Joint P and P-S seismic inversion*

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ABSTRACT

Linear equations relate small changes in rock properties across an interface to seismic reflectivity (Aki and Richards, 1980). These equations for P-wave and P-S wave reflectivity can be inverted exactly or in a least-squares sense to provide estimates of relative changes in density, P-wave velocity and S-wave velocity. By using two observations (P and P-S reflectivity), this inversion promises better rock property estimates.

INTRODUCTION

In the attempt to understand subsurface lithologies, it is useful to have not just P-wave properties but those of the S wave (e.g. Danbom and Domenico, 1986). Amplitude-versus-offset (AVO) analysis tries to infer S-wave velocities (or Poisson's ratio) from the change of P-wave reflectivity R^{pp} with varying angles of incidence: The change in R^{pp} is partially controlled by the conversion of P-wave into S-wave energy, according to the S-wave velocities. On the other hand, converted-wave (P-to-S) reflectivity is generally more dependent on the S-wave velocity. So if our goal is to find S-wave properties, it is reasonable to try to use converted-wave reflectivity, R^{ps}.

METHODS

A good place to start is with the equations of Aki and Richards (1980) for P-wave reflectivity and P-S reflectivity which assume small changes in elastic-wave properties across an interface:

$$R^{pp}(\theta) \sim \frac{1}{2}\left(1 - 4\frac{\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\rho} + \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\alpha} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\,\frac{\Delta\beta}{\beta}, \tag{1}$$

$$R^{ps}(\theta) \sim \frac{-\alpha\tan\varphi}{2\beta}\left[\left(1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho}\right.$$
$$\left. - \left(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\beta}{\beta}\right], \tag{2}$$

where

θ is the average of the P-wave angle of incidence at and transmission through the interface,

φ is the average of the S-wave angle of reflection and its associated transmission angle,

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$\alpha,\ \beta,\ \rho$ are the average P-wave and S-wave velocities, and density across the interface,

$\Delta\alpha,\ \Delta\beta,\ \Delta\rho$ are the P-wave and S-wave velocity changes, and density change across the interface.

We can attempt to simplify these equations by using an empirical relationship between velocity and density. The Gardner et al. (1974) relationship

$$\rho \sim k\alpha^{1/4}, \tag{3}$$

can be written in differential form as

$$\frac{\Delta\rho}{\rho} \sim \frac{1}{4}\frac{\Delta\alpha}{\alpha}. \tag{4}$$

Similarly, the Lindseth (1982) relationship

$$\rho\alpha = l\alpha - m, \tag{5}$$

written in differential form is

$$\frac{\Delta\rho}{\rho} = \frac{1}{\left(\frac{l\alpha}{m} - 1\right)}\frac{\Delta\alpha}{\alpha}, \tag{6}$$

where k, l, m, are constants.

Perhaps an even more useful equation could be developed which would relate density to both α and β.

Substituting (4), say, into (1) and (2) gives

$$R^{pp}(\theta) \sim a\frac{\Delta\alpha}{\alpha} + b\frac{\Delta\beta}{\beta}, \tag{7}$$

$$R^{ps}(\theta) \sim c\frac{\Delta\alpha}{\alpha} + d\frac{\Delta\beta}{\beta}, \tag{8}$$

where

$$a = \frac{1}{8}\left(1 - \frac{4\beta^2}{\alpha^2}\sin^2\theta + \frac{4}{\cos^2\theta}\right),$$

$$b = -\frac{4\beta^2}{\alpha^2}\sin^2\theta,$$

$$c = -\frac{\alpha \tan\varphi}{8\beta}\left(1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\varphi\right),$$

$$d = \frac{\alpha \tan\varphi}{2\beta}\left(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\varphi\right).$$

Smith and Gidlow (1987) estimate $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ using only (7) and a least-squares filtering approach. However, given an appropriately processed P section and a carefully processed and correlated P-S section, we should be able to estimate $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ using (7) and (8). Again, by using two independent observations R^{pp} and R^{ps}, we have the possibility of better estimating the velocities. On appropriately gathered and stretched data, we could directly solve for the velocities as

$$\frac{\Delta\alpha}{\alpha} = \frac{dR^{pp} - bR^{ps}}{ad - bc}, \qquad \frac{\Delta\beta}{\beta} = \frac{aR^{ps} - cR^{pp}}{ad - cb}. \tag{9}$$

More likely though, there will be noise in the data so a least-squares method might be more useful. In this case, let's set up a value to be minimized:

$$\varepsilon = \sum_{i=1}^{\#offsets}\left(R^{pp} - a\frac{\Delta\alpha}{\alpha} - b\frac{\Delta\beta}{\beta}\right)^2 + \left(R^{ps} - c\frac{\Delta\alpha}{\alpha} - d\frac{\Delta\beta}{\beta}\right)^2 \tag{10}$$

Setting $\dfrac{\partial\varepsilon}{\partial\frac{\Delta\alpha}{\alpha}}, \dfrac{\partial\varepsilon}{\partial\frac{\Delta\beta}{\beta}} = 0$

and solving for $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$, with sums over the trace offsets, gives

$$\frac{\Delta\alpha}{\alpha} = \frac{\Sigma(d^2 - b^2)\Sigma(aR^{pp} - cR^{ps}) - \Sigma(cd - ab)\Sigma(bR^{pp} - dR^{ps})}{\gamma}, \tag{11}$$

$$\frac{\Delta\beta}{\beta} = \frac{\Sigma(c^2 - a^2)\Sigma(bR^{pp} - dR^{ps}) - \Sigma(cd - ab)\Sigma(aR^{pp} - cR^{ps})}{\gamma}, \tag{12}$$

where $\gamma = \Sigma(c^2 - a^2)\Sigma(d^2 - b^2) - [\Sigma(cd - ab)]^2$.

Equations (11) and (12) give us a method to jointly process P and P-S data to extract compressional and shear properties.

CONCLUSION

This paper has outlined equations which relate rock properties (ρ, α, β) to elastic-wave reflectivities. Using both P-wave and P-S wave reflectivities provides observations which can be jointly inverted to estimate the relative changes in rock properties across an interface.

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