Extending Sommerfeld integral based spherical wavefield computations beyond two layers

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Outline

- Introduction
- Sommerfeld Integrals and seismic wave fields
- Point sources in layered half spaces: Ewing, Jardetzky and Press
- Interfaces and boundary conditions
- P-wave multiples in a three-layer system
- Class 1 AVO-response of a three-layer system
- Conclusions
- Acknowledgements
Direct-Wave Particle Motion Based on Sommerfeld Integral (Aki and Richards, 1980)

\[ u_p(\omega) = i\omega e^{-i\omega t} \int_0^\infty \left[ \frac{p^2}{\xi} J_1(\omega pr) \sin(\theta) - ipJ_0(\omega pr) \cos(\theta) \right] e^{i\omega \xi} dp \]
\[ \varphi_j = \int_0^\infty Q_j' J_0(kr)e^{-v_jz} \, dk + \int_0^\infty Q_j'' J_0(kr)e^{v_jz} \, dk \]

\[ \psi_j = \int_0^\infty S_j' J_0(kr)e^{-v'_jz} \, dk + \int_0^\infty S_j'' J_0(kr)e^{v'_jz} \, dk \]

Potential of Down-Going and Up-Going Waves in Layer \( j \) (Ewing, Jardetzky and Press, 1957)
Application of Boundary Conditions at the First Interface \((z_1 = 0)\)

\[
A_{R1} \alpha_1 p + B_{R1} \beta_1 \eta_1 - (A_{T1} + A_{R2} e^{-i\omega \xi_2 z_2}) \alpha_2 p - (B_{T1} + B_{R2} e^{-i\omega \eta_2 z_2}) \beta_2 \eta_2 = -A_{I1} \alpha_1 p
\]

\[
-A_{R1} \alpha_1 \xi_1 + B_{R1} \beta_1 p - (A_{T1} - A_{R2} e^{-i\omega \xi_2 z_2}) \alpha_2 \xi_2 + (B_{T1} - B_{R2} e^{-i\omega \eta_2 z_2}) \beta_2 p = -A_{I1} \alpha_1 \xi_1
\]

\[
-2A_{R1} \alpha_1 \beta_1^2 \rho_1 p \xi_1 - B_{R1} \beta_1 \rho_1 (1 - 2\beta_1^2 p^2) - (A_{T1} - A_{R2} e^{-i\omega \xi_2 z_3}) 2\alpha_2 \beta_2^2 \rho_2 p \xi_2
\]

\[-(B_{T1} - B_{R2} e^{-i\omega \eta_2 z_2}) \beta_2 \rho_2 (1 - 2\beta_2^2 p^2) = -2A_{I1} \alpha_1 \beta_1^2 \rho_1 p \xi_1
\]

\[
A_{R1} \alpha_1 \rho_1 (1 - 2\beta_1^2 p^2) - 2B_{R1} \beta_1^3 \rho_1 p \eta_1 - (A_{T1} + A_{R2} e^{-i\omega \xi_2 z_2}) \alpha_2 \rho_2 (1 - 2\beta_2^2 p^2)
\]

\[+(B_{T1} + B_{R2} e^{-i\omega \eta_2 z_2}) 2\beta_2^3 \rho_2 p \eta_2 = -A_{I1} \alpha_1 \rho_1 (1 - 2\beta_1^2 p^2)
\]
Reservoir Model Geometry

$\alpha_1, \beta_1, \rho_1$

$\alpha_2, \beta_2, \rho_2$

$\alpha_1, \beta_1, \rho_1$
Plane-Wave Class 1 Elastic Reflection Coefficient
P-Wave Response of a 100 m Reservoir (top at 500 m)
Plane-Wave $R_{pp}(\omega)$ (vertical incidence, 100 m reservoir)
Spher.-Wave $R_{pp}(\omega)$ (vertical incidence, 100 m reservoir)
Plane-Wave $R_{pp}(\omega)$ (vertical incidence, 50 m reservoir)
Spher.-Wave $R_{pp}(\omega)$ (vertical incidence, 50 m reservoir)
Plane-Wave $R_{pp}(\omega)$ (100 m reservoir)
Plane-Wave $R_{pp}(\omega)$ (single interface)
Spherical-Wave $R_{pp}(\omega)$ (100 m reservoir)
Spherical-Wave $R_{pp}(\omega)$ (single interface)
Spherical-Wave PP-Response (single reflector at 500m)
Spherical-Wave PP-Response (50 m reservoir)
Conclusions

- Reverberations and spherical spreading are modelled by the Ewing-method.
- Small-offset Class 1 AVO-responses show reservoir layer reverberations in the frequency-domain but at larger offsets the response is obscured.
- Time-domain responses show an amplitude build-up because of far-offset tuning.
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