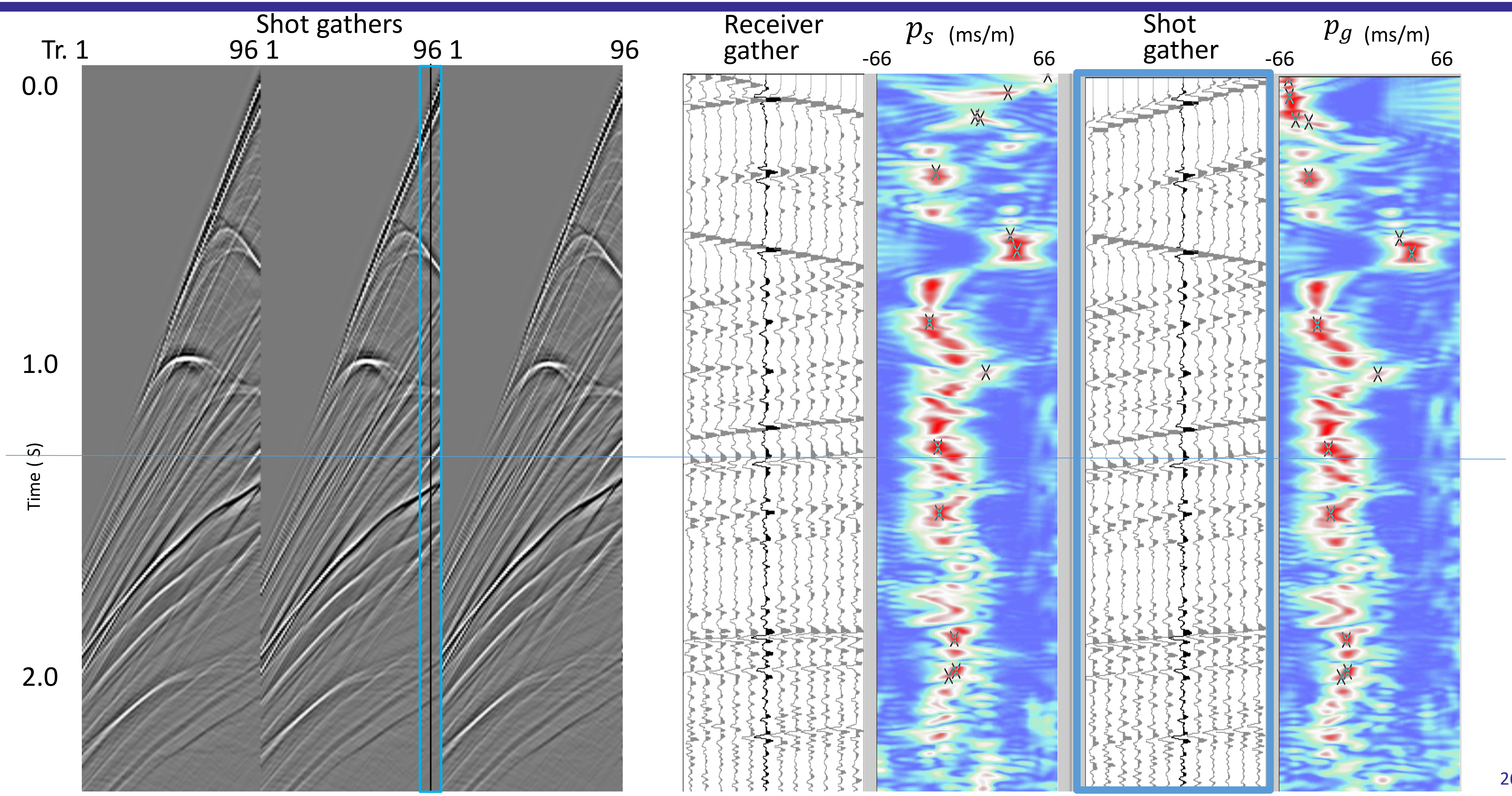
Synthetic example



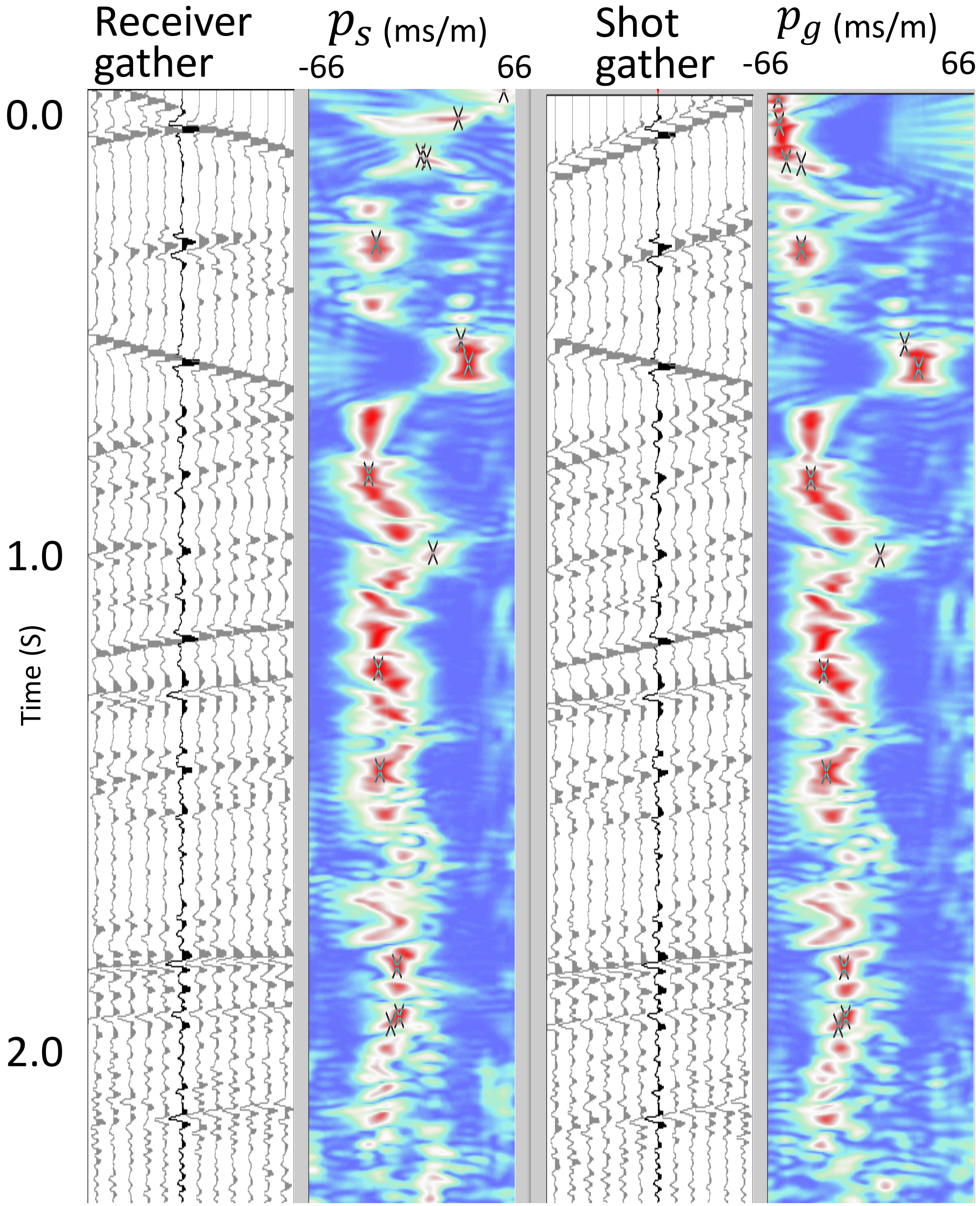


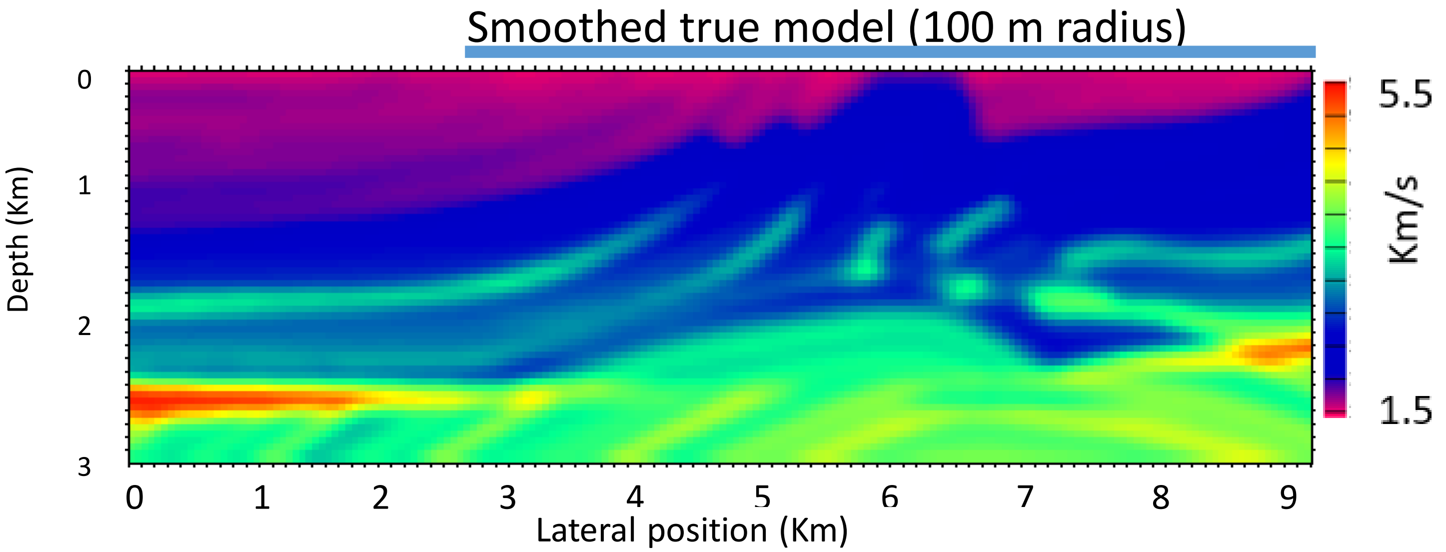
300	shots
96	channels per shot
25 m	shot and receiver spacing

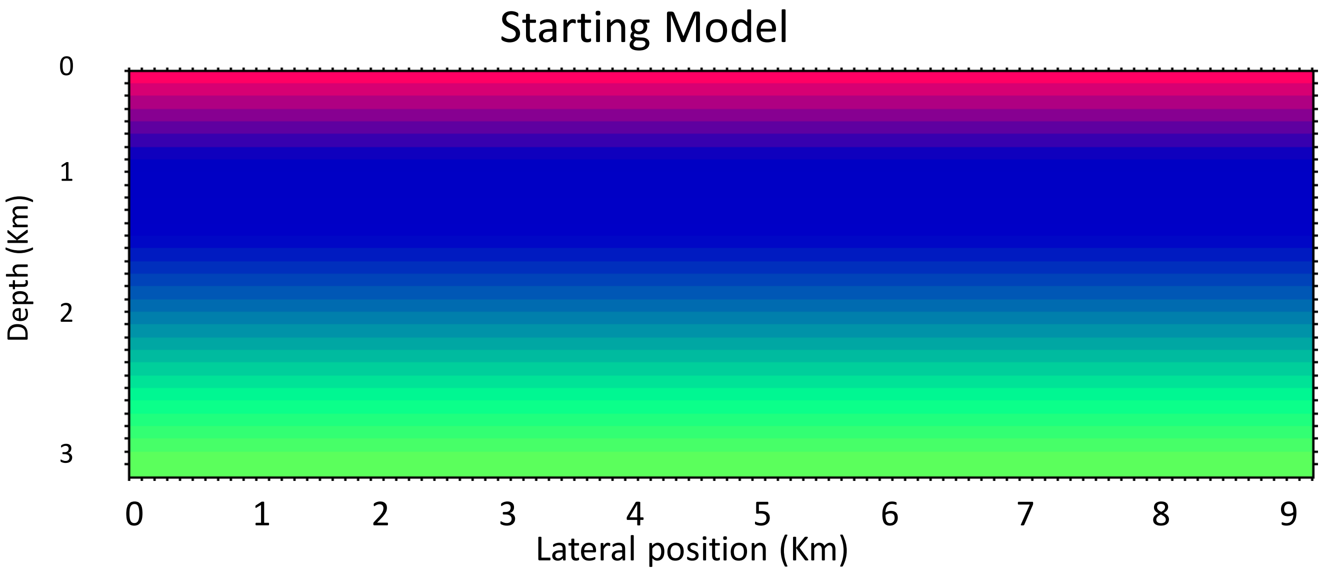
Synthetic example



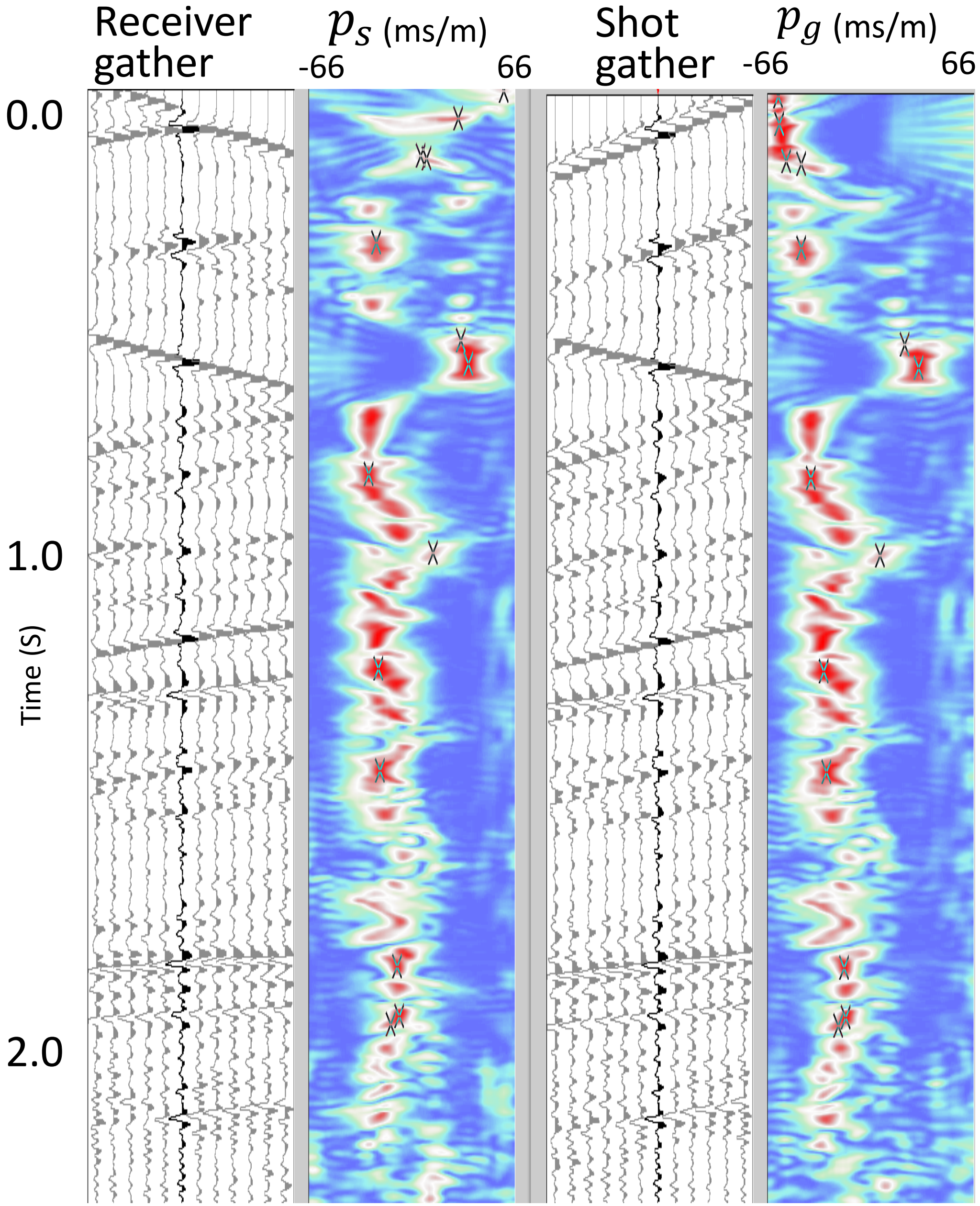
Synthetic example

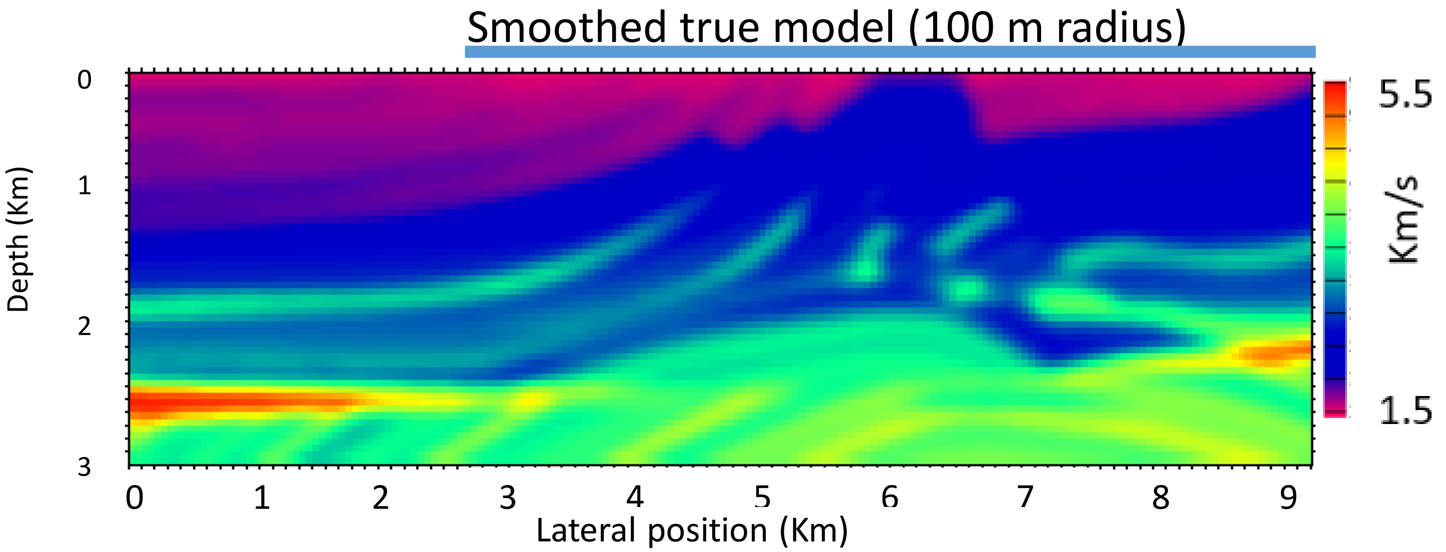


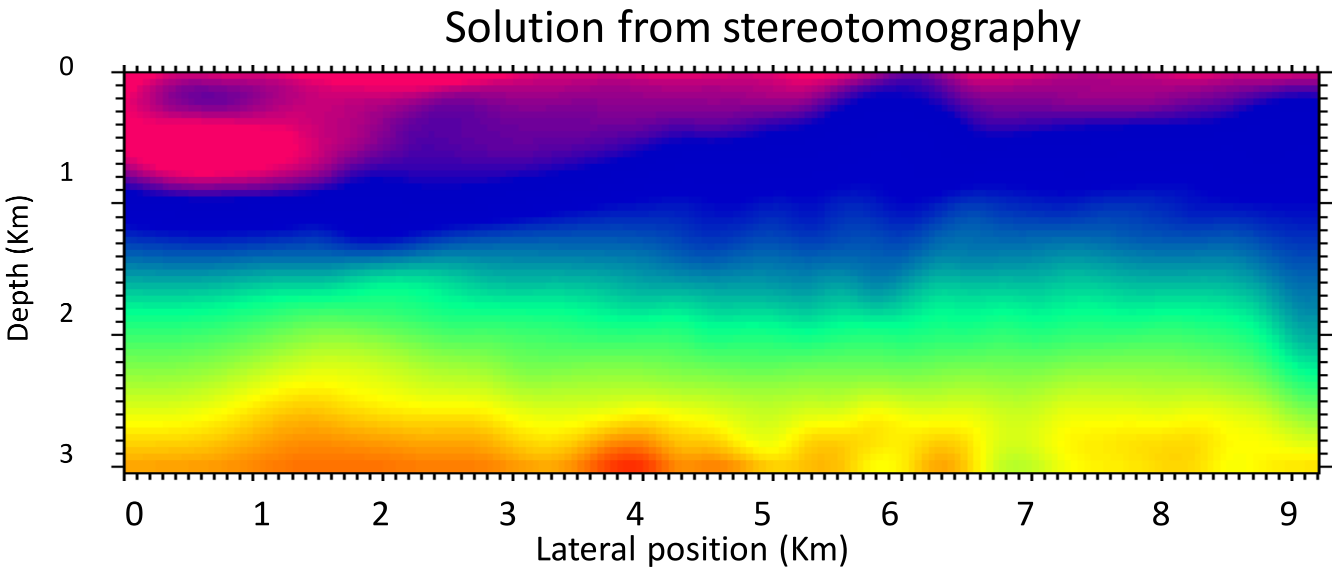




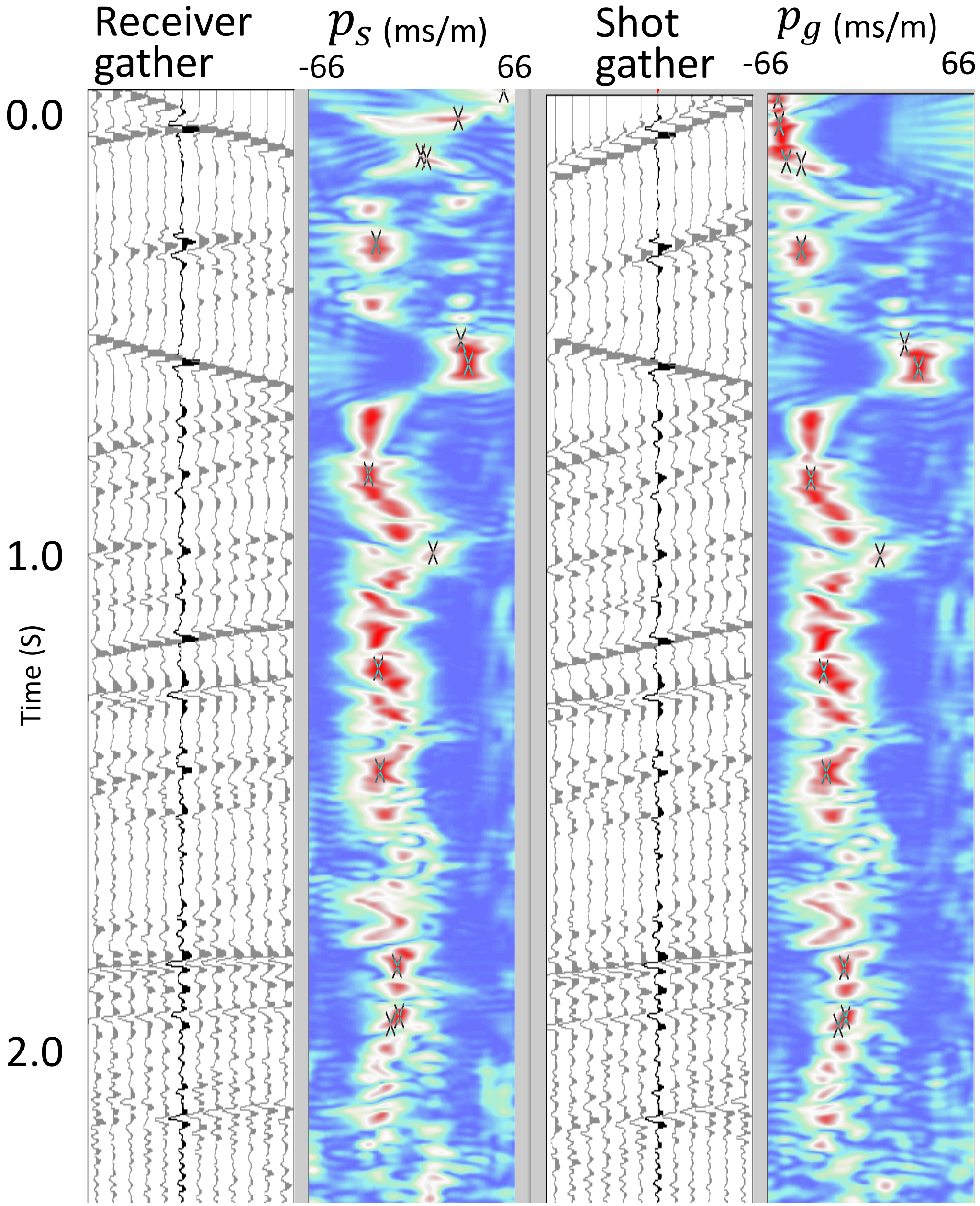
Synthetic example Stereotomography

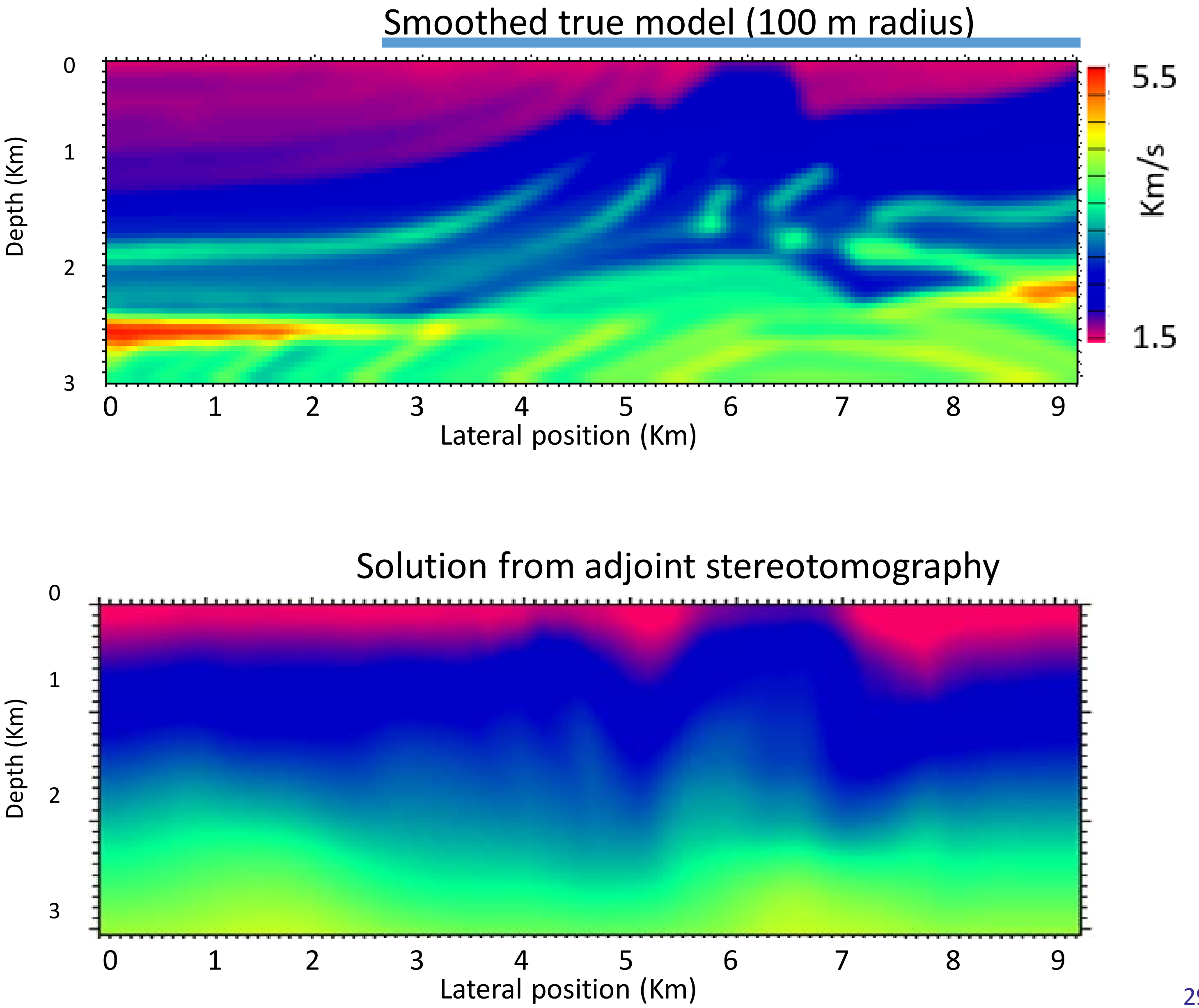






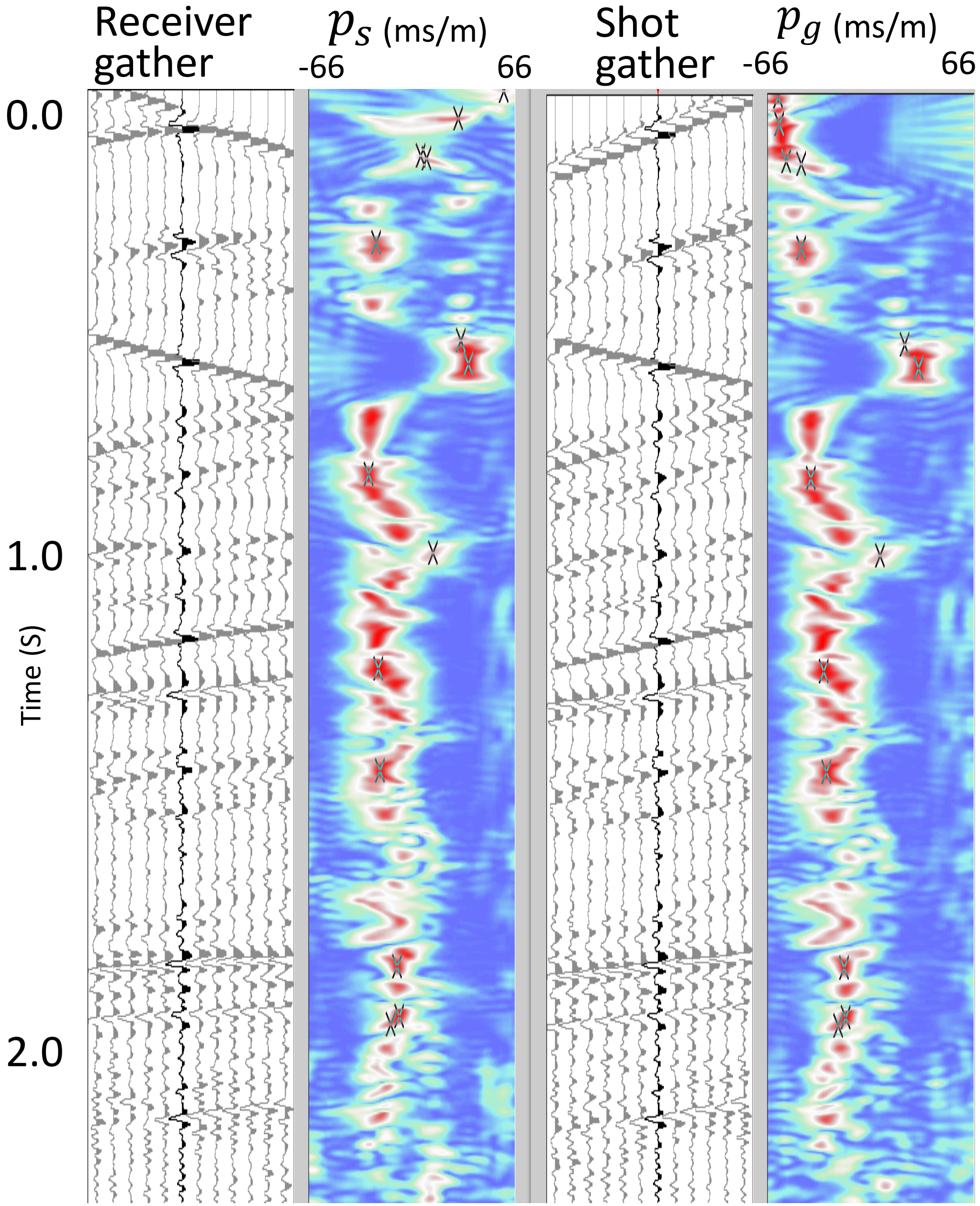
Synthetic example — Adjoint stereotomography

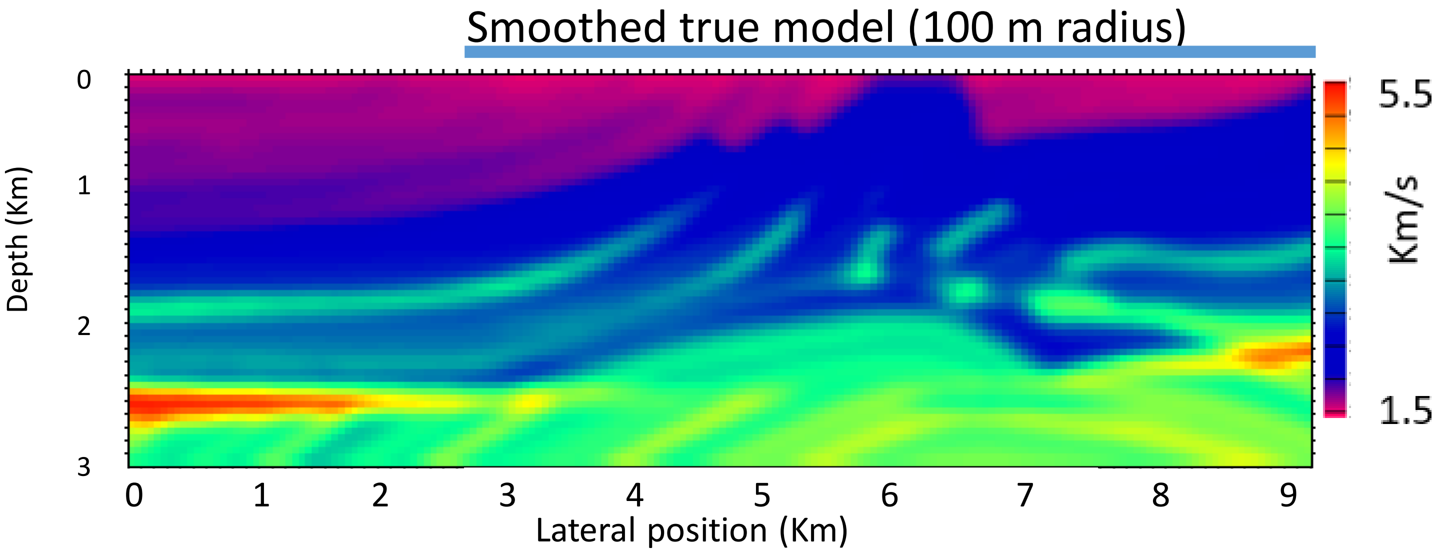


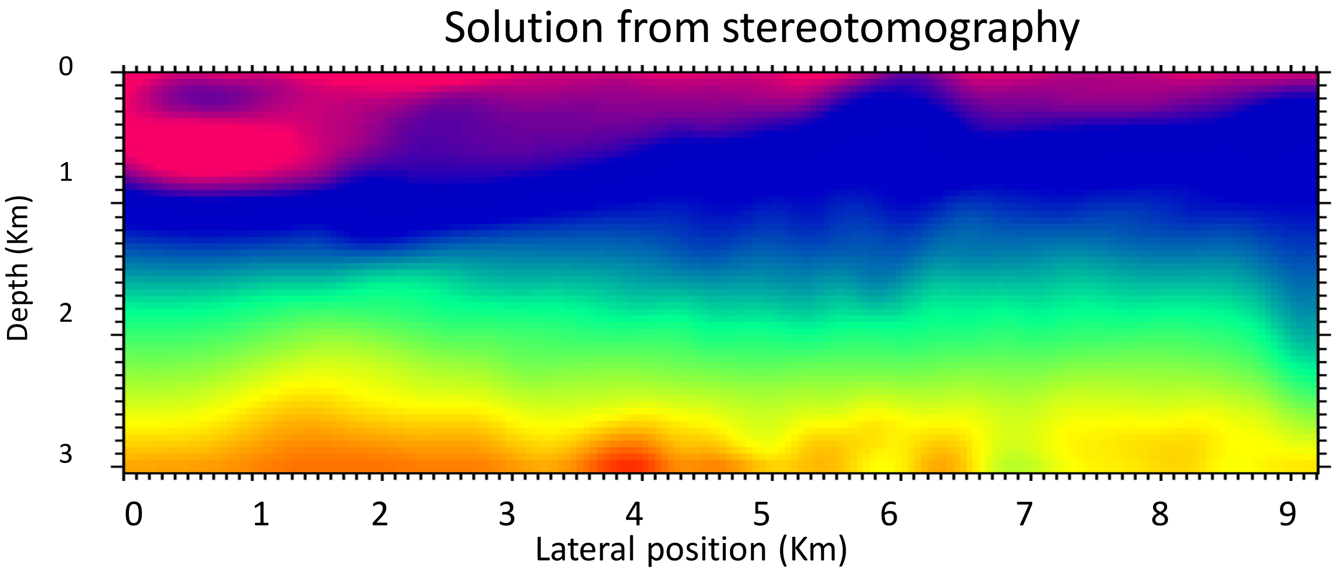


Synthetic example

Stereotomography







Conclusions

- Reviewed the classical reflection tomography, PSDM tomography and stereotomography
- PSDM tomography and stereotomography has picking advantage over the classical reflection tomography
- Stereotomography methods may be more efficient in building a macro velocity model than PSDM tomography, because depth migration step is not required
- We found the solution from our implementation of adjoint stereotomography matches the long wavelength component of the reference model
- Solution from classical stereotomography seems to have better resolution

Future work

- Improving the resolution of adjoint stereotomography
- Using solution from classical stereotomography and adjoint stereotomography as starting model for FWI
- Including anisotropic parameters in adjoint stereotomography
- Tests on real data including multi-component data

Acknowledgments

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NSERC

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